06/04
CLEARANCE
$ .25

STRAYER-UPTON

JUNIOR MATHEMATICS

BOOK TWO

My own Copy

Mc Shaine.

STRAYER-UPTON
JUNIOR MATHEMATICS

BOOK TWO

BY

GEORGE DRAYTON STRAYER, PH.D
Professor of Education, Teachers College
Columbia University

AND

CLIFFORD BREWSTER UPTON, A.M.
Professor of Mathematics, Teachers College
Columbia University

AMERICAN BOOK COMPANY
NEW YORK CINCINNATI CHICAGO
BOSTON ATLANTA

COPYRIGHT, 1929, BY
AMERICAN BOOK COMPANY
All rights reserved

STRAYER-UPTON JUNIOR MATHEMATICS, BOOK TWO

W. P. 15

MADE IN U.S.A.

PREFACE

THIS series of three books offers for the junior high school a modern course in mathematics, which aims to acquaint the pupil with the importance and the larger uses of this subject in everyday life. These books present only methods and materials that have been thoroughly tested in the classroom by many experienced teachers; in no place do they introduce untried theories. They also incorporate the most valuable findings of modern experimentation in the teaching of mathematics, including the results of important researches conducted by the authors.

These books meet the requirements of the leading curricula in junior high school mathematics. They also follow the recommendations of the National Committee on Mathematical Requirements and the Year-books of the National Council of Teachers of Mathematics and the Department of Superintendence.

In this book, which covers the work of the second year of the junior high school, or of the eighth school year, the following features are especially noteworthy:

1. *Scope.* There is a balanced offering in arithmetic, intuitive geometry, the formula, and the equation. Each of these topics is treated interestingly and adequately and none is unduly emphasized at the expense of the others.

2. *Arithmetic.* The applications of arithmetic to modern life are fully presented in a way that is both real and interesting to boys and girls. The problem material has been selected with unusual care and is in harmony with modern business practice. Much new material is offered in banking, thrift, installment buying, taxation, stocks and bonds, and insurance.

An efficient means of perfecting skill in computation is provided by including 47 Improvement Tests. See Chapter II.

3. *Intuitive Geometry.* In order that there may be a gradual growth in the pupil's ideas concerning geometric form, the work in intuitive geometry is distributed over all three books of this series. In the present book special attention is given to the common geometric solids, to indirect measurement, and to the notions of congruence, similarity, and symmetry.

4. *Algebra.* The formula and the equation, two of our most effective mathematical instruments, are fully developed as important aids in arithmetic and intuitive geometry. The formula is presented in Chapter I and is applied throughout the work in mensuration. The equation is introduced in Chapter III and is liberally employed in ratio and proportion and in the indirect cases in mensuration and percentage. The everyday uses of positive and negative numbers are also included.

5. *Grading.* The work in each topic has been graded very carefully. The unusually large number of graded exercises insures complete mastery of each topic and also provides fully for the individual differences of pupils.

6. *Problems.* The book includes an abundance of interesting problems and projects, which have been so selected as to afford a constant challenge to the pupil's initiative.

7. *Selection.* The liberal amount of material offered in this book permits a selection of topics in accordance with local needs. Changes may also be made in the order of presenting topics without disturbing the unity of the work. For example, a teacher may begin the year's work with Chapter III, IV, or V. Likewise, a single unit in intuitive geometry may be formed by combining Chapter IV with Chapters XIII to XVI.

George Drayton Strayer
Clifford Brewster Upton

CONTENTS

CHAPTER		PAGE
I.	FORMULAS	
	Using Formulas to Find Areas	1
	Making and Graphing Formulas	7
II.	IMPROVEMENT TESTS	
	Taking Improvement Tests	21
	Keeping Records in Improvement Tests	25
III.	EQUATIONS	
	Solving Equations	31
	Solving Problems by Equations	42
IV.	MEASUREMENTS	
	Finding Volumes and Surfaces	51
	Solving Formulas as Equations	74
V.	PERCENTAGE	
	Three Types of Problems	83
	More Difficult Per Cents	87
	Applications of Percentage	98
	Using Equations in Percentage	107
VI.	BANKING	
	Checks and Drafts	113
	Borrowing Money from Banks	124
VII.	THRIFT AND COMPOUND INTEREST	
	Keeping Money in a Savings Bank	135
	Depositing Money Regularly	141
VIII.	INSTALLMENT BUYING	
	Buying on the Installment Plan	147
	Monthly Payments on a Home	153
	Building and Loan Associations	157

CHAPTER PAGE

IX. STOCKS AND BONDS
Buying and Selling Stocks 161
Investing in Bonds 170

X. INSURANCE
Fire Insurance 179
Life Insurance 182
Other Forms of Insurance 190

XI. TAXES
Town and City Taxes 197
Federal Taxes 203

XII. METRIC SYSTEM
Metric Length 205
Metric Capacity and Weight 209

XIII. STUDYING GEOMETRIC FIGURES
Geometric Constructions 213
Congruence and Symmetry 223

XIV. RATIO AND PROPORTION
Comparing Numbers 231
Solving Proportions 235

XV. SIMILAR TRIANGLES
Studying Similar Triangles 241
Using Similar Triangles 244

XVI. SQUARE ROOT
Finding Square Roots 251
Using Square Root in Right Triangles 256

XVII. POSITIVE AND NEGATIVE NUMBERS
Adding Signed Numbers 261
Subtracting Signed Numbers 267

XVIII. REVIEW 271

TABLES FOR REFERENCE 279

INDEX 281

BOOK TWO

CHAPTER I

FORMULAS

THE IMPORTANCE OF FORMULAS

Last year you learned to write certain rules in a kind of shorthand called a **formula,** in which letters represented numbers. Thus, to find the area of a rectangle you used this formula:

$$A = bh$$

This is a very brief way of writing the rule that *the area of a rectangle equals its base multiplied by its height.*

In the above formula A stands for *area,* b for *base,* and h for *height.* When two letters are written side by side they are to be multiplied; thus, bh means $b \times h$.

To find the area of a rectangle having a base of 10 ft. and a height of 3 ft. you *substitute* these values in the formula $A = bh$, putting 10 in place of b and 3 in place of h, thus:

$$A = 10 \times 3 = 30$$

Hence the area of the rectangle is 30 sq. ft.

When you substitute numbers for letters in the above formula to find the value of A, you *evaluate* the formula.

Formulas are so important that people in all parts of the world need them to express the rules of mathematics and the laws of science. They are particularly useful to business men, bankers, engineers, mechanics, electricians, navigators, astronomers, and surveyors. Without formulas it would be impossible to guide a ship safely at sea or to build a bridge or a skyscraper.

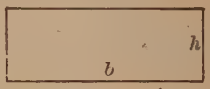
SOME FORMULAS FOR AREAS

Below you will find brief rules and formulas, which you learned last year, for finding the areas of four important geometric figures. In each formula, A stands for the *area* of the figure, b for its *base*, and h for its *height*, or *altitude*.

The **base** of a figure is the side on which it stands. The **height** of a figure is a line *perpendicular* to the base. The height of a rectangle is the same as one of its sides; but in the triangle, the parallelogram, and the trapezoid, as shown below, the height of each figure is shorter than either of its slanting sides.

Rectangle: $Area = base \times height$

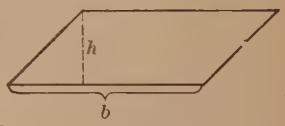
or $A = bh$



If the base is 10 in. and the height is 4 in., then $b = 10$ and $h = 4$. Hence $A = 10 \times 4$, or 40. The area is 40 sq. in.

Parallelogram: $Area = base \times height$

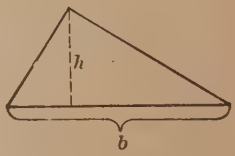
or $A = bh$



If the base is 14 in. and the height is 6 in., then $b = 14$ and $h = 6$. Hence $A = 14 \times 6$, or 84. The area is 84 sq. in.

Triangle: $Area = \dfrac{base \times height}{2}$

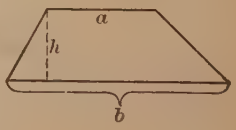
or $A = \dfrac{bh}{2}$, or $\frac{1}{2}\,bh$



If the base is 12 in. and the height is 8 in., then $b = 12$ and $h = 8$. Hence $A = \frac{1}{2} \times 12 \times 8$, or 48. The area is 48 sq. in.

Trapezoid: $Area = \dfrac{height \times sum\ of\ bases}{2}$

or $A = \dfrac{h(a+b)}{2}$, or $\frac{1}{2}h(a+b)$



If the bases of the trapezoid are 16 in. and 12 in. and the height is 10 in., then $A = \frac{1}{2} \times 10 \times (16 + 12)$, or 140. The area is 140 sq. in.

USING RECTANGLES AND PARALLELOGRAMS

Before working each problem, tell which formula you will use.

1. Mr. Turner has a rectangular lawn measuring 66 ft. by 24 ft. How much will it cost to sod it at \$.08 a square foot?

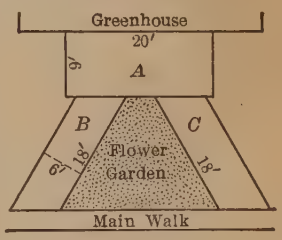
2. If 1 gal. of varnish covers 300 sq. ft., how many gallons are needed to varnish a floor 35 ft. by 30 ft.?

3. Mr. Thomas is building a concrete sidewalk $2\frac{1}{2}$ ft. wide and 90 ft. long. Find the cost at \$2.10 a square yard.

4. Mrs. Andrews bought a lot 100 ft. by 125 ft. Find, to the nearest hundredth, what decimal part of an acre she has. There are 43,560 sq. ft. in an acre.

5. Oriental rugs larger than 9 ft. by 6 ft. are often sold to stores by the square foot. At \$2.15 a square foot, how much does a store pay for 2 rugs, each $8\frac{1}{2}$ ft. by 10 ft.? for 3 rugs, each 8 ft. 9 in. by 12 ft.? for 3 rugs, each $9\frac{1}{2}$ ft. by 11 ft.?

6. Mr. Fuller is laying three new pieces of cement walk, *A*, *B*, and *C*, in the park. *B* and *C* are each 6 ft. wide. The other dimensions are given in the picture. What kinds of figures are *B* and *C*? Find the total cost of laying these walks at \$2.10 a square yard.



Find the areas of rectangles having these dimensions:

7. 8 ft. × 6 ft.

8. 5 yd. × 1 yd.

9. $6\frac{1}{2}$ ft. × $3\frac{1}{2}$ ft.

10. $7\frac{1}{4}$ yd. × $1\frac{3}{4}$ yd.

11. 4.5″ × 3.1″

12. 7.2″ × 7.2″

Find the areas of the following parallelograms. The first number gives the base and the second number, the height:

13. 7 in.; 6 in.

14. 8 in.; $2\frac{1}{2}$ in.

15. $5\frac{1}{2}$ ft.; 3 ft.

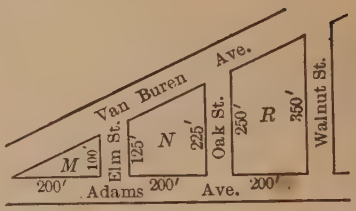
16. $1\frac{1}{2}$ yd.; $1\frac{1}{4}$ yd.

17. 18 ft.; 7.3 ft.

18. 9.2 ft.; 6.8 ft.

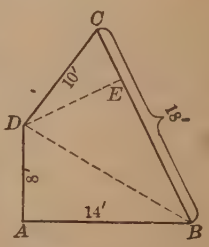
USING TRIANGLES AND TRAPEZOIDS

1. On this map, Elm, Oak, and Walnut streets are each perpendicular to Adams Avenue. What kind of figure is block M? block N? block R? Find the areas of M, N, and R, using the dimensions given on the map.



In the trapezoid N, $h = 200$ ft., $b = 225$ ft., and $a = 125$ ft. Substituting these values in $A = \frac{1}{2}h(a + b)$, you get $A = \frac{1}{2} \times 200 \times 350$, which gives the area in square feet. What is the area of N?

2. This picture shows the sail of Bert's catboat, drawn to the scale of $\frac{1}{16}'' = 1'$. Show that this sail is not a trapezoid. Find the area of the sail by adding the areas of triangles ABD and BCD.



Line DE shows the height of triangle BCD. Measure this line carefully and find the number of feet it represents, from the scale $\frac{1}{16}'' = 1'$. What line do you take as the base of triangle BCD?

Angle A is a right angle, hence line AD is the height of triangle ABD.

Find the areas of the following triangles. The first number gives the base and the second, the height:

3. 5 ft.; 3 ft.	**6.** $6\frac{1}{2}$ in.; 4 in.	**9.** 2.5′; 3.1′
4. 7 ft.; 4 ft.	**7.** $4\frac{1}{4}$ ft.; 3 ft.	**10.** 6.6′; 2.8′
5. 6 ft.; 5 ft.	**8.** $5\frac{3}{4}$ yd.; 2 yd.	**11.** 5.8′; 2.5′

Find the areas of the following trapezoids:

	Bases	Height		Bases	Height
12.	5 yd., 7 yd.	3 yd.	**16.**	75 ft., 49 ft.	60 ft.
13.	8 yd., 5 yd.	$6\frac{1}{3}$ yd.	**17.**	$15\frac{1}{2}$ ft., $10\frac{1}{2}$ ft.	7 ft.
14.	18 ft., 16 ft.	20 ft.	**18.**	$9\frac{1}{2}$ ft., $11\frac{1}{2}$ ft.	$8\frac{1}{2}$ ft.
15.	50 ft., 45 ft.	35 ft.	**19.**	$14\frac{1}{4}$ ft., $15\frac{3}{4}$ ft.	10 ft.

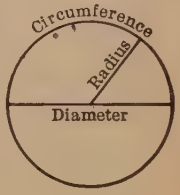
THE CIRCUMFERENCE OF A CIRCLE

You have learned this rule for the circumference of a circle:

Circumference $= 3\frac{1}{7} \times$ *Diameter*

This rule may be written as a formula, thus:

$$C = \pi D$$



Here the symbol π (pronounced "pie") stands for $3\frac{1}{7}$, or 3.14.

Since a diameter equals two radii ($D = 2r$), the above formula may also be written thus:

$$C = 2\pi r$$

Exercises

In the following exercises, let $\pi = 3\frac{1}{7}$.

1. The wheels on Fred's bicycle have a diameter of 28 in. What is the circumference, in inches, of each wheel?

2. How many feet does Fred's bicycle travel while each wheel makes one complete revolution? In a trip of 3 mi., how many times does each wheel go around?

3. The wheels of Dolly's bicycle are each 26 in. in diameter. What is the circumference of these wheels in inches? in feet?

4. In a trip of 1 mi., a wheel on Dolly's bicycle makes how many more revolutions than one on Fred's? (See ex. 1 and 3.)

5. A circus ring has a radius of 21 ft. A pony running around the outside edge travels how far in one trip around it? How many times must the pony run around the ring to go 1 mi.?

Find the circumferences of circles with these radii:

6. 7 in.	**8.** 28 yd.	**10.** 1 yd.	**12.** $1\frac{1}{6}$ ft.
7. 14 ft.	**9.** 25 ft.	**11.** 49 in.	**13.** $4\frac{2}{3}$ ft.

Find the circumferences of circles with these diameters:

14. 35 in.	**16.** 77 ft.	**18.** 42 ft.	**20.** $8\frac{3}{4}$ rd.
15. 10 mi.	**17.** 18 rd.	**19.** 100 ft.	**21.** $5\frac{3}{5}$ mi.

THE AREA OF A CIRCLE

Last year you learned this rule for the area of a circle:

Area of circle $= 3\frac{1}{7} \times$ *square of the radius*

If r stands for *radius*, the rule becomes this formula:

$$A = \pi r^2$$

Here r^2 is read "r square" and means $r \times r$.

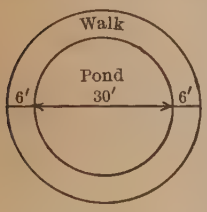
Exercises

In the following exercises, let $\pi = 3\frac{1}{7}$.

1. A cow is tethered to a post in the middle of a field by a rope 28 ft. long. How many square feet has the cow to graze in? If the rope is lengthened 6 ft., how many more square feet will the cow have?

Using the formula $A = \pi r^2$, you get $A = 3\frac{1}{7} \times 28 \times 28$, which gives the area of the circle in which the cow grazes.

2. A forest ranger watches for forest fires from a tower on a mountain. He can see 35 mi. in all directions. How many square miles of land does he guard?



3. In Sunset Park the children sail toy boats on a circular pond that is 30 ft. in diameter. A circular cement walk, 6 ft. wide, is being laid all around the edge of this pond. How much will this cement walk cost at $2.25 a square yard?

To find the area of the walk, subtract the area of the circle that forms its inside edge from the area of the circle that forms its outside edge.

In finding the cost, call a fraction of a square yard another square yard.

Find the areas of circles with these radii:

4. 7 in.	**7.** 1 ft.	**10.** 21 in.	**13.** 56 in.
5. 2 ft.	**8.** 14 ft.	**11.** 28 in.	**14.** 49 in.
6. 3 in.	**9.** 63 ft.	**12.** 10 ft.	**15.** 42 in.

EXPRESSING RULES AS FORMULAS

1. Mr. Adams is ordering some readers for his school. He knows that he can find the cost of all the readers by multiplying the price of one reader by the number of readers bought. His rule may be stated more briefly as follows:

Cost = number of readers bought × price of one reader

If you let C stand for the *cost*, n for the *number* of readers bought, and p for the *price* of one reader, you can write the above rule as a formula, thus:

$$C = np$$

Using this formula, find the cost of 60 readers at \$.88 each.

Using the formula in ex. 1, find the cost of the following:

2. 75 spellers @ \$.48

3. 80 geographies @ \$1.10

4. 95 readers @ \$.78

5. 60 histories @ \$.96

6. Mr. Park averages 30 mi. an hour in his car. To estimate the distance he will travel on a proposed trip, he multiplies 30 by the number of hours he plans to drive. Write Mr. Park's rule in brief form, as in ex. 1. Then write the rule as a formula, letting D stand for the *distance* and n for the *number* of hours.

7. In ex. 6, 30 miles an hour may be called the *rate*, while the *number of hours* may be called the *time*. Using these new terms, you may write Mr. Park's rule thus:

Distance = Rate × Time

Express this rule as a formula, letting R stand for the *rate* and T for the *time*.

Using the formula in ex. 7, find the distance in these cases:

8. R = 30 mi. an hour; T = 5 hr.

9. R = 25 mi. an hour; T = 7 hr.

10. R = 40 mi. an hour; $T = 1\frac{1}{2}$ hr.

11. $R = 36$; $T = 4\frac{1}{2}$

12. $R = 24$; $T = 2\frac{1}{4}$

13. $R = 26$; $T = 10\frac{1}{2}$

MAKING TABLES FROM FORMULAS

1. The pupils of the Lincoln Junior High School are selling tickets to a school play at 35¢ each. Alice Bell writes the formula $A = .35\,n$ to show the amount that she should collect from the sale of a certain number of tickets. What do A and n represent in her formula? Is her formula correct?

2. In order to know quickly the amount of money each pupil should turn in from the sale of the tickets assigned to him, Alice uses the formula in ex. 1 to make a table like the one shown here. Complete this table up to 25 tickets. In this table the numbers 1 and \$.35 are called **corresponding values.** Name some other pairs of corresponding values.

Tickets Sold	Amount Received
1	\$ .35
2	.70
3	1.05

3. Chester finds that it costs \$.036 per mile to ride on the railroad. How much will it cost to travel 15 mi.? 25 mi.? 130 mi.? 0 mi.? If n stands for the number of miles traveled, show that a formula for finding the cost of a trip is $C = .036\,n$.

4. Using the formula in ex. 3, make a table like the one at the right, showing the cost of trips for every 5 miles, beginning with 0 mi. and ending with 125 mi.

Number of Miles	Cost of Trip
0	\$.00
5	.18
10	.36

When you substitute 0 for n in the formula $C = .036\,n$, you get $C = .036 \times 0$, which equals 0. Hence $C = 0$ when $n = 0$. This means that it costs nothing if you travel 0 miles.

5. The table at the right has been made for bananas sold at 30¢ a dozen. Tell what the table shows and what should be written at the top in place of A and of B. Write the formula from which this table was made; then use this formula to extend the table to 12 under column A, with corresponding values under B.

A	B
0	\$.00
1	.30
2	.60
3	.90

MAKING TEMPERATURE GRAPHS

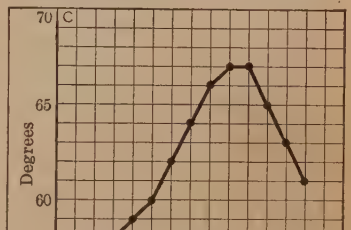
If you want to know how the temperature changes during the day, you can read the thermometer every hour and record the temperatures in a table like the following:

Hour	7	8	9	10	11	12	1	2	3	4	5	6
Degrees	57	58	59	60	62	64	66	67	67	65	63	61

You can get a much better idea of these temperature changes, however, by picturing them on a *graph*, as shown below. Such a graph is made as follows:

First draw line *AB* and mark the hours on it; this is the *horizontal scale*. Next draw line *AC* and mark the degrees on it; this is the *vertical scale*. Since the temperature was 57° at 7 A.M., place a point on the graph directly above 7 A.M. and directly to the right of 57°. In the same way place points showing the temperatures at each of the other hours given in the table above. Then connect the 12 points with a broken line, as shown at the right.

Temperature During a Day



Exercises

1. In the graph above, at what hour was the temperature lowest? At what hours was it highest?

2. How much warmer was it at 6 P.M. than at 7 A.M.?

3. Can you tell from the graph what the temperature was at 10.30 A.M.? at 2.30 P.M.? at 4.30 P.M.?

4. Draw a line graph showing these changes in temperature:

Hour	6	7	8	9	10	11	12	1	2	3	4	5	6
Degrees	57	59	63	66	69	70	73	74	75	76	73	71	69

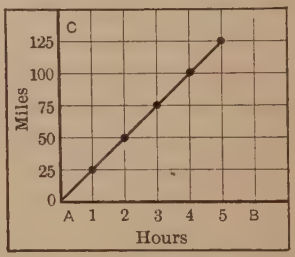
MAKING A GRAPH OF A FORMULA

You can graph the facts which you get from a formula just as you graphed the changes in temperature on page 9.

Suppose that Mr. Case drives his car at the rate of 25 mi. an hour. To find the distance that he has driven at the end of each hour you use the formula $D = 25\,n$, in which D stands for the *distance* and n for the *number of hours* he drives.

Using this formula, you can make the following table:

Hours	Distance
0	0
1	25
2	50
3	75
4	100
5	125



To draw a graph of the facts given in this table, first mark off the number of hours on line AB. Then mark off the number of miles on line AC, letting each space equal 25 mi.

From the table you find that Mr. Case drove 25 mi. in 1 hr. To show this fact on the graph, place a point *directly above* 1 hr. and *directly to the right* of 25 mi. Where will you place the point to show that he drove 50 mi. in 2 hr.? 75 mi. in 3 hr.? 0 mi. in 0 hr.? Tell how to place the points to represent the other pairs of corresponding values in the table. If you connect the points, you will find that they all lie on the *same straight line*. This straight line is the graph of the formula $D = 25\,n$.

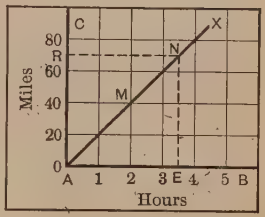
Exercise

Draw a graph showing the distances covered by Mr. Case at the end of each hour if he drives his car for 5 hr. at the rate of 22 mi. an hour. First write the formula and from it make a table like the one above.

On the vertical scale of the graph let each space equal 22 mi.

HOW TO READ A GRAPH

1. This graph shows Mr. Case's trip on another day. Read the graph and tell how many miles he had traveled at the end of 2 hr.



On line AB, find the point representing 2 hr. Then follow the vertical line above this point until it meets the graph line AX at point M. Since point M is also on the horizontal line that is marked 40 (at the left), Mr. Case had traveled 40 mi. at the end of 2 hr.

2. From the graph in ex. 1, tell how far Mr. Case had traveled at the end of 3 hr.; at the end of 4 hr.

3. On the graph above, find the distance traveled in $3\frac{1}{2}$ hr.

First, on line AB, find the point E, representing $3\frac{1}{2}$, which is halfway between 3 and 4. Then imagine a vertical line drawn upward from this point until it cuts the graph line at point N. From N imagine a horizontal line drawn to the left until it cuts the line AC at point R. Since R is halfway between 60 and 80, it represents a distance of 70 mi. Hence Mr. Case had traveled 70 mi. at the end of $3\frac{1}{2}$ hr.

How far did Mr. Case travel in $2\frac{1}{2}$ hr.? in $1\frac{1}{2}$ hr.? in $\frac{1}{2}$ hr.?

4. By reading the graph above, tell how many hours it took Mr. Case to travel 80 mi.; 60 mi.; 40 mi.; 50 mi.; 30 mi.

Find the point where the horizontal line marked 80 cuts the graph line AX. You see that this point of intersection is also on the vertical line that is marked 4 (at the bottom); hence it took 4 hr. to cover 80 mi.

5. On squared paper copy the graph in ex. 1. Then extend the horizontal scale to 10 hr., the vertical scale to 200 mi., and the graph line AX, which is a straight line, until it cuts the vertical line representing 10 hr. From this graph read the distance traveled in 6 hr.; in 7 hr.; in 8 hr.; in 9 hr.; in $7\frac{1}{2}$ hr.; in $6\frac{1}{2}$ hr.; in $8\frac{1}{2}$ hr.; in $9\frac{1}{2}$ hr.; in $4\frac{1}{2}$ hr.

6. From the graph in ex. 5, tell how many hours it took to travel 140 mi.; 180 mi.; 160 mi.; 70 mi.; 90 mi.; 170 mi.; 130 mi.

MAKING AND READING GRAPHS

1. Draw a graph like the one on page 11, showing the distances traveled by a train that goes at the rate of 30 mi. an hour for a period of 6 hr. Let each space on the vertical scale equal 30 mi. From your graph read the distance traveled in $2\frac{1}{2}$ hr.; in $3\frac{1}{2}$ hr.; in $4\frac{1}{2}$ hr.; in $5\frac{1}{2}$ hr.

First write the formula and from it make a table of corresponding values.

2. From the graph in ex. 1, tell how many hours it takes the train to travel 150 mi.; 90 mi.; 120 mi.; 75 mi.

3. In the graph in ex. 1, extend the hour scale to 10 hr. and the distance scale to 300 mi.; also extend the graph line until it cuts the vertical line representing 10 hr. From this larger graph read the distances covered in 7 hr.; in $7\frac{1}{2}$ hr.; in 8 hr.; in 9 hr.; in $9\frac{1}{2}$ hr. Also read the number of hours required to travel 240 mi.; 180 mi.; 195 mi.; 285 mi.; 225 mi.

4. Draw a graph showing the distances traveled by a train that goes at the rate of 40 mi. an hour for 7 hr. Let 4 spaces on the vertical scale equal 40 mi. and 4 spaces on the horizontal scale equal 1 hr. From your graph tell the distance traveled in $6\frac{1}{2}$ hr.; in $4\frac{1}{2}$ hr.; in $5\frac{1}{4}$ hr.; in $2\frac{3}{4}$ hr.; in $4\frac{3}{4}$ hr.; in $\frac{1}{2}$ hr. Also tell the number of hours required to travel 220 mi., 100 mi., 270 mi., 150 mi., 170 mi., 120 mi., 180 mi., 70 mi., 30 mi.

5. If a man earns \$8 a day, show that the formula $W = 8n$ will give the total amount of his wages (W) for any number of days (n). From this formula make a table of corresponding values and a graph of the man's wages for a period of 10 days.

6. If lace sells at 80¢ a yard, make a graph showing the prices of from 0 yd. to 5 yd. of lace. On the vertical scale let each space stand for 20¢. On the horizontal scale let 4 spaces = 1 yd. From your graph tell the price of 3 yd.; 4 yd.; $4\frac{1}{2}$ yd.; $2\frac{1}{4}$ yd.; $1\frac{3}{4}$ yd.; $3\frac{1}{2}$ yd.; $\frac{3}{4}$ yd. Also tell how many yards of lace you can buy for \$2.00; for \$3.20; \$1.60; \$2.80; \$3.00.

USING DIVISION IN FORMULAS

1. To find the cost (C) of 1 orange when oranges are sold by the dozen, Miss Davis divides the price per dozen (d) by 12. This gives the formula $C = \frac{d}{12}$. By this formula find the cost of 1 orange when $d = \$.60$; when $d = \$.54$; when $d = \$.40$.

2. Using the formula in ex. 1, complete the table given at the right, finding the cost of 1 flower when 1 doz. flowers are selling at the prices given. Include these prices per dozen in the table: \$4.00, \$3.00, \$2.50, \$2.00, \$1.80, \$1.50, \$1.00.

Cost of 12	Cost of 1
\$6.00	
5.40	
4.80	

3. To find the average number of miles an hour, or the *rate*, that he drives his car, Mr. Bird divides the total number of miles traveled, or the *distance*, by the number of hours he drives, or the *time*. This gives the formula $R = \frac{D}{T}$. What do R, D, and T stand for? Using this formula, find the rate at which Mr. Bird drives if the distance is 192 mi. and the time is 6 hr.; if $D = 195$ mi. and $T = 5$ hr.; if $D = 300$ mi. and $T = 12$ hr.

Using the formula in ex. 3, find R in these cases:

4. $D = 200$ mi.; $T = 8$ hr.

5. $D = 225$ mi.; $T = 7\frac{1}{2}$ hr.

6. $D = 294$ mi.; $T = 10\frac{1}{2}$ hr.

7. $D = 120$; $T = 3\frac{1}{3}$

8. $D = 98.7$; $T = 3$

9. $D = 20$; $T = \frac{1}{2}$

10. To find the average number of miles (M) that he can drive his car on each gallon of gasoline, Mr. Macy divides the total number of miles traveled (D) by the number of gallons (G) that he uses. Express this rule as a formula.

11. Using the formula in ex. 10, find how many miles per gallon Mr. Macy gets if he drives 200 mi. on 8 gal. of gasoline; if he drives 140 mi. on 7 gal.; if he drives 9 mi. on $\frac{1}{2}$ gal.

AN IMPORTANT FORMULA IN ELECTRICITY

You will now see how an important formula is used in electricity. This formula shows the relation between the units for measuring electricity with which you must first become acquainted.

The **volt** is the unit for measuring the *pressure* or *force* with which electricity goes through a wire. A radio "B" battery sends a current through a wire with a force of 45 volts. In cities the electric current used for lighting buildings is usually 110 volts or 220 volts. The **ampere** is the unit for measuring the *strength* of an electric current. A flatiron requires a *strong* current, usually about 6 *amperes,* while an ordinary electric light bulb requires only about $\frac{1}{2}$ ampere. The **watt** is the unit for measuring the total amount of electric power furnished. An ordinary electric light bulb is often marked " 50 W," which indicates that it requires 50 watts of electricity to light it.

The formula connecting the above measures is as follows:

$$W = VA$$

which means that $watts = volts \times amperes$

Exercises

1. Find the number of watts produced by an electric current having a pressure of 110 volts and a strength of 6 amperes.

Using the formula $W = VA$, you find that $W = 110 \times 6 = 660$. Hence 660 watts is the amount of power produced.

2. Find the number of watts produced by a current of 220 volts and 3 amperes; by a current of 660 volts and 1 ampere.

Using the formula $W = VA$, find the number of watts produced by each of the following currents:

3. 6 volts, 10 amperes

4. 32 volts, 120 amperes

5. 120 volts, 15 amperes

6. 45 volts, 2 amperes

7. 110 volts, 10 amperes

8. 220 volts, 10 amperes

9. A 50-watt lamp requires 50 watts of electricity to light it. Two 50-watt lamps require 2×50 watts, or 100 watts. How many watts will six 50-watt lamps require? Will a current having a pressure of 110 volts and a strength of 3 amperes furnish enough watts to light these six lamps?

10. How many watts will be produced by a current of 110 volts and 10 amperes? Will such a current furnish enough watts to operate eight 75-watt lamps and a 660-watt flatiron, all at the same time? If not, how many 75-watt lamps can be operated on that current at the same time as the flatiron?

How to Prevent Fuses from Blowing Out

1. It is dangerous to allow too strong an electric current to pass over a wire, because it might cause a fire. To prevent this, a *fuse* is attached to each electric circuit so that the current must pass through the fuse before it reaches the lamp or other device that is being operated. These fuses are made of special metal, which melts and thus shuts off the electricity if too strong a current passes through the fuse. A 15-ampere fuse is one that will melt or "blow out" if a current stronger than 15 amperes passes through it. What is the strongest current that can pass through a 10-ampere fuse? a 5-ampere fuse?

2. In a house that is supplied with a 110-volt electric current, 15-ampere fuses are often used on each branch line. What is the greatest number of watts that can be furnished by each branch line without blowing out a fuse?

3. In ex. 2, could a 500-watt toaster, a 660-watt heater, and a 600-watt flatiron all be used on the same line at the same time without blowing out a fuse?

4. In ex. 2, how many 75-watt lamps, together with a 660-watt electric flatiron, could be used at the same time on the same branch line without blowing out a fuse?

A FORMULA FOR TESTING AUTOMOBILE BRAKES

Mr. Holt thinks that the 2-wheel brakes on his car are not working well. If he applies his brakes when he is driving 20 mi. an hour, his car travels 50 ft. before stopping. He thinks it should stop within a shorter distance than that.

Mr. Day tells Mr. Holt that he can find out whether his brakes are working properly or not by means of this formula:

$$D = .09\,S^2$$

In this formula D = the number of feet (*distance*) in which the car should stop and S = the number of miles an hour (*speed*) at which the car is going.

To use this formula, suppose that Mr. Holt is driving 20 mi. an hour when he applies his brakes; then $S = 20$. Putting 20 for S in the formula and remembering that S^2 means $S \times S$, you get $D = .09 \times 20 \times 20 = 36$. Hence the car should stop in 36 ft. Are Mr. Holt's brakes working properly?

Exercises

Speed in Miles per Hour	Stopping Distance in Feet
0	0
5	$2\frac{1}{4}$
10	9
15	$20\frac{1}{4}$

1. Mr. Holt uses the above formula to make a table of corresponding values, as shown here. Continue this table for every 5 mi. up to 40 mi.

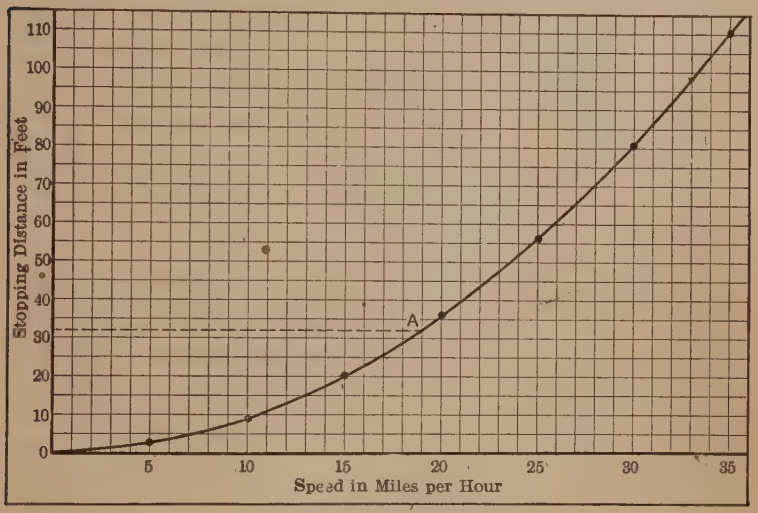
2. Since a speed of 20 mi. an hour is twice as fast as a speed of 10 mi., you might expect the stopping distance for a speed of 20 mi. to be twice as far as that for 10 mi. Study the table you made in ex. 1 and tell whether this is so or not. If not, how many times as great is the stopping distance?

3. As in ex. 2, compare the stopping distances when the speeds are 10 mi. and 5 mi.; when the speeds are 30 mi. and 15 mi.; when they are 40 mi. and 20 mi.

4. Raymond said, "If you double the speed of a car and then try to stop it, the car will go four times as far before stopping as it would at the lower speed." Was Raymond right?

GRAPHING THE FORMULA FOR 2-WHEEL BRAKES

A graph of the " brakes " formula $D = .09\,S^2$ is shown below:



In this graph the points represented by the heavy dots were located by using the values given in the table made for ex. 1, page 16. Since these points do not lie on a straight line, a smooth curved line was drawn so that it would pass through each point.

To find the stopping distance when the speed is 19 mi. an hour, find the point (A) where the vertical line representing 19 cuts the curved line. Imagine a horizontal line drawn from A to the left, as indicated by the dotted line. Since this dotted line cuts the vertical scale at about 32, the stopping distance is about 32 ft.

Exercises

1. From the graph find the stopping distance when the speed per hour is 21 mi.; 29 mi.; 26 mi.; 17 mi.; 33 mi.; 9 mi.

2. Extend the table in ex. 1, page 16, for each 5 mi. up to 60 mi. Then graph $D = .09\,S^2$ for speeds up to 60 mi. From the graph find the stopping distance when the speed per hour is 38 mi.; 42 mi.; 45 mi.; 47 mi.; 50 mi.; 53 mi.; 58 mi.; 60 mi.

HOW MUCH BETTER ARE 4-WHEEL BRAKES THAN 2-WHEEL BRAKES?

1. The formula $D = .09\,S^2$ is for 2-wheel brakes. For a car with 4-wheel brakes, the stopping distance is given by the formula $D = .06\,S^2$. Using this formula, find the number of feet in which a car with 4-wheel brakes should stop if it is traveling 20 mi. an hour when the brakes are applied. At the same speed, what is the stopping distance for a car with 2-wheel brakes? The 4-wheel brakes will stop the car in how many feet less than the 2-wheel brakes?

2. Using the new formula $D = .06\,S^2$, make a table like the one in ex. 1, page 16, in which you find the stopping distances corresponding to speeds for every 5 mi. up to 60 mi. an hour.

3. Using the values found for the table in ex. 2, make a graph of the formula $D = .06\,S^2$ for speeds of from 0 mi. to 60 mi. an hour. Draw this graph on the same sheet that contains the graph of $D = .09\,S^2$, which you made in ex. 2, page 17. This will enable you easily to compare the stopping distances for 4-wheel brakes with those for 2-wheel brakes.

In graphing $D = .06\,S^2$, use the same vertical and horizontal scales that you used for graphing $D = .09\,S^2$. This new graph for 4-wheel brakes will be a curved line similar to the one for 2-wheel brakes already drawn on the same sheet. The first point on this new curved line will be the point representing 0 on each scale, which was also the first point of your preceding curved line for 2-wheel brakes. This first point, however, is the only one that is the same on both curved lines. If the new curved line is correctly drawn, it will lie below the other one.

4. From the graph that you made in ex. 3, which contains both curves, tell how many feet greater the stopping distance for 2-wheel brakes is than that for 4-wheel brakes when the speed is 10 mi. an hour; when the speed is 20 mi. an hour; 30 mi. an hour; 40 mi. an hour; 50 mi. an hour; 60 mi. an hour. Is it wise to have 4-wheel brakes on a car?

A MATCHING TEST

In ex. 1 *a rule is given in words. Find in the right-hand column the formula that states the same rule and tell what each letter in the formula stands for. Do the same for ex.* 2 *to* 9.

Rule	Formula
1. The area of a circle equals $3\frac{1}{7}$ times the square of the radius.	(*a*) $W = VA$
2. The area of a trapezoid equals half the height times the sum of the parallel sides.	(*b*) $f = \frac{i}{12}$
3. The circumference of a circle equals $3\frac{1}{7}$ times the diameter.	(*c*) $N = L - D$
4. The number of watts of electricity equals the number of volts times the number of amperes.	(*d*) $A = \pi r^2$
5. To change yards to feet, multiply the number of yards by 3.	(*e*) $M = \frac{D}{G}$
6. To change inches to feet, divide the number of inches by 12.	(*f*) $D = RT$
7. To find the average number of miles a car makes on each gallon of gasoline, divide the total distance (in miles) that the car covers by the number of gallons used.	(*g*) $A = \frac{1}{2}h(a + b)$
8. To find the distance covered by a train, multiply the number of miles it goes per hour by the number of hours it travels.	(*h*) $f = 3y$
9. To find the net price, subtract the discount from the list price.	(*i*) $C = \pi D$

REVIEW

1. Make a formula showing the price of any number of gallons of gasoline at 22¢ a gallon. Then make a graph showing the price of from 0 gal. to 10 gal. of gasoline. Let each space on the vertical axis stand for 22¢. From your graph tell the price of 5 gal.; of 7 gal.; 9 gal.; $6\frac{1}{2}$ gal.; $2\frac{1}{2}$ gal.

Draw graphs showing the cost of the following:

2. From 0 lb. to 10 lb. of tea at 80¢ a pound.

3. From 0 tickets to 20 tickets at 50¢ each.

4. From 0 baseballs to 10 baseballs at 75¢ each.

5. From 0 yd. to 10 yd. of silk at $1.50 a yard.

6. From 0 lb. to 15 lb. of sugar at 8¢ a pound.

Evaluate each of the following formulas:

7. $A = \frac{1}{2}\,bh$ when $h = 5$ and $b = 6$; when $h = 2\frac{1}{4}$ and $b = 4$.

8. $W = VA$ when $V = 45$ and $A = 6$; when $V = 6$ and $A = 25$.

9. $A = \frac{1}{2}\,h(a + b)$ when $h = 4$, $a = 8$, and $b = 6$.

10. $R = \dfrac{D}{T}$ when $D = 126$ and $T = 3\frac{1}{2}$.

11. $A = \pi r^2$ when $r = 35$; when $r = 20$. Let $\pi = 3\frac{1}{7}$.

12. $D = .09\,S^2$ when $S = 32$; when $S = 65$; when $S = 70$.

13. When David bought some new "A" dry batteries for his radio set, he asked the salesman to test them to see if they were good. The test showed that each battery gave $1\frac{1}{2}$ volts and 35 amperes. How many watts did each battery give?

14. David tested six of his old "A" batteries and found that one of them gave only 12 amperes; the other five gave 22, 18, 15, 20, and 24 amperes, respectively. If each battery gave $1\frac{1}{2}$ volts, what was the number of watts given by each battery? How many fewer watts did each battery give than a new "A" battery like those in ex. 13?

CHAPTER II

IMPROVEMENT TESTS

GAINING SKILL IN COMPUTATION

George is trying to win a place on the school basket-ball team. Every day he practices shooting the ball into the basket and records carefully the number of baskets he makes. Yesterday he threw the ball in on 13 out of 30 trials; to-day he tries to get at least 14 out of 30. Only by constantly watching and improving his record can he gain real skill.

In the same way you can gain skill in computing by watching your record and always trying to improve it. Begin by taking Improvement Test 1 A, on page 23. The teacher will time you, telling you when to start and when to stop. She will then read the correct answers and you should make a record of your score, using the table on the inside of the back cover of this book to find your score. When you take the test again, in a day or two, try hard to raise your mark.

For each Improvement Test you should make a graph in your notebook, showing your scores (see page 25). These graphs will show whether your skill is increasing. They will also show on which tests you need more practice. You should also compare your daily scores on each test with the median scores of the entire class, which will be kept on the board. On each test, *try to get every example right.* If you cannot do this, your score on any day will be satisfactory only if it is equal to or above the median score of the class for that day.

The median score of a class (see page 27) is like the average score; but it is much easier to compute the median score.

TAKING AN IMPROVEMENT TEST

In taking an Improvement Test in the addition or subtraction of whole numbers, such as Test 1 A or Test 1 C on the next page, do not copy the examples. Instead, lay a sheet of paper on the book with its edge directly under the examples. Write only the answers on the paper, as shown in the picture. If a test has more than one row of examples, like Test 1 C, first write the answers to the top row; then fold them under and write the answers to the second row along the folded edge.

In an Improvement Test in the multiplication or division of whole numbers, begin by copying all the examples. The time allowed for these tests does not include the time for copying.

Work on each test only the number of minutes permitted for that test and *stop all work immediately at the signal for stopping*, even if you are just finishing an example. You cannot compare your scores from day to day on the same test, to show whether you are improving, unless you work on that test exactly the same number of minutes each time you take it.

The teacher will usually have you do the same test on several successive days, until the class "hits the roof" by making a median score of 10. Then she will start a new test and continue with it until the class again reaches a median score of 10. If your score is not up to the class median when the class leaves a test, practice the test by yourself. Then, when the class tries that test again after a few weeks, you should be able to equal the class median.

IMPROVEMENT TEST No. 1

Test 1 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
879	498	891	516	636
154	183	675	588	698
718	627	539	934	959
495	478	736	746	313
366	295	373	599	165
583	778	977	648	486
957	461	196	869	773
715	527	821	424	274
184	236	615	365	823
261	747	360	953	151
928	599	254	544	678
373	373	884	625	454

Test 1 B — Addition (Time 3 min.)

1.	2.	3.	4.
43736	81865	79234	76838
47088	53928	40975	44436
26925	34376	82144	50728
30849	89969	56398	67469
56246	39100	58789	53543
69580	52672	12720	49670
54857	60528	34556	63349

Test 1 C — Subtraction (Time 3 min.)

1. 957142 − 769834	**2.** 785000 − 96023	**3.** 156020 − 78609	**4.** 800000 − 25482
5. 753810 − 168852	**6.** 808239 − 114672	**7.** 679139 − 392140	**8.** 892377 − 459397
9. 840909 − 368579	**10.** 927859 − 546863	**11.** 638636 − 379257	**12.** 475282 − 139708

FINDING SCORES IN IMPROVEMENT TESTS

This table will help you to find your score on a test:

NUMBER OF EXAMPLES IN TEST	NUMBER OF EXAMPLES RIGHT																
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
3	0	3	7	10													
4	0	3	5	8	10												
5	0	2	4	6	8	10											
6	0	2	3	5	7	8	10										
A → 8	0	1	3	4	5	6	8	9	10								
9	0	1	2	3	4	6	7	8	9	10							
10	0	1	2	3	4	5	6	7	8	9	10						
12	0	1	2	3	3	4	5	6	7	8	8	9	10				
15	0	1	1	2	3	3	4	5	5	6	7	7	8	9	9	10	
16	0	1	1	2	3	3	4	4	5	6	6	7	8	8	9	9	10

B

C

Suppose that you take a test having 8 examples and get 5 examples right. First find in the table the horizontal row beginning with 8 (as shown by Arrow A). Then look *along that row* until you reach the *column* with 5 at the top (shown by Arrow B) and you find the number 6 (shown by Arrow C), which means that your score on the test is 6.

In finding your score, *count as correct only the examples that are entirely right.* An unfinished problem should be counted wrong.

USING THE SCORING TABLE

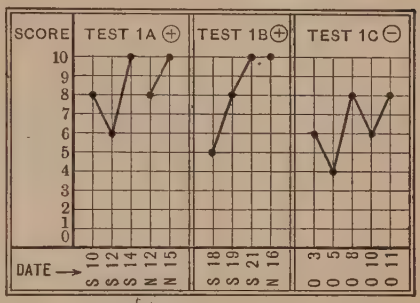
1. In Test 1 C, show that your score is 8 if you get 10 examples right. What is your score if you get 7 examples right? if you get 5 examples right? 12 examples right?

2. Use the above table to find the scores in these tests:

Number in test	5	5	8	10	12	4	5	16	12	15
Number right	4	5	4	7	11	3	3	13	8	14

KEEPING RECORDS IN IMPROVEMENT TESTS

The best way to keep your record is to make a picture, or *graph*, of your daily scores, as Paul did at the right.



Along the lowest line of the graph Paul wrote the dates on which he tried the tests. He used the abbreviations given below. S 10 means Sept. 10, which is the first day on which Paul tried Test 1 A. The heavy black dot above S 10, on the horizontal line numbered 8, shows that on Sept. 10 his score was 8.

S = Sept.	F = Feb.
O = Oct.	M = Mar.
N = Nov.	A = Apr.
D = Dec.	My = May
J = Jan.	Ju = June

You should have a notebook of squared paper in which to keep your record, as shown above.

Reading the Graph of Paul's Record

1. What score did Paul get on Sept. 12? Did he do as well on Sept. 12 as on Sept. 10? Did he improve again on Sept. 14?

2. How long after he first scored 10 on Test 1 A did he try the test again? Did he do as well on Nov. 12 as on Sept. 14?

3. The first time Paul tried Test 1 B, did he do as well as he did the first time on Test 1 A? What was his lowest score on this test? his highest score?

4. On Test 1 B, on what date did Paul first get a score of 10? How long after that did he wait before trying Test 1 B again?

5. Did Paul do as well on Test 1 C as he did on the other two tests? What was his first score on Test 1 C? his lowest score? his highest score? his last score?

FINDING AND RECORDING SCORES

1. How many examples are there in Test 1 A? Dorothy had 2 examples right in Test 1 A on Sept. 11, 4 examples right on Sept. 12, and 5 right on Sept. 16. Use the table on page 24 to find her score for each of these days.

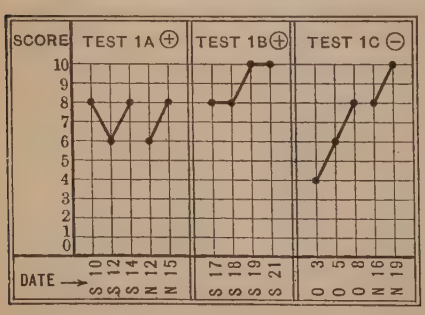
Make a graph to show Dorothy's record on Test 1 A, using the abbreviations given on page 25 for the names of months. Did Dorothy improve steadily on this test?

2. On a test containing 6 examples, Arthur had 5 examples right on Sept. 22, 4 right on Sept. 24, 5 right on Oct. 6, and 6 right on Oct. 8. Use the table on page 24 to find Arthur's score on each trial. Make a graph of Arthur's record. Does the graph show a steady improvement?

3. On a test of 12 examples, Leon had 8 examples right on Nov. 4, 9 right on Nov. 5, 9 right on Nov. 10, and 12 right on Nov. 13. On the same test, Claude had 9 examples right on Nov. 4, 7 right on Nov. 5, 8 right on Nov. 10, and 9 right on Nov. 13. Find each boy's daily scores and make a graph of each record.

Whose graph shows the greater improvement, Leon's or Claude's? Which boy made the better record on this test?

4. Here are the graphs of Elsie's records on three tests.

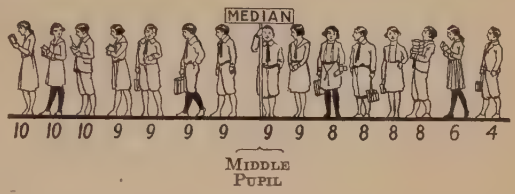


On Sept. 10 Elsie scored 8 on Test 1 A. What was her score each of the other times she tried Test 1 A? Give her score on each day that she tried Test 1 B; Test 1 C.

On which test did Elsie do the best? On which test did she improve the most? On which does she need more practice?

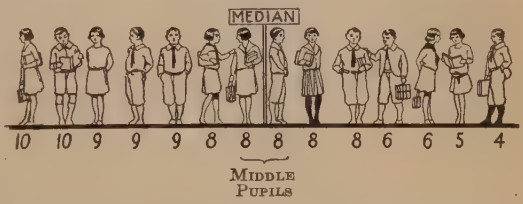
HOW TO FIND THE MEDIAN SCORE

This picture shows the members of a class arranged according to their marks on an Improvement Test. The numbers under the pupils show their scores. Those with the highest scores come first; those with the lowest come last. The middle pupil in the line has the middle score, or the *median score*, of the class.



There are 15 pupils in this class. The middle pupil is the 8th, there being 7 pupils before him and 7 pupils after him. His mark of 9 is the median score of the class for this test. All those who have less than 9 must work very hard to get up to, or ahead of, the median next time.

Here is another class in similar order. When there are 14 pupils in the class, as in this case, there are 2 middle pupils instead of only one. The 2 middle pupils here both have 8 as their mark; therefore the middle mark of the class is also 8.



Below are the marks of another class of 14 pupils. The two middle pupils have marks of 9 and 8. The median score is

10 10 9 9 9 9 9 8 8 8 8 8 6 6
MEDIAN

halfway between the two middle scores, or halfway between 9 and 8. This gives a median of $8\frac{1}{2}$; but as Improvement Tests are scored by whole numbers, the median is called 9.

WHICH SIDE WILL YOU BE ON?

The median divides a class into two equal groups, representing the upper and lower halves of the class. In which half do you want to be? Will you be before the median flag or behind it? The median is often a high mark; but since at least half of your class has reached it, you also can reach it.

Always try to be among those whose marks are above the median, so that you will be in the upper half of the class.

A Short Way to Find the Class Median

Here is a short way to find the class median:

Consider the first class of 15 pupils, which is pictured on page 27. After the papers are scored, the teacher need not take the time to arrange the pupils in line. She simply asks how many have a score of 10, and 3 pupils raise their hands. She then asks how many have 9, and 6 pupils raise their hands. She writes on the board the number of pupils having each score, as shown above.

Score	*Number of Pupils*	
10	3	
9	6	← Median score in this group
8	4	
6	1	
4	1	
	15	

Since the 8th score is the middle score, the teacher counts down in the column marked "Number of Pupils" until she reaches the 8th pupil. In the first group there are 3 pupils; to these she adds 5 pupils from the next group in order to reach the 8th pupil. Since the 8th pupil belongs to the group of 6 pupils who received a mark of 9, the median score is 9.

NOTE. To find which is the middle score, add 1 to the total number of scores and divide by 2. In the example above, add 1 to 15, which gives 16. Dividing 16 by 2, you get the 8th as the middle score. For a class of 14 pupils, add 1 to 14, which gives 15. Since $15 \div 2 = 7\frac{1}{2}$, the $7\frac{1}{2}$th score, which is halfway between the 7th and the 8th score, is the middle score.

FINDING CLASS MEDIANS

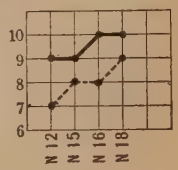
1. In a class, 4 pupils had a score of 10 on an Improvement Test, 7 pupils had 9, 7 had 8, 4 had 7, 2 had 6, and 1 had 3. How many pupils took the test? Arrange the marks in a table, and find the median by the short method given on page 28.

2. On Monday, in Amy's class, 5 pupils had a score of 10 on an Improvement Test, 6 had 9, 9 had 8, 4 had 7, 2 had 6, and 2 had 5. Find the class median.

3. On Tuesday Amy's class repeated the test. This time 7 pupils had a score of 10, 7 had 9, 9 had 8, 3 had 7, 1 had 6, and 1 had 5. Find the class median for Tuesday. Had it gone up since Monday?

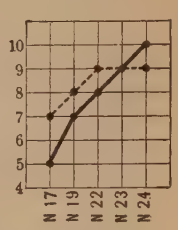
Comparing Your Work with That of Your Class

In this graph the heavy line shows Anna's record on a test; the dotted line shows the median scores of the entire class on that test. What was the median on Nov. 12? on Nov. 18? What were Anna's scores on those days? Was Anna's work always satisfactory?



The second graph shows Joe's record by a heavy line and that of his class by a dotted line. Joe was far below the class on Nov. 17. On Nov. 19 he did better, but the class median also rose. What was the class score on Nov. 22? What was Joe's score?

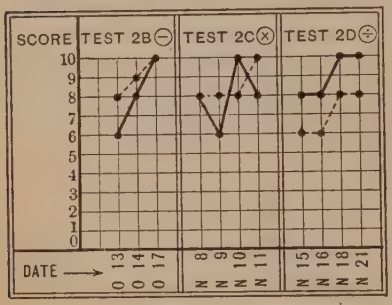
Joe worked hard and continued to improve. How did his score compare with the class median on Nov. 23? on Nov. 24?



For every Improvement Test your class medians will be kept on the board. In your record book, make a graph of the class medians in red and of your own scores in black. Try to keep your line above the class line.

COMPARING RECORDS WITH THE CLASS RECORD

This graph shows Philip's scores (heavy lines) and the medians of his class (dotted lines) on Tests 2 B, 2 C, and 2 D.



1. Was Philip above or below the class median when he tried Test 2 B on Oct. 13? Did he improve on Oct. 14? Did the class also improve on Oct. 14? On what day did Philip come up to the class median on Test 2 B?

2. On Test 2 C, how did Philip's score compare with the class median on Nov. 8? on Nov. 9? on Nov. 10? on Nov. 11? Was his work always satisfactory on Test 2 C? Does he need more practice on this test?

3. Compare Philip's score with the class score on each of the days Test 2 D was tried. Did he always make a satisfactory score on Test 2 D? Which needs more practice on this test, Philip or the rest of the class?

4. On Test 2 B, Nora's marks were 8 on Oct. 21, 6 on Oct. 24, 8 on Oct. 26, and 10 on Oct. 28. The medians of her class were 8 on Oct. 21, 8 on Oct. 24, 9 on Oct. 26, and 9 on Oct. 28. Draw a graph of Nora's scores in black and of the class medians in red. On which days was Nora's score below the class median? On which days was her score equal to the class median? above the median?

5. Draw a graph showing Bertha's scores on a test, if her score was 8 on Oct. 5, 10 on Oct. 8, 9 on Oct. 26, and 10 on Oct. 29. On the same graph show, in red, that the class medians on that test were 7 on Oct. 5, 9 on Oct. 8, 8 on Oct. 26, and 10 on Oct. 29. Compare Bertha's graph with the class graph.

CHAPTER III

EQUATIONS

SOLVING PROBLEMS BY SHORTHAND

Mr. Harris, who runs a shoe store, is deciding at what price he should sell shoes that cost him \$3.40 a pair. He knows from experience that he must allow 32% of the selling price to cover his expenses and profit together. He asks his clerks, Mr. Ford and Mr. Lane, each to compute the selling price for him. Mr. Ford does the problem by arithmetic, thinking out the steps as shown below. S. P. stands for *selling price*.

Expenses and profit are 32% of S. P.
Hence *cost* is 100% − 32%, or 68%, of S. P.
If 68% of S. P. is \$3.40,
then 1% of S. P. is \$3.40 ÷ 68, or \$.05
and 100% of S. P. is 100 × \$.05, or \$5.00.

Mr. Lane does his thinking by algebra, writing each step in a kind of shorthand called an *equation*, as shown below. You see that Mr. Lane's work is shorter than Mr. Ford's, even though you do not know enough about equations yet to understand it. You will learn more about equations in this chapter.

$$\begin{aligned} &\text{Let } x = \text{S. P.} \\ &\text{Then } x = \$3.40 + \tfrac{32}{100}x \\ &\tfrac{68}{100}x = \$3.40 \\ &x = \$5.00 \end{aligned}$$

For solving difficult problems, the *equation method* which Mr. Lane uses is employed by business men, engineers, and scientists all over the world. It is the shortest as well as the most useful single method of solving problems that has ever been invented.

A NUMBER GAME

1. A class was playing a number game. Jane said, "If you add 4 to the number I am thinking of, you get 9."

The teacher wrote this statement on the board thus:

Jane's number plus 4 *makes* 9.

Then she wrote the same statement in shorthand, letting n stand for the number Jane was thinking of, thus:

$$n + 4 = 9$$

The class saw that Jane's number was 5 because $5 + 4 = 9$.

2. Then it was Edna's turn. She said, "Take 10 from the number I am thinking of and you get 19."

Letting n stand for Edna's number, the teacher wrote:

$$n - 10 = 19$$

The class saw that n must be 29 because $29 - 10 = 19$.

3. Next Jack said, "Six times my number is 48." This was also written on the board in the shorthand way, thus:

$$6\,n = 48$$

Here $6\,n$ means "6 times some number." Can you tell what Jack's number was? How do you know that you are right?

Using Letters for Unknown Numbers

The letter n or any other letter may be used to represent an unknown number. Often x stands for the unknown number, as in Mr. Lane's work on page 31.

In writing "6 times some number" by this short method, you omit the sign of multiplication and write $6\,n$. Likewise, $4\,x$ means 4 times x. In the same way, in your work with formulas, you wrote two letters side by side to indicate multiplication. In the formula for the area of a rectangle, $A = lw$, you know that lw means *length times width.*

UNKNOWN NUMBERS

In the game on page 32, other pupils made these statements. Write each in the short way and find the unknown number:

1. 7 times my number gives 21.

2. Add my number to 4 and you get 7.

3. Add 7 to my number and you get 20.

4. Multiply my number by 2 and you get 18.

5. If you take 4 from my number, you get 13.

6. If you find $\frac{1}{4}$ of my number, the answer is 6.

7. If you take my number from 5, you have 1 left.

Here are the short forms in which several statements were written on the board. Tell what each original statement was and what number each letter stands for:

8. $y + 2 = 7$	**12.** $8 + n = 15$	**16.** $\frac{1}{2}m = 4$
9. $d + 3 = 4$	**13.** $3 + x = 12$	**17.** $3n = 21$
10. $9 - x = 7$	**14.** $n - 7 = 11$	**18.** $6x = 30$
11. $11 - n = 5$	**15.** $z + 3 = 14$	**19.** $\frac{1}{2}x = 12$

20. Make up other statements like those above, writing them in the short way, and let the class tell the unknown numbers.

The Equation

A statement like $x + 5 = 9$, in which one expression is shown to equal another, is called an **equation.**

You *solve* the equation $x + 5 = 9$ when you find that $x = 4$. Here 4 is the *value* of x because x stands for 4.

In these equations, find the value of x mentally:

1. $x + 1 = 7$	**4.** $15 = x + 8$	**7.** $2x = 12$
2. $x + 7 = 9$	**5.** $26 = x + 6$	**8.** $12 = 3x$
3. $2 - x = 1$	**6.** $13 = 15 - x$	**9.** $7x = 42$

THE NATURE OF EQUATIONS

It is not always easy to solve an equation at a glance. In an equation like $3x + 51 = 348$, it would be difficult to find the value of x mentally. It is therefore necessary to study the nature of equations more closely and to learn some simple principles for use in solving them.

Every equation has two sides separated by the sign $=$. Thus, the equation $x + 5 = 9$ has a left side, $x + 5$, and a right side, 9, with the sign $=$ between them.

Left side		Right side
$x + 5$	$=$	9

The entire left side of an equation is exactly equal to the entire right side. Therefore an equation is like a perfectly balanced seesaw, in which the weight on one end is exactly equal to that on the other end. The sign $=$ is like the wooden horse that supports the board of the seesaw. In solving an equation, you find a value for x that will keep the balance perfect.

The sides of an equation are sometimes called its *members*. The left side is the *first member* and the right side is the *second member*. In the equation $x + 5 = 9$, the first member is $x + 5$ and the second member is 9.

Balancing Equations

If these equations were balanced on a seesaw, tell which part of each would be set on the left end of the board and which on the right end. Solve each equation mentally:

1. $x + 4 = 16$

2. $5 + x = 12$

3. $x - 7 = 13$

4. $n - 2 = 17$

5. $3n = 9$

6. $\frac{1}{4}x = 6$

7. $4m = 8$

8. $\frac{1}{10}x = 8$

9. $5x = 55$

10. $2x = 48$

11. $x - 9 = 20$

12. $3 + x = 25$

KEEPING THE BALANCE

Here John, who weighs 100 lb., and Sam, who weighs 40 lb., are on one end of a seesaw, while Mary, who weighs 100 lb., is on the other end with a 40-pound bag of sand. The seesaw balances because there is the same weight on each end of the board.

If Sam gets off, what weight is taken from his end of the seesaw? What must be taken from the other end to keep the balance? What weight is then left on each end?

This story may be briefly written in equation form, thus:

Start with $100 + 40 = 100 + 40$

Then subtract 40 from both sides, like this:

$$100 + 40 - 40 = 100 + 40 - 40$$

This leaves $100 = 100$

If you subtract the same number from both sides of an equation, the sides are still equal.

Solving an Equation by Subtraction

Now try the above plan on the following equation:

$$x + 347 = 925$$

Subtract 347 from both sides, as shown below:

$$x + 347 - 347 = 925 - 347$$

This gives $x = 578$

To make sure that 578 is the value of x, *check it* by putting 578 in place of x in the original equation, thus:

$$578 + 347 = 925$$

This gives $925 = 925$

Since both sides are still equal, 578 is the correct value of x.

SOLVING EQUATIONS BY SUBTRACTION

You have seen that if 347 is subtracted from both sides of the equation $x + 347 = 925$, the sides are still equal and the result is $x = 578$. Hence by subtracting the same number from both sides of the equation, you get x on one side and its value on the other. Thus the rule on page 35 may be used in solving certain equations.

To solve an equation by subtraction, subtract from both sides of the equation the number that will leave only x on one side and its value on the other.

Exercises

1. In the equation $x + 3 = 12$, what number do you subtract from both sides to get the value of x? What does x equal?

Find the value of x by subtraction. Check by putting the answer in place of x in the given equation:

2. $x + 7 = 13$
3. $x + 9 = 14$
4. $x + 6 = 48$
5. $x + 8 = 57$
6. $x + 5 = 20$
7. $x + 7 = 34$
8. $x + 256 = 524$
9. $x + 347 = 623$
10. $x + 562 = 730$
11. $x + 777 = 900$
12. $600 + x = 801$
13. $240 + x = 680$
14. $x + 1.15 = 2.11$
15. $x + 3.65 = 7.09$
16. $x + 2.25 = 4.15$
17. $x + 3.74 = 5.23$
18. $x + 41.1 = 89.2$
19. $2.41 + x = 4.45$

20. A certain number plus 9 equals 34. Find the number.
First write the statement as an equation, letting x stand for the number.

21. Will is thinking of a number. If he adds 16 to it, he gets 45. Write this as an equation and find the number.

22. Add 340 to the number which Anna has in mind and you obtain 657. Express this as an equation and solve it.

23. Ida lives on Oak Street. If you add 412 to the number of her house, you get 1000. Find the number of her house.

SOLVING EQUATIONS BY ADDITION

If Tom and Jane, each weighing 100 lb., are on opposite ends of a seesaw, the seesaw balances. But if Tom's dog, which weighs 20 lb., gets on with Tom, a 20-pound weight is needed on Jane's side to keep the balance.

You can write these facts in equation form, thus:

Start with	$100 = 100$
Add 20 to both sides:	$100 + 20 = 100 + 20$
This gives	$120 = 120$

If the same number is added to both sides of an equation, the sides are still equal.

This principle is used in solving an equation like $x - 3 = 5$, in which $x - 3$ means that 3 has been taken away from x. If you add 3 to $x - 3$, you put back what was taken away and get x again. Hence $x - 3 + 3$ is the same as x.

Start with	$x - 3 = 5$
Add 3 to both sides:	$x - 3 + 3 = 5 + 3$
This gives	$x = 8$

To check, put 8 in place of x in the original equation.

This gives	$8 - 3 = 5$
Then	$5 = 5$

Since the sides are still equal, the work is right.

Exercises

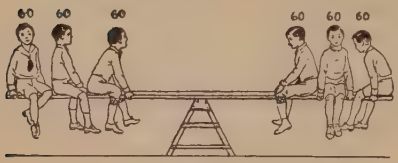
Solve the following equations. Then check your work:

1. $x - 5 = 7$ **2.** $x - 8 = 5$ **3.** $x - 7 = 15$ **4.** $x - 1 = 18$ **5.** $x - 2 = 43$

6. $p - 217 = 186$ **7.** $h - 356 = 245$ **8.** $x - 900 = 250$ **9.** $c - 255 = 846$ **10.** $n - 180 = 900$

11. $n - \frac{1}{2} = 4\frac{1}{2}$ **12.** $x - 2.5 = 2$ **13.** $c - 12\frac{1}{4} = 10\frac{1}{2}$ **14.** $x - .20 = 1.20$ **15.** $x - .30 = 3.00$

SOLVING EQUATIONS BY MULTIPLICATION



If Henry and James, each weighing 60 lb., are on opposite ends of a seesaw, the seesaw balances.

If two more boys, each weighing 60 lb., jump on Henry's side, and two other boys, each weighing 60 lb., get on with James, the seesaw still balances.

You can express the above in equation form, thus:

Start with	$60 = 60$
Multiply both sides by 3:	$3 \times 60 = 3 \times 60$
This gives	$180 = 180$

If both sides of an equation are multiplied by the same number, the sides are still equal.

This principle may be used to solve an equation like $\frac{1}{2}\,n = 5$, in which $\frac{1}{2}\,n$ means $\frac{n}{2}$. To change $\frac{1}{2}\,n$ to n, you multiply by 2.

Start with	$\frac{1}{2}\,n = 5$
Multiply both sides by 2:	$2 \times \frac{1}{2}\,n = 2 \times 5$
This gives	$n = 10$
Check.	$\frac{1}{2} \times 10 = 5$

Exercises

Solve the following equations. Check your results:

1. $\frac{1}{4}\,x = 9$

2. $\frac{1}{3}\,x = 8$

3. $\frac{1}{2}\,x = 6$

4. $\frac{b}{4} = 30$

5. $\frac{1}{4}\,x = 7$

6. $\frac{1}{5}\,x = 4$

7. $\frac{1}{7}\,x = 8$

8. $\frac{c}{5} = 85$

9. $\frac{1}{4}\,a = 3.25$

10. $\frac{1}{3}\,x = 0.45$

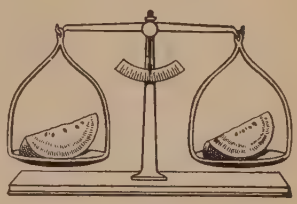
11. $\frac{1}{10}\,x = 10\frac{1}{2}$

12. $\frac{x}{2} = 75.5$

SOLVING EQUATIONS BY DIVISION

If you have two 18-pound watermelons and put one on each side of a scale, the scale balances.

If you cut each melon into six equal parts and leave one part on each side of the scale, the scale still balances. Tell why it balances.



You can express the above in equation form, as follows:

Start with	$18 = 18$
Divide both sides by 6:	$\frac{18}{6} = \frac{18}{6}$
This gives	$3 = 3$

If both sides of an equation are divided by the same number, the sides are still equal.

This principle is used to solve an equation like $4x = 28$, in which $4x$ means 4 times x. To change $4x$ to x, divide $4x$ by 4; also divide 28 by 4 to keep both sides equal.

Start with	$4x = 28$
Divide both sides by 4:	$\frac{4x}{4} = \frac{28}{4}$
This gives	$x = 7$
Check.	$4 \times 7 = 28$

Exercises

1. By what number do you divide $7x$ to change it to x?

Solve the following equations. Check your results:

2. $7x = 42$

3. $12b = 36$

4. $15n = 45$

5. $13n = 13$

6. $4x = 2$

7. $5x = 1$

8. $4r = 7$

9. $6x = 3$

10. $1\frac{1}{2}x = 4\frac{1}{2}$

11. $2.5x = 7.5$

12. $32s = 96.96$

13. $115x = 6.90$

USING THE RIGHT PRINCIPLE IN EQUATIONS

You have seen that the sides of an equation remain equal

(1) if you add the same number to both sides;
(2) if you subtract the same number from both sides;
(3) if you multiply both sides by the same number;
(4) if you divide both sides by the same number.

Of the above principles, use the one which leaves only x (or the letter that stands for the unknown number) on one side of the equation and only known numbers on the other side.

Exercises

1. In solving $\frac{x}{8} = 12$, you first multiply both sides of the equation by 8, in order to have only x left on the left side. What do you do to $8x = 16$ to have only x left on one side?

Tell what you would do in each example to have only x left:

2. $x + 4$	**5.** $x - 16$	**8.** $6x$	**11.** $\frac{1}{2}x$	**14.** $x - 5$
3. $x - 2$	**6.** $x + 18$	**9.** $3x$	**12.** $\frac{1}{4}x$	**15.** $5 + x$
4. $x - 7$	**7.** $x + 54$	**10.** $19x$	**13.** $\frac{1}{10}x$	**16.** $27 + a$

Solve, being careful to use the correct principle:

17. $x - 1 = 8$	**26.** $\frac{1}{4}x = 8$	**35.** $12 = y + 8$
18. $x - 9 = 5$	**27.** $\frac{1}{2}x = 17$	**36.** $\frac{1}{3}r = 12\frac{1}{4}$
19. $x - 8 = 27$	**28.** $3x = 30$	**37.** $\frac{1}{25}x = 51.4$
20. $x + 13 = 22$	**29.** $8x = 22$	**38.** $67.5 = 15m$
21. $p - 16 = 17$	**30.** $\frac{1}{5}t = 125$	**39.** $x + 11\frac{1}{4} = 15$
22. $r - 12 = 15$	**31.** $24 = 12x$	**40.** $n - 18.3 = 504$
23. $a + 10 = 56$	**32.** $100 = \frac{1}{8}c$	**41.** $35.7 + x = 96.5$
24. $14 + b = 18$	**33.** $120 = 5r$	**42.** $c - 18\frac{1}{2} = 21\frac{3}{4}$
25. $20 + m = 21$	**34.** $150 = 6m$	**43.** $43\frac{1}{4} = 25\frac{3}{4} + b$

USING THE LANGUAGE OF ALGEBRA

1. Edward has x stamps. If he buys 15 stamps more, how many stamps will he have then?

In the language of algebra, we say that the number of stamps he will then have will be $x + 15$.

2. Josephine had m beads and lost 6 of them. How many beads did she have left?

3. Kate paid y dollars for a hat. If Mary paid twice as much for her hat, how many dollars did Mary's hat cost? Carolyn paid half as much for her hat as Kate did. How many dollars did Carolyn's hat cost?

4. Paul paid x dollars for a dog. How many dollars would he receive for the dog if he sold it at a loss of \$3? if he sold it at a gain of \$5? if he sold it for twice as much as he paid for it? if he sold it for half as much as he paid for it?

5. Henry saved d dollars last summer. Joe saved $\frac{1}{3}$ as much as Henry. How many dollars did Joe save? Will saved twice as much as Henry. How many dollars did Will save?

6. If you are n years old now, how old will you be 5 yr. from now? 1 yr. from now? 10 yr. from now?

7. If you are x years old now, how old were you 2 yr. ago? 5 yr. ago? 7 yr. ago? x years ago?

8. Tom had x dollars and spent \$2.50 for a tennis racket. How many dollars did he have left?

9. Will had y dollars and spent half of that sum for a bicycle. How many dollars did he pay for the bicycle?

10. If $x = 15$, find the value of $x + 2$; of $x - 10$; of $2x$.

11. Mr. Foster's salary has been raised \$5 a week so that he now receives x dollars a week. What was his weekly salary before it was increased?

HOW TO SOLVE PROBLEMS

Sam needs 15 more stamps to give him a collection of 100 stamps. How many stamps has he now?

This problem can be solved by an equation, as follows:

Let x = the number of stamps Sam has now

If he gets 15 more stamps, he will have $(x + 15)$ stamps. The problem also states that he will then have 100 stamps. Hence $x + 15$ and 100 both stand for the same thing and you can write this equation:

$$x + 15 = 100$$

Subtract 15: $x = 85$

Since $x = 85$, you know that Sam now has 85 stamps.

Check. $85 + 15 = 100$

In working a problem by an equation, follow these steps:

(1) *Let x (or any other letter) stand for the unknown number.*

(2) *By studying the facts of the problem, find two expressions that equal the same thing; then write the equality sign (=) between them to form the equation.*

(3) *Solve the equation and check the work.*

THE IMPORTANCE OF THE EQUATION

Before you could form the equation in solving the above problem, you had to find two expressions that represented the same quantity. This method of working a problem differs from the one you would use if you did it by arithmetic. It is very important to learn this new method of solving problems, which is called the *equation method,* since you will use it frequently throughout your study of algebra. The study of equations makes up a very large part of algebra.

You started with the easy problem given above in order to learn the equation method of solving a problem. In the case of this simple problem, you might find the arithmetic method easier; but you will soon discover that many problems can be solved much more easily by equations than by arithmetic.

SOLVING PROBLEMS BY EQUATIONS

1. George sold 235 stamps from his large collection and then had 2250 stamps left. How many stamps did he have at first?

2. If Katherine saves \$23 more, she will then have \$50. How many dollars has she now?

3. Albert sold a dog for \$18, which was \$7 less than the dog cost. How much did the dog cost?

4. Pauline lives in Watertown, which has 3 times as large a population as Belleville. If the population of Watertown is 27,000, what is the population of Belleville?

Let x = the number of people that live in Belleville.

5. Mary's collection of sea shells is $\frac{1}{4}$ the size of Jane's collection. If Mary has 63 shells, how many has Jane?

Let x = the number of shells in Jane's collection.

6. Raymond has \$17.50 more than Elizabeth. If Raymond has \$26.00, how much has Elizabeth?

7. If Roy earns \$8 a week, how much can he earn in x weeks? How many weeks must he work to earn \$96? In working this problem by an equation, what two expressions represent Roy's total earnings in x weeks?

8. If Mr. McCall drives his car an average of 24 mi. an hour, how far does he drive in x hours? At that rate how many hours will it take him to drive 132 mi.?

9. Jules spent \$2.48 for a book and then found that he had \$7.52 left. How much did he have before buying the book?

10. Stephen cut a board 10 ft. long into 4 equal pieces for shelves for a bookcase. How long was each shelf?

Let x = the length in feet of each shelf.

11. Bert cut a board x feet long into 5 equal pieces for shelves for his bookcase. If each shelf was 3 ft. long, how long was the board before he cut it? In making the equation, what two expressions represent the same thing?

ADDING *X'S* IN SOLVING AN EQUATION

In solving the equation $3x + x = 28$, you find that x appears twice; hence you begin by adding the x's. When x is written alone, it means $1x$; hence $3x + x$ means $4x$. Therefore the equation becomes $4x = 28$. What must you do to both sides of $4x = 28$ to find the value of x?

$$3x + x = 28$$
$$4x = 28$$
$$x = 7$$
Check. $21 + 7 = 28$

Exercises

Write each of the following in another way:

1. $2x + 3x$ **4.** $x + 2x$ **7.** $x + x$ **10.** $2x + \frac{1}{2}x$

2. $7x - 4x$ **5.** $4b - b$ **8.** $x - x$ **11.** $\frac{2}{3}x - \frac{1}{3}x$

3. $3r - \frac{1}{2}r$ **6.** $2c - c$ **9.** $m + m$ **12.** $\frac{1}{2}x + \frac{1}{2}x$

Solve the following and check the work:

13. $x + 4x = 20$ **21.** $n + n = 30$ **29.** $x - \frac{1}{2}x = 10$

14. $2x + x = 18$ **22.** $6x - x = 5$ **30.** $b - \frac{3}{4}b = 12$

15. $p + 5p = 12$ **23.** $9x - x = 28$ **31.** $c - \frac{2}{3}c = 11$

16. $6m + m = 56$ **24.** $2x - x = 17$ **32.** $\frac{1}{2}x + \frac{1}{2}x = 4$

17. $3r + 5r = 20$ **25.** $4x - 2x = 8$ **33.** $\frac{1}{3}q + \frac{2}{3}q = 9$

18. $7x - 2x = 15$ **26.** $7x - 5x = 2$ **34.** $\frac{1}{4}r + \frac{3}{4}r = 5$

19. $5x + 5x = 40$ **27.** $2\frac{1}{2}b - \frac{1}{2}b = 6$ **35.** $\frac{2}{3}d - \frac{1}{3}d = 50$

20. $8a - 2a = 24$ **28.** $1\frac{1}{2}c + \frac{1}{2}c = 5$ **36.** $2\frac{1}{2}m + \frac{1}{2}m = 6$

37. Ed said, "The number I am thinking of, plus 3 times the same number, gives 8." Of what number was he thinking?

Show that the equation is $x + 3x = 8$. What does x represent?

38. Ellen said, "Five times the number I am thinking of minus 3 times the same number equals 10." Find the number.

39. Mary asked Susan how old she was. Susan replied, "If you add 4 times my age to my age, the result is 65." How did Mary find out Susan's age?

USING MORE THAN ONE PRINCIPLE

In solving an equation, you must sometimes use more than one of the principles you have learned, as shown below:

Given:	$5x - 7 = 38$	Given:	$\frac{3x}{2} = 15$
Add 7:	$5x = 45$	Multiply by 2:	$3x = 30$
Divide by 5:	$x = 9$	Divide by 3:	$x = 10$
Check.	$45 - 7 = 38$	*Check.*	$\frac{30}{2} = 15$

When more than one principle is used, first do the addition or subtraction, then the multiplication, if any, and then the division.

Exercises

What two things must you do in each example to have only x left?

1. $2x + 1$ **3.** $3x - 4$ **5.** $x + 3x$ **7.** $\frac{1}{2}x - 6$

2. $\frac{x}{2} + 7$ **4.** $\frac{3x}{4}$ **6.** $\frac{5x}{8}$ **8.** $\frac{175x}{100}$

9. To solve the equation $\frac{3}{8}x = 15$, first write $\frac{3}{8}x$ as $\frac{3x}{8}$.

In solving an equation, how would you write $\frac{2}{3}x$? $\frac{7}{8}x$?

10. To solve $1\frac{1}{2}x = 12$, first write $1\frac{1}{2}x$ as $\frac{3}{2}x$. In solving an equation, how would you write $1\frac{2}{3}x$? $4\frac{1}{2}x$? $2\frac{1}{4}x$? $1\frac{3}{4}x$?

Solve these equations and check your results:

11. $\frac{3x}{4} = 90$ **18.** $\frac{2x}{3} = 18$ **25.** $\frac{80x}{100} = \$16.80$

12. $4x + 9 = 29$ **19.** $\frac{3}{4}x = 15$ **26.** $\frac{52}{100}x = \$20.80$

13. $5x + 8 = 48$ **20.** $\frac{2}{3}x = 12$ **27.** $\frac{19}{100}x = \$13.30$

14. $8x - 30 = 2$ **21.** $\frac{3}{4}x = 27$ **28.** $2 + \frac{1}{4}x = 122$

15. $4x - 12 = 20$ **22.** $\frac{5}{6}x = 35$ **29.** $x + \frac{1}{2}x = 210$

16. $24 + 5x = 44$ **23.** $\frac{15}{100}x = 75$ **30.** $\frac{124}{100}x + 8 = 256$

17. $18 + 3x = 51$ **24.** $\frac{1}{5}x + 3 = 5$ **31.** $100x + 20x = 60$

USING EQUATIONS IN PROBLEMS

1. Jack was asked how old he was. He said, "Multiply my age by 7, add 9, and you get 100." How old was he?

2. Suppose that Jack (ex. 1) had said, "Multiply my age by 10, add 20, and you get 150." What would his age have been then? Is this the same age as you found in ex. 1?

3. Agnes said, "I am thinking of a number. If I multiply this number by 5 and subtract 5 from the product, the result is 5." Find the number.

4. If 4 is subtracted from 4 times a certain number, the result is 4. Find the number.

5. Mr. Morris is now earning $2700 a year. He says that this is $500 more than twice as much as he earned 15 yr. ago. How much did he earn 15 yr. ago?

6. Fred said to his Uncle Nat, "I can guess your age if you will do what I tell you." Then Fred gave him these directions: "Write 4 times your age on a piece of paper, but do not tell me what you have written; from this number subtract 10; then subtract twice your age and tell me only the result." Uncle Nat said the result was 60. Fred mentally added 10 to 60 and took half of the result, which gave 35. He then told Uncle Nat that he was 35 years old. Was Fred right?

7. Ask one of your friends to follow the same directions that Fred gave to Uncle Nat. Add 10 to the number he gives you and take half of the result. Does this give your friend's correct age? Does this method always work?

8. In ex. 6, let x stand for Uncle Nat's age. Then form an equation from the facts given and solve it for x. Does this explain why Fred's rule for guessing ages works?

9. Mrs. Adams said, "The price of sugar per pound is now 8¢, which is 2¢ less than half the price I paid for it in 1919." How much per pound did she pay for sugar in 1919?

HOW MUCH IS EACH BOY'S SHARE?

Jack and his brother Bert take care of Mr. Howe's tennis court. Mr. Howe pays them \$15, of which Jack is to get twice as much as Bert. How much does each boy get?

Here you have to find two quantities, Bert's share and Jack's share. Jack's share is twice as great as Bert's.

Let	x = Bert's share, in dollars
Then	$2x$ = Jack's share, in dollars
Both together get \$15:	$x + 2x = 15$
Add the x's:	$3x = 15$
Divide by 3:	$x = 5$

Since $x = 5$, $2x = 10$. Hence Bert gets \$5 and Jack gets \$10.

Check. $5 + 10 = 15$

When two numbers are to be found in a problem, let x represent one of them and express the other in terms of x.

Problems

1. Jane bought a tennis racket for x dollars and a sweater for twice as much. How can you express the cost of the sweater? the cost of both?

2. If the sweater and tennis racket together (ex. 1) cost \$12, form an equation and find the cost of each.

3. A book and pencil together cost \$1.40. If the book cost 13 times as much as the pencil, find the cost of each.

4. A house and lot together cost \$48,000. The house cost \$18,000 more than the lot. Find the cost of each.

5. To make cocoa, Ada uses twice as much milk as water. How many cups of each does she use for 9 cups of cocoa?

6. Alice divides 5 yd. of silk into two pieces, one of which is 4 times as long as the other. How long is each piece?

7. Together Mary and Jane have \$172. Mary has \$4 more than 3 times as much as Jane. How much has each?

HOW OLD IS JOHN?

John said to Ruth, "Add my age now to my age 4 yr. from now and you get 30." How can Ruth figure out John's age?

Let	x = John's present age, in years
Then	$x + 4$ = his age 4 yr. from now
The equation is	$x + x + 4 = 30$
Add the x's:	$2x + 4 = 30$
Subtract 4:	$2x = 26$
Divide by 2:	$x = 13$

Since $x = 13$, you see that John's present age is 13 yr.

Check. $13 + 13 + 4 = 30$

Age Puzzles

1. Martha is now 12 yr. old. How old will she be 5 yr. from now? x years from now? How old was she x years ago?

2. Julia is now z years old. Express her age 2 yr. ago.

3. Betty is 3 times as old as Frank, who is x years old. Express Betty's age. Express the sum of their ages. If the sum of their ages is 16, how old is Frank? How old is Betty?

4. Tom said to Ann, "If I were 7 yr. younger than my present age, I should be 8 yr. old." How old is Tom?

5. Henry's age is 4 times Susan's. The sum of their ages is 30 yr. How old is each?

6. Edna says that her age is $\frac{1}{4}$ of her mother's, and that their ages together total 40 yr. Find how old each is.

7. Harry's age is 3 yr. less than 3 times Joe's age. Their ages total 29 yr. Find how old they are.

8. Hugh is now twice as old as Lena. Two years ago the sum of their ages was 17 yr. How old is each of them now?

9. Mark and Molly are twins. If Mark were 4 times as old as he is and Molly were 4 yr. older than she is, the sum of their ages would be 59. Find how old they are.

IMPROVEMENT TEST No. 2

Test 2 A — Addition (Time 3 min.)

1.	2.	3.	4.
296.41	906.32	21.312	95.826
373.85	84.98	39.595	9.712
69.92	424.76	68.471	31.482
847.26	630.33	5.046	97.678
53.18	575.65	16.717	73.981
408.95	25.44	25.877	46.789
218.64	894.19	35.835	2.028

Test 2 B — Subtraction (Time 3 min.)

1.	2.	3.	4.
2684.51	69.7348	7.00000	111.300
1132.84	8.9364	2.53691	2.243
5.	**6.**	**7.**	**8.**
866.315	58.2842	522.621	7369.80
141.467	5.2667	55.273	684.48
9.	**10.**	**11.**	**12.**
76309.7	5129.23	81.6731	877.654
58524.9	2741.89	19.0475	18.459

Test 2 C — Multiplication (Time 3 min. after copying)

1.	2.	3.	4.
600	.051	5.47	549
.59	18	0.6	.900
5.	**6.**	**7.**	**8.**
863	.009	1.39	.018
.98	.07	.65	500
9.	**10.**	**11.**	**12.**
2.74	.906	62	8.07
17	12	.001	4000

Test 2 D — Division (Time 3 min. after copying)

Find each quotient correct to the nearest hundredth:

1. $5.8\overline{)3.909}$ **2.** $42\overline{)199.9}$ **3.** $.77\overline{)70.8}$

REVIEW OF EQUATIONS

1. The profits of Mr. Johnson's grocery store were \$8200 last year. This was \$200 less than 4 times the profits of the store 5 yr. ago. Find the profits 5 yr. ago.

2. There are 261 pupils in the Whittier School, which has $\frac{9}{10}$ as many pupils as the Leonard School. Find the number of pupils in the Leonard School.

3. How would you divide \$9.60 between two children so that one child gets twice as much as the other?

4. Our class is raising money for a set of books that costs \$36. Mr. Davis has promised to contribute 3 times as much as the class contributes. How much must the class give in order to have the entire contribution equal the cost of the books?

5. Tom, Joe, and Roy together bought a boat for \$45. Toward this amount Tom paid $\frac{1}{2}$ as much as Roy and Joe paid $\frac{3}{4}$ as much as Roy. How many dollars did each boy pay?

6. Helen was asked how much money she had. She replied, "If you will give me \$2, I will then have 3 times as much as I have now." How much has she now?

Solve these equations and check your results:

7. $\frac{5x}{6} = \$.35$

8. $\frac{3}{2}x = 12$

9. $\frac{3}{4}x = 24$

10. $\frac{2}{3}x = 15$

11. $\frac{3}{10}x = 48$

12. $2\frac{1}{2}x = 10$

13. $1\frac{1}{4}x = 20$

14. $\frac{27}{100}x = 9$

15. $\frac{7x}{10} = \$25.20$

16. $2x + 21 = 31$

17. $12 + \frac{1}{2}x = 13$

18. $4x - 2x = 17$

19. $\frac{1}{8}x - 70 = 22$

20. $\frac{1}{3}x - 15 = 27$

21. $\frac{3}{5}x + 18 = 96$

22. $\frac{1}{2}x + 13 = 18$

23. $\frac{75x}{100} = \$37.50$

24. $x - \frac{1}{4}x = 630$

25. $5x - 110 = 50$

26. $\frac{2}{3}x - 100 = 30$

27. $x + \frac{30}{100}x = 78$

28. $x + \frac{6}{100}x = 53$

29. $x - \frac{15}{100}x = 68$

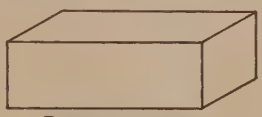
30. $x - \frac{16}{100}x = 63$

CHAPTER IV

MEASUREMENTS

THE VOLUME OF A RECTANGULAR SOLID

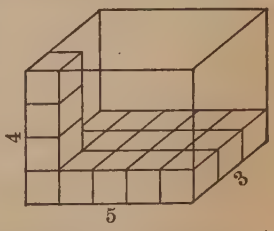
A solid shaped like a box is called a **rectangular solid** or a **rectangular prism,** because its six faces are all rectangles. If its six faces are all squares, it is a **cube.** A cube 1 in. long, 1 in. wide, and 1 in. high contains 1 *cubic inch* and is called a 1-inch cube. What is a cubic foot? a cubic yard?



RECTANGULAR PRISM

CUBE

The **volume** of a solid is the number of cubic units it contains. This picture shows a box 5 in. long, 3 in. wide, and 4 in. high. It would take 5×3, or 15, 1-inch cubes to cover the bottom of the box, and 4×15, or 60, 1-inch cubes to fill the box. Hence the volume of the box is $5 \times 3 \times 4$ cu. in., or 60 cu. in. in all.



Volume of rectangular solid = length × width × height

This rule may be written as a formula, thus:

$$V = lwh$$

In finding the volume (V), first express all dimensions in inches or all in feet or all in yards, etc.

Exercises

Find the volumes of rectangular solids having these dimensions:

1. 7 in. × 5 in. × 3 in.
2. 8 in. × 8 in. × 8 in.
3. 1 ft. 6 in. × 6 in. × 4 in.
4. 3 ft. 6 in. × 2 ft. × 2 ft.

FINDING VOLUMES

Review this table of *cubic measure:*

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.) 27 cubic feet = 1 cubic yard (cu. yd.)

Exercises

1. A cubic foot is how many inches long? how many inches wide? how many inches high? Show that 1 cu. ft. contains 1728 cu. in.

2. Show that 1 cu. yd. contains 27 cu. ft.

3. A schoolroom is 35 ft. by 30 ft. by 12 ft. How many cubic feet of air space does it contain? If 200 cu. ft. of air space are necessary for each pupil, for how many pupils is this room large enough?

4. How many cubic yards of earth, to the nearest whole yard, will be removed in digging a tunnel 50 ft. long, 10 ft. wide, and 9 ft. high? Find the cost at 65¢ per cubic yard.

5. Granite weighs 170 lb. per cubic foot. How much does a granite block $6\frac{1}{2}$ ft. square and 3 ft. high weigh?

6. A swimming pool is 60 ft. long and 24 ft. wide, and is filled to an average depth of 4 ft. 3 in. How many cubic feet of water does it contain? How much does it cost to fill the pool, as above, if the water costs 27¢ per 100 cu. ft.?

7. An aquarium 18 in. long and 12 in. wide is 11 in. high. How many cubic inches of water does it hold when filled to within 4 in. of the top? A cubic foot of water weighs $62\frac{1}{2}$ lb. What is the weight of the water in the aquarium?

8. A box 15 in. long, 12 in. wide, and 12 in. high is filled with cans of cocoa. If each can is 5 in. by 3 in. by 2 in., how many cans does the box contain?

MEASURING LIQUIDS

Memorize these useful equivalents in *liquid measure:*

1 gal. = 231 cu. in.
$31\frac{1}{2}$ gal. = 1 barrel
$7\frac{1}{2}$ gal. = about 1 cu. ft.

Exercises

1. Bob wants a can that will hold 3 gal. of oil. He finds one can that measures 7 in.×10 in.×6 in., another that is 8 in.×12 in.×$6\frac{1}{2}$ in., and a third that is 9 in.×11 in.×7 in. Which can should he take?

In ex. 1 to 3, use 1 gal. = 231 cu. in.; in ex. 4 and 5, use $7\frac{1}{2}$ gal. = 1 cu. ft.

2. Will a rectangular can that measures $3\frac{1}{2}$ in. by $2\frac{3}{4}$ in. by 4 in. hold more or less than a pint of olive oil? how many cubic inches more or less?

3. Ann has an aquarium 28 in. long, 18 in. wide, and 13 in. high. How many gallons of water will it hold when filled to within 2 in. of the top?

4. A swimming pool 90 ft. long and 35 ft. wide is filled with water to an average depth of 6 ft. How many gallons of water does this pool hold?

5. During a storm $\frac{3}{4}$ in. of rain fell. How many cubic feet of rain fell on a flat roof 48 ft. by 35 ft.? If all this rain ran off the roof into a cistern, how many gallons of water went into the cistern? how many barrels of water?

Tell how many gallons these rectangular cans or tanks hold:

6. 4 ft.×2 ft.×5 ft.
7. $1\frac{1}{2}$ ft.×4 ft.×7 ft.
8. $3\frac{1}{2}$ ft.×5 ft.×4 ft.
9. $3\frac{1}{2}$ in.×6 in.×11 in.
10. 6 in.×$8\frac{3}{4}$ in.×11 in.
11. 60 ft.×30 ft.×$5\frac{1}{2}$ ft.

HOW ACCURATELY CAN YOU MEASURE A GALLON?

You will now learn why, for most purposes, it is sufficiently accurate to say that $7\frac{1}{2}$ gal. = 1 cu. ft. instead of using the exact equivalent, which is 1 gal. = 231 cu. in.

1. How many cubic inches are there in 1 gal.? in $7\frac{1}{2}$ gal.? Show that $7\frac{1}{2}$ gal. contain $4\frac{1}{2}$ cu. in. more than 1 cu. ft.

When you say that $7\frac{1}{2}$ gal. = 1 cu. ft., you make an error of $4\frac{1}{2}$ cu. in. to a cubic foot, or $\frac{4.5}{1728}$ of a cubic foot. Dividing both numerator and denominator of this large fraction by 4.5, or $4\frac{1}{2}$, you get $\frac{1}{384}$, which is about equal to $\frac{1}{400}$. An error of $\frac{1}{400}$ cu. ft. is called an error of 1 in 400 and shows how accurately the measurement has been made.

2. Jack has a rectangular can, which measures 6 in. by $5\frac{1}{2}$ in. at the bottom and which is 7 in. high. Show that it will hold exactly 1 gal. of oil if it is filled to the top.

If Jack fills his can to within $\frac{1}{16}$ in. of the top, how many cubic inches less than 1 gal. does it contain? Show that it is short about $\frac{1}{100}$ gal. and that the error is about 1 in 100.

3. Show that Jack makes an error of about 1 in 200 if he fills the can to within $\frac{1}{32}$ in. of the top and that he makes an error of about 1 in 400 if he fills it to within $\frac{1}{64}$ in. of the top.

Now look at your ruler. Are $\frac{1}{32}$ in. and $\frac{1}{64}$ in. very small? Can you measure accurately to $\frac{1}{64}$ in. with your ruler?

4. In the ordinary measuring of liquids, do you think that dealers get as close to an exact gallon as Jack did in ex. 3? Do you think the error in measuring liquids is generally nearer to 1 in 100 or to 1 in 400? Is an error of 1 in 100 a larger or a smaller error than one of 1 in 400?

In using $7\frac{1}{2}$ *gal.* = 1 *cu. ft., you have an error of only* 1 *in* 400, *while most measurements of liquids in everyday life give an error greater than* 1 *in* 400.

MEASURING BINS

Memorize these useful equivalents in *dry measure:*

1 bu. = about 2150 cu. in.
1 bu. = about $1\frac{1}{4}$ cu. ft.
.8 bu. = about 1 cu. ft.
1 T. hard coal = about 35 cu. ft.

A bushel contains exactly 2150.42 cu. in. This exact value is not used in everyday measurements because it is impossible to measure so accurately with ordinary instruments. In finding the capacities of small boxes, you may let 2150 cu. in. = 1 bu.; but in measuring bins, it is more practical to use $1\frac{1}{4}$ cu. ft. = 1 bu. or 1 cu. ft. = .8 bu.

Exercises

1. If you know that $1\frac{1}{4}$ cu. ft. = 1 bu., how can you show that 1 cu. ft. = $\frac{4}{5}$ bu., or .8 bu.?

2. A corn bin 10 ft. long, 8 ft. wide, and 5 ft. deep will hold how many bushels of corn when filled to a height of $2\frac{1}{2}$ ft.? when filled to a height of 4 ft. 3 in.?

3. Mr. Dane's coal bin is 7 ft. wide and $9\frac{1}{2}$ ft. long. How many tons of hard coal does it contain when filled to a depth of 5 ft.? when filled to a depth of 6 ft.?

4. Mr. Ford is building a bin 6 ft. long and 5 ft. wide. He wants it to hold 120 bu. of feed. How many feet high should he make the bin?

Find the capacity of each of these bins in bushels:

5. $10' \times 8\frac{1}{2}' \times 6'$
6. $12' \times 8\frac{1}{4}' \times 5'$
7. $7\frac{3}{4}' \times 7\frac{1}{2}' \times 4'$
8. $9\frac{1}{3}' \times 7\frac{1}{2}' \times 6'$
9. $9\frac{1}{2}' \times 9' \times 5'$
10. $6\frac{3}{4}' \times 5' \times 3\frac{1}{2}'$
11. $7' \times 6\frac{1}{4}' \times 4\frac{1}{2}'$
12. $10\frac{1}{2}' \times 8\frac{2}{3}' \times 5'$
13. $6.5' \times 4.5' \times 5'$
14. $3.5' \times 1.5' \times 4'$
15. $6' \times 2.6' \times 6.5'$
16. $7' \times 5.7' \times 8.5'$

SOME DIFFICULTIES IN MEASURING GRAIN

In these problems, consider 2150 *cu. in. as* 1 *exact bushel.*

1. How many cubic inches are there in $1\frac{1}{4}$ cu. ft.? In using $1\frac{1}{4}$ cu. ft. for 1 bu., there is an error of about how many cubic inches in each bushel? Is this an error of about 1 in 100 or 1 in 200 or 1 in 400?

Now let us see whether the error in the ordinary measuring of grain is as small as 1 in 200.

2. Paul has a box that measures, on the inside, $12\frac{1}{2}'' \times 16''$ on the bottom, and $10\frac{3}{4}''$ in height. What is its volume in cubic inches? You see that this box holds just 1 bushel.

If Paul fills this box with grain to within $\frac{1}{4}$ in. of the top, what is the volume of the part of the box that is not yet filled? How many cubic inches less than a full bushel has Paul? Is his error about 1 in 40 or 1 in 50 or 1 in 100?

3. If Paul fills the box to within $\frac{1}{8}$ in. of the top, the exact error is 1 in how many? If he fills it to within $\frac{1}{16}$ in. of the top, is the error greater or less than 1 in 200?

4. If Paul fills the box to the top and then shakes it, so that the grain settles $\frac{3}{16}$ in. below the top, is the error considerably more or less than 1 in 200? What error did you find in ex. 1?

Since the error in measuring grain is seldom less than 1 *in* 200, *it is sufficiently accurate to let* $1\frac{1}{4}$ *cu. ft.* = 1 *bu.*

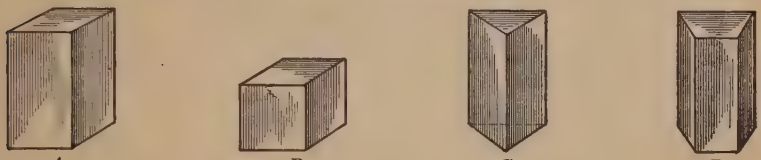
5. A farmer has boxes with the following inside measurements. Tell which box holds nearest to an exact bushel:

(*a*) $15'' \times 16'' \times 9''$
(*b*) $16'' \times 16'' \times 8\frac{3}{8}''$
(*c*) $14'' \times 14'' \times 11''$
(*d*) 14 in. $\times$ 15 in. $\times 10\frac{1}{4}$ in.
(*e*) $13\frac{1}{2}$ in. $\times 13\frac{1}{4}$ in. $\times$ 12 in.
(*f*) 14 in. $\times$ 14 in. $\times 10\frac{31}{32}$ in.

6 Does a box measuring 10 in. $\times$ 10 in. $\times 10\frac{3}{4}$ in. or one measuring $10\frac{1}{2}$ in. $\times 10\frac{1}{2}$ in. $\times 9\frac{3}{4}$ in. hold nearer to an exact half bushel? The dimensions given are inside measurements.

HOW TO FIND THE VOLUME OF A PRISM

You learned on page 51 that a solid like A is called a rectangular prism. In a **prism** the upper and lower bases are parallel, and the sides are all rectangles.



A *B* *C* *D*

If a prism has squares for all its bases and sides, as in B, it is called a **cube.** If its bases are triangles, as in C, it is called a **triangular prism.** A prism may also have bases of other shapes, as in D. In each of the above prisms the *height is perpendicular to the base.*

On page 51 you learned this rule:

Volume of rectangular prism = length × width × height

In prism A above, you see that *the length × the width* is the same as *the area of the base.* Hence the rule becomes:

Volume of prism = area of base × height

If you let B stand for the *area of the base,* the above rule may be written as a formula, thus:

$$V = Bh$$

When you study geometry, you will learn that this formula may be used to find the volume of any prism, whether its base is a rectangle, a triangle, a trapezoid, or a figure of some other shape.

If the base of the triangular prism shown at the right has an area of 5 sq. in. and the height of the prism is 4 in., then, by the formula, the volume of the prism is 5×4, or 20, cu. in.

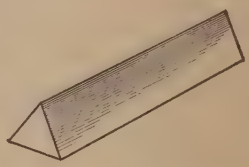
FINDING VOLUMES OF PRISMS

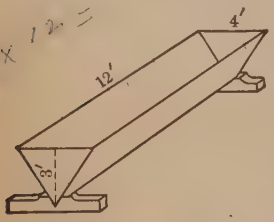
1. Find the volume of a rectangular prism whose base has an area of 28 sq. in. and whose height is 7 in.

2. Find the volume of a triangular prism whose base has an area of $4\frac{1}{2}$ sq. ft. and whose height is 6 ft.

3. Mary has a glass prism 7 in. high. Its base has an area of $1\frac{1}{2}$ sq. in. Find its volume.

4. When a prism is lying on its side, as shown here, think of the ends as the bases and of the length of the prism as its height. What is the volume of this prism if it is 14 in. long and if its end has an area of 2 sq. in.?

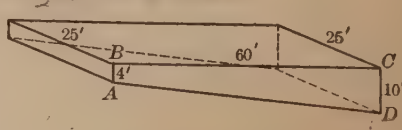




5. In the water trough shown at the left, first find the area of the triangle which forms the end (base); then find the volume of the trough.

6. In ex. 5, find how many gallons of water the trough will hold, using 1 cu. ft. = $7\frac{1}{2}$ gal.

7. The swimming tank shown at the right is 60 ft. long and 25 ft. wide. It is 10 ft. deep at one end and 4 ft. deep at the other end. Find its volume in cubic feet. How many gallons of water will it hold when filled to the very top?



Think of the trapezoid *ABCD*, whose height is 60 ft., as the base of the prism and find its area. Then 25 ft. is the height of the prism.

8. In ex. 7, how many gallons of water will the tank hold when filled to within 1 ft. of the top? How much will the water cost at 32¢ per 100 cu. ft.?

FINDING THE VOLUME OF A CYLINDER

This tin can is a cylinder. Peaches, tomatoes, and corn are usually canned in cylindrical cans. The bases of a cylinder are circles.

You know that the volume of a prism equals the area of its base multiplied by its height. You may use the same rule for the volume of a cylinder:

Volume of cylinder = area of base × height

Since the base is a circle, its area $= \pi r^2$. The rule above may therefore be written as a formula, thus:

$$V = \pi r^2 h$$

Suppose that the above can is 5 in. high and its base has a diameter of 4 in., or a radius of 2 in. To find the volume (V) of the can, substitute these values in the formula, as follows:

$$V = 3\tfrac{1}{7} \times 2 \times 2 \times 5 = 62\tfrac{6}{7}$$

Hence the volume of the can is $62\frac{6}{7}$ cu. in.

Exercises

Find the volumes of these cylinders, letting $\pi = 3\frac{1}{7}$:

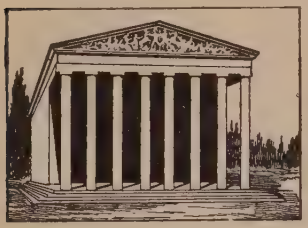
	Area of Base	Height		Radius of Base	Height
1.	8 sq. in.	4 in.	**5.**	2 ft.	6 ft.
2.	7 sq. in.	6 in.	**6.**	3 in.	8 in.
3.	9 sq. ft.	$5\frac{1}{2}$ ft.	**7.**	$1\frac{1}{2}$ in.	4 in.
4.	5 sq. ft.	$2\frac{1}{2}$ ft.	**8.**	7 in.	45 in.

9. A can of chicken soup is 4 in. high. It has a diameter of 3 in. Find its volume in cubic inches.

10. A cylindrical gasoline tank is 7 ft. in diameter and 5 ft. high. Find its volume. How many gallons of gasoline, to the nearest whole gallon, will it hold?

FINDING VOLUMES OF CYLINDERS

In these problems, let $\pi = 3\frac{1}{7}$.



1. The stone pillars in front of a building are cylinders 35 ft. high and 3 ft. in diameter. How many cubic feet of stone does each pillar contain? What is the weight of each pillar if the stone weighs 140 lb. per cubic foot?

2. A cylindrical pail is 4 in. in radius and 11 in. high. What is its volume? How many gallons of water (to the nearest tenth of a gallon) will it hold?

Since the volume is found in cubic inches, use 1 gal. = 231 cu. in. If the volume were found in cubic feet, as in ex. 5 below, you would use 1 cu. ft. = $7\frac{1}{2}$ gal.

3. How many gallons of water will a cylindrical hot-water tank hold if it is 60 in. long and 14 in. in diameter? if it is 70 in. long and 16 in. in diameter?

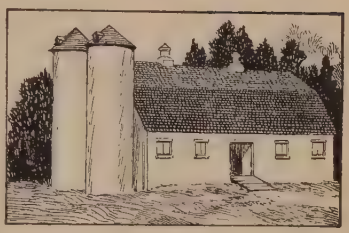
4. A cylindrical gas tank is 90 ft. in diameter and 112 ft. high. How many cubic feet of gas can it hold? Find the value of this gas at \$1.25 per 1000 cu. ft.

5. How many gallons of water will a cylindrical cistern 6 ft. in diameter contain if it is filled to a depth of 10 ft. 6 in.? (First express all dimensions as feet.)

6. A street-sprinkling wagon has a cylindrical tank 4 ft. in diameter and 10 ft. long. Find its volume. How many gallons of water will it hold?

7. A cylindrical gasoline can has a diameter of 7 in. It is 18 in. high. Find out how many gallons of gasoline this can will hold.

CANNING FOOD FOR COWS



Every modern dairy farm has a building, called a *silo*, in which finely cut green corn or other green food is tightly packed to preserve it. This food, called *silage* or *ensilage*, is fed to the cows during the winter in place of grass and other fresh foods. To give it greater strength and capacity and to avoid corners where the silage is apt to spoil, the silo is usually built in the form of a tall cylinder.

Finding the Capacity of Silos

Find answers to the nearest whole number. Let $\pi = 3\frac{1}{7}$.

1. The inside diameter of a silo is 10 ft. Find its volume if its height is 20 ft.; 25 ft.; 30 ft.; 35 ft.

2. Find the volume of the silage in a silo 12 ft. in diameter and filled to a height of 25 ft.; of 30 ft.; of 40 ft.

3. Find how many cubic feet of silage there are in a silo 14 ft. in diameter when the depth of the silage is 30 ft.; when the depth is 35 ft.; 40 ft.; in a silo 16 ft. in diameter when the depth is 36 ft.; when the depth is 38 ft.; 42 ft.

4. Mr. Fain has a silo 16 ft. in diameter and 45 ft. high. It is entirely filled with silage. How many cows can he feed for 180 da. if each cow needs 1 cu. ft. of silage per day?

5. Mr. Jones has a silo 18 ft. in diameter and 40 ft. high. How many days' food will it hold for 64 cows, allowing 1 cu. ft. a day for each cow?

6. A farmer builds two silos, each 10 ft. in diameter and 30 ft. high, in place of one silo 14 ft. in diameter and 30 ft. high. How much does this increase his silo volume?

KEEPING SILAGE FROM SPOILING

Find all answers to the nearest whole number.

To prevent silage from spoiling, it is necessary to remove from the top an average of 3 in. a day.

1. How many cubic feet of silage are there in a 3-inch layer from the top of a silo 14 ft. in diameter? Allowing 1 cu. ft. of silage to each cow, how many cows would it take to use such a layer every day?

2. Would 30 cows, each eating 1 cu. ft. a day, be able to eat a 3-inch layer of silage daily from a silo 12 ft. in diameter? Would 20 cows be able to do this?

3. Mr. Lee feeds his 20 cows each 1 cu. ft. of silage a day. Is a silo 10 ft. in diameter or one 20 ft. in diameter more economical for him to build?

Other Problems

1. If silage averages 40 lb. per cubic foot, how many acres of corn, yielding 15 tons of silage per acre, are needed to fill a silo 12 ft. in diameter and 40 ft. high?

2. A farm bulletin gives the following daily silage rations: dairy cows, 40 lb.; calves, 20 lb.; sheep, 4 lb. How many tons of silage are needed to feed 18 dairy cows for 220 da., 10 calves for 180 da., and 60 sheep for 180 da.?

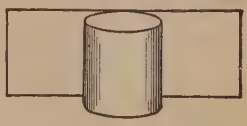
3. A farmer uses the upper 14 ft. of a concrete silo 16 ft. in diameter for a water tank, putting a concrete floor under the tank. How many gallons of water will this tank hold?

4. A coal company constructs 6 silo-like buildings, each 18 ft. in diameter and 40 ft. high, for holding coal. Find the total volume in cubic feet. If 1 cu. ft. of hard coal weighs 57 lb., how many tons of coal will the 6 buildings hold?

5. Two cylindrical cans have the same height, but the diameter of one is twice that of the other. Compare the volumes of these two cans.

FINDING THE AREA OF A CYLINDRICAL SURFACE

The entire surface of a cylinder is made up of the curved surface plus the areas of the two bases, which are circles. You will now learn how to find the area of the curved surface.



If the label on a can of corn exactly covers the curved surface, its area equals that of the curved surface. By moistening the label, you can remove it and spread it out flat to form a rectangle. The height of this rectangle equals the height of the can and its base equals the circumference of the can. Since the curved surface of the can is equal in area to the rectangle, you have this rule:

Area of curved surface = circumference of base × height

Letting S stand for *area of curved surface* and remembering that *circumference* $= 2\pi r$, you get from the rule this formula:

$$S = 2\pi rh$$

Exercises

1. A can of corn 4 in. high has a radius of $1\frac{1}{2}$ in. How large a label is needed to cover the curved surface?

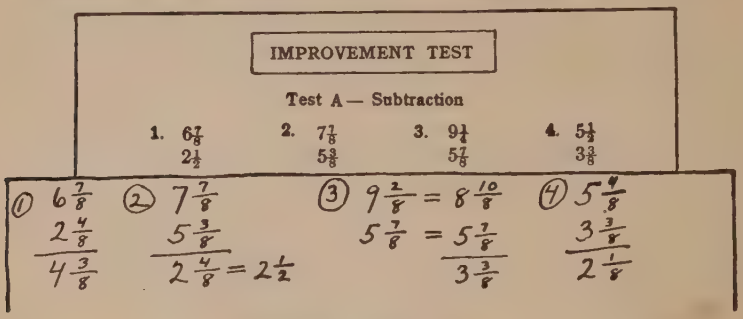
Substituting in the formula, you get $S = (2 \times 3\frac{1}{7} \times 1\frac{1}{2} \times 4)$ sq. in.

2. At 18¢ per square yard, find the cost of painting two smokestacks of a ship if each stack is 9 ft. in diameter and 35 ft. high; if each stack is 12 ft. in diameter and 40 ft. high.

Using the formula, find the curved surfaces of these cylinders:

	Circumference of Base	Height of Cylinder		Radius of Base	Height of Cylinder
3.	15 in.	6 in.	**6.**	7 in.	20 in.
4.	16 ft.	$1\frac{1}{2}$ ft.	**7.**	5 in.	21 in.
5.	65 in.	40 in.	**8.**	6 ft.	25 ft.

TAKING AN IMPROVEMENT TEST IN FRACTIONS



In taking an Improvement Test in the addition or subtraction of common fractions, place the edge of a wide sheet of paper just under the top row of examples. Copy ex. 1 on your paper under ex. 1 of the test; but instead of copying the example exactly as it stands in the test, change the fractions, as you copy, to fractions having a common denominator. When you have worked the first example, slide your paper slightly to the left and copy ex. 2 in the same way. Write the number of each example in a circle, as shown in the illustration above.

If a test has more than one row of examples, fold the top row under on your paper as soon as you finish it. Do the next row along the folded edge of the paper. After finishing the second row, fold your work under again, and so on.

The illustration at the top of this page shows how the first row of examples in a subtraction test in fractions should look on your paper. Study it carefully and tell how each example was changed as it was copied.

A test in the multiplication or division of fractions should be copied as it stands before the test is given.

NOTE. The time allowed for tests in the addition and subtraction of fractions includes the copying of each example in the common denominator form. The time on multiplication and division tests does not include the time required for copying.

IMPROVEMENT TEST No. 3

Test 3 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
$7\frac{5}{16}$	$6\frac{2}{3}$	$2\frac{3}{5}$	$4\frac{1}{4}$	$8\frac{5}{6}$
$9\frac{1}{2}$	$4\frac{1}{6}$	$1\frac{1}{4}$	$\frac{1}{8}$	$3\frac{1}{3}$
$\frac{3}{4}$	$3\frac{5}{6}$	$1\frac{1}{5}$	$7\frac{1}{3}$	$2\frac{3}{4}$
$1\frac{7}{16}$	$8\frac{1}{2}$	$1\frac{3}{10}$	$5\frac{1}{4}$	$9\frac{5}{12}$

Test 3 B — Subtraction (Time 4 min.)

1.	2.	3.	4.
$5\frac{1}{8}$	$4\frac{3}{4}$	$9\frac{3}{16}$	$6\frac{3}{4}$
$2\frac{5}{8}$	$3\frac{1}{2}$	$5\frac{1}{2}$	$2\frac{1}{3}$
5.	**6.**	**7.**	**8.**
4	$7\frac{2}{3}$	$9\frac{2}{5}$	$8\frac{5}{8}$
$1\frac{5}{6}$	$3\frac{1}{2}$	$6\frac{1}{2}$	$5\frac{1}{4}$
9.	**10.**	**11.**	**12.**
6	$6\frac{2}{5}$	$8\frac{5}{6}$	$3\frac{5}{16}$
$2\frac{7}{10}$	$3\frac{1}{3}$	$6\frac{7}{8}$	$2\frac{1}{8}$

Test 3 C — Multiplication (Time 4 min. after copying)

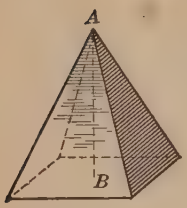
1.	2.	3.
48	\$1.75	\$1.37$\frac{1}{2}$
$7\frac{5}{8}$	$6\frac{3}{4}$	23

4. $\frac{5}{8}$ of 35	**5.** $\frac{15}{16}\times\frac{4}{5}$	**6.** $8\frac{1}{8}\times\frac{4}{15}$
7. $\frac{5}{6}$ of 64	**8.** $\frac{8}{9}$ of $4\frac{5}{16}$	**9.** $2\frac{4}{5}\times5\frac{5}{6}$

Test 3 D — Division (Time 4 min. after copying)

1. $\frac{7}{8}\div 2$	**2.** $18\div\frac{3}{8}$	**3.** $3\frac{1}{2}\div5\frac{1}{4}$
4. $\frac{9}{16}\div\frac{5}{12}$	**5.** $6\frac{3}{8}\div 3$	**6.** $1\frac{2}{3}\div8\frac{1}{3}$
7. $2\frac{1}{2}\div 4$	**8.** $6\div1\frac{4}{5}$	**9.** $12\frac{3}{8}\div\frac{3}{4}$
10. $9\frac{3}{8}\div 6$	**11.** $\frac{15}{16}\div2\frac{1}{12}$	**12.** $13\frac{1}{5}\div3\frac{1}{7}$

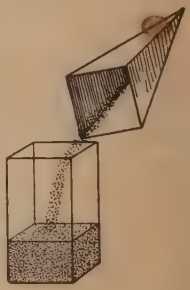
THE VOLUME OF A PYRAMID



The solid shown here is a **pyramid.** The sides of a pyramid are all triangles. Its base may be a triangle, a rectangle, or a figure of some other shape.

The point A is the **vertex.** The line AB, which is perpendicular to the base, shows the **height** of the pyramid.

Take a hollow prism and a pyramid, each having the same base and the same height. Fill the pyramid with sand and pour the sand into the prism. The contents of the pyramid will fill exactly $\frac{1}{3}$ of the prism; hence the volume of the pyramid is $\frac{1}{3}$ of the volume of the prism. But the volume of a prism equals the area of the base times the height; hence



Volume of pyramid $= \frac{1}{3} \times$ *area of base* $\times$ *height,*

or $V = \frac{1}{3} Bh$

Exercises

Find the volumes of the following pyramids:

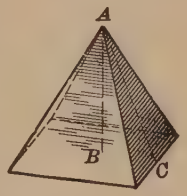
	Area of Base	Height		Area of Base	Height
1.	6 sq. in.	5 in.	**4.**	7 sq. in.	3 in.
2.	10 sq. ft.	4 ft.	**5.**	15 sq. ft.	6 ft.
3.	14 sq. ft.	$4\frac{1}{2}$ ft.	**6.**	20 sq. ft.	$6\frac{1}{2}$ ft

7. The largest Egyptian pyramid, Cheops, is 461 ft. high. Its square base measures 746 ft. on a side. How many cubic feet does this pyramid contain?

8. At the county fair some prize apples are piled in a pyramid 4 ft. high and covering 25 sq. ft. How many bushels of apples are there in the pile if 1 bu. $= 1\frac{1}{4}$ cu. ft.?

FINDING THE LATERAL AREA OF A PYRAMID

The **lateral area** of the pyramid shown in the picture below is the area of its sides. These sides are all triangles of the same size. Hence to find the lateral area of the pyramid, you find the area of one triangle and multiply this by the number of triangles.



To find the area of one triangle, multiply half the height of the triangle by its base. In doing this, notice that the height of the triangle, AC, is *not* the same as the height of the pyramid, AB. AC is called the **slant height** of the pyramid to distinguish it from AB, the *real height* of the pyramid. The height of each triangle is the *slant height* of the pyramid.

Exercises

1. Find the lateral area of a pyramid whose base is a square measuring 8 in. on a side and whose slant height is 12 in.

2. The roof of this garage is a pyramid with 4 equal sides, each measuring 14 ft. in slant height and 20 ft. at the base. How many gallons of paint (to the nearest whole gallon) will it take to paint this roof if 1 gal. covers 200 sq. ft.?

3. If each side of the roof of the garage in ex. 2 had a base of 30 ft. and a slant height of 21 ft., how many gallons of paint would it take to paint it?

4. Last night John dreamed that he was painting the Great Pyramid of Egypt, which has 4 equal sides, each having a slant height of 593 ft. and a base of 746 ft. If its sides were entirely smooth, what would its lateral area be? If John took 1 hr. to paint 100 sq. ft. and worked 8 hr. a day, how many days would he take to paint the Great Pyramid?

THE VOLUME OF A CONE

The solid shown here is a **cone.** An ice-cream cone gets its name from its cone shape.

The base of a cone is a circle. The point A is the **vertex** and the line AB, which is perpendicular to the base, shows the **height** of the cone.

The rule for finding the volume of a cone is just like the rule for the volume of a pyramid:

$$\textit{Volume of cone} = \tfrac{1}{3} \times \textit{area of base} \times \textit{height}$$

The base of a cone, being a circle, has an area of πr^2. Hence you may write the rule as a formula, thus:

$$V = \tfrac{1}{3} \pi r^2 h$$

Exercises

1. A pile of gravel falls naturally into the shape of a cone. If a pile is $4\frac{1}{2}$ ft. high and 12 ft. in diameter, how many cubic yards of gravel does it contain?

2. Edna bought an ice-cream cone, which had a diameter of 2 in. at the top and was $3\frac{1}{2}$ in. deep. If it was exactly filled, how many cubic inches of ice cream did it hold? How many such cones could be filled with 1 gal. of ice cream?

3. How many bushels of grain are there in a conical pile of grain 14 ft. in diameter and $4\frac{1}{2}$ ft. high?

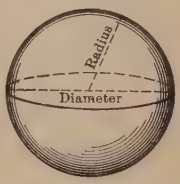
4. A conical pile of coal is 60 ft. in diameter and 21 ft. high. How many cubic feet of coal does it contain? How many tons is that, to the nearest whole ton, if 1 T. = 35 cu. ft.?

Find the volumes of cones with the following dimensions:

	Radius of Base	Height		Radius of Base	Height
5.	6 in.	15 in.	**7.**	28 ft.	45 ft.
6.	21 in.	30 in.	**8.**	70 in.	150 in.

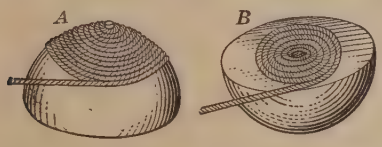
THE AREA OF A SPHERE

The **sphere** is a very common figure. Tennis balls and baseballs are spheres. Can you name other common objects that are spheres?



The radius of a sphere is a line from its center to any point on its surface. The diameter is twice the length of the radius.

Cut a sphere, such as a wooden ball, into halves. Each half is a **hemisphere.** Wind heavy cord around one of the hemispheres, as in *A*, and around the circular base of the other hemisphere, as in *B*.



It takes twice as much cord to cover the hemisphere in *A* as to cover the circle in *B*. The area of the circle in *B* is πr^2. Hence the area of the hemisphere in *A* must be $2\,\pi r^2$ and the area of the whole sphere is $4\,\pi r^2$. Thus you get this formula:

$$A = 4\,\pi r^2$$

Exercises

Find the areas of spheres with the following radii:

1. 7 in. **3.** 5 in. **5.** 21 in. **7.** 35 ft. **9.** 116 in.

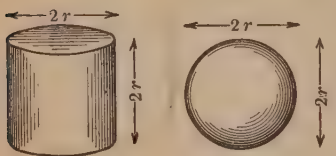
2. $3\frac{1}{2}$ in. **4.** 9 ft. **6.** 14 in. **8.** 42 ft. **10.** 350 in.

11. The earth is almost a perfect sphere. If its radius is about 4000 mi. long, what is the approximate area of the entire surface of the earth?

12. The gilded dome on top of a state capitol is in the shape of a hemisphere with a diameter of 21 ft. What is the total area of the surface of this dome? How much does it cost to gild this dome at 75¢ a square foot?

FINDING THE VOLUME OF A SPHERE

Ted has a wooden cylinder whose base has a diameter of 4 in. and whose height is 4 in. He also has a sphere of the same kind of wood, with a diameter of 4 in. Hence the sphere has the same height as the cylinder.



Ted finds that the sphere weighs 4 oz. and the cylinder, 6 oz. Hence the sphere weighs $\frac{2}{3}$ as much as the cylinder. Since both objects are made of the same kind of wood, the volume of the sphere must be $\frac{2}{3}$ the volume of the cylinder.

The formula for the volume of a cylinder is $V = \pi r^2 h$. In Ted's cylinder, the height equals the diameter; that is, $h = 2r$. Hence the formula for the volume of Ted's cylinder may be written $V = \pi r^2 \times 2r$, or $2\pi r^3$. Remember that $r^3 = r \times r \times r$.

Since the volume of the sphere is $\frac{2}{3}$ the volume of the cylinder, or $\frac{2}{3} \times 2\pi r^3$, you have this formula:

$$V = \tfrac{4}{3}\pi r^3$$

Exercises

1. A solid rubber ball has a diameter of 3 in. How many cubic inches of rubber does it contain?

Since the diameter is 3 in., the radius is $\frac{3}{2}$ in. Substituting in $V = \frac{4}{3}\pi r^3$, you get $V = \frac{4}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$, or $14\frac{1}{7}$. Hence the ball contains $14\frac{1}{7}$ cu. in.

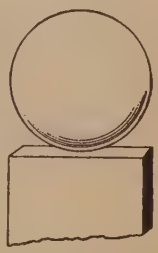
Find the volumes of spheres having these radii:

2. 14 in.	**5.** 21 in.	**8.** 3.5 ft.	**11.** 1 ft.	**14.** $3\frac{1}{2}$ in.
3. 28 in.	**6.** 10 in.	**9.** 4.2 ft.	**12.** $\frac{3}{4}$ in.	**15.** $5\frac{1}{4}$ in.
4. 30 in.	**7.** 12 in.	**10.** 2.8 ft.	**13.** $\frac{7}{8}$ in.	**16.** $4\frac{3}{8}$ in.

17. Find the volume of a croquet ball whose diameter is 4 in.; of a spherical balloon whose diameter is 15 in.

FINDING VOLUMES OF SPHERES

1. A granite sphere 2 ft. in diameter rests on the top of a granite column. How much does this sphere weigh if 1 cu. ft. of granite weighs 168 lb.?



2. How much would a granite sphere 4 ft. in diameter weigh (see ex. 1)? A sphere 4 ft. in diameter is how many times as heavy as a sphere 2 ft. in diameter?

3. A kettle used for boiling maple sirup is a hemisphere 2 ft. in diameter. How many gallons will it hold?

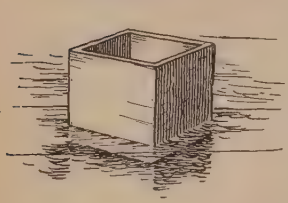
4. A stack of hay is approximately a hemisphere 18 ft. in diameter. How many cubic feet of hay does it contain? If 1 ton of hay occupies 500 cu. ft., about how many tons of hay are there in this stack?

5. What is the volume of an orange whose diameter is $2\frac{1}{2}$ in.? of one whose diameter is $3\frac{1}{4}$ in.? If the oranges are equally juicy, the second orange will give about how many times as much juice as the first?

6. Helen says that the second orange (ex. 5) gives twice as much juice as the first one. Joe says that if the first orange is $2\frac{1}{2}$ in. in diameter, the second orange would have to be 5 in. in diameter to give twice as much juice. Is Joe right?

7. In the lunchroom of a large school, soup is made in a kettle that consists of a cylindrical part 18 in. in diameter and 18 in. deep, with a hemisphere 18 in. in diameter for a bottom. How many gallons does this kettle hold when filled to within 6 in. of the top? when filled to within 1 ft. of the top?

A FLOATING BOX



1. The box at the left weighs a little over 10.4 lb. when out of water. Its base is 1 ft. square and its sides are 1 ft. high. When it floats, its sides sink 2 in. into the water. Show that $\frac{1}{6}$ cu. ft. of the box is under water. What part of a cubic foot of water is displaced by the part of the box under water?

2. Water weighs 62.5 lb. per cubic foot. Show that the water displaced by the above box weighs as much as the box.

3. If you put a stone weighing 5.2 lb. into the box, the box sinks 1 in. further into the water. What volume of water does the box displace then? How many pounds of water does it displace? Does the displaced water weigh as much as the box and the stone together?

The weight of the water displaced by a floating object equals the weight of the object.

Exercises

1. A box whose base is 1 ft. square sinks 4 in. into the water. What part of a cubic foot of water does it displace?

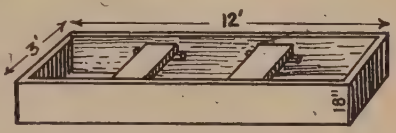
2. What is the weight of the water displaced by the box in ex. 1? What is the weight of the box?

3. When a stone is put into it, the box (ex. 1) sinks 2 in. further into the water. Find the volume and the weight of the water displaced then. What is the weight of the stone?

4. A box whose base is 3 ft. by 4 ft. sinks 1 in. into the water. How many pounds must be put into it to make it sink into the water 6 in.? 1 ft.? 18 in.? 2 ft.? If the box is 30 in. high, how many pounds of load will make it sink completely under water?

HOW MUCH WILL A PUNT CARRY?

1. Harry built a scow like the one shown here. When it floated, its sides sank 2 in. into the water. How heavy was the scow?

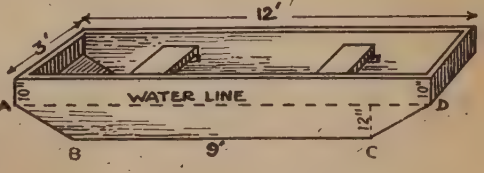


2. When Harry entered the scow, it sank $\frac{1}{2}$ in. further into the water. What was the total weight of the scow and Harry? What was Harry's weight?

3. Four other boys also got into the scow with Harry. Its sides were then in the water to a depth of 4 in. What was the total weight of the scow and the five boys? What was the total weight of the four boys who had just climbed in? (See ex. 1 and 2.)

4. Harry's scow is 18 in. deep. What must be the sum of its weight and the weight of its load to make it sink? How heavy a load will cause it to sink?

5. Roger has a punt of the shape and dimensions shown at the right. It weighs 245 lb. He wants to know how many pounds of load will cause it to sink up to the water line, AD. Can you tell him?



Consider the section below the water line as a solid whose base is the trapezoid $ABCD$ and whose height is the width of the boat, 3 ft. Find the volume of this portion of the boat; then find the weight of an equal volume of water. Next tell what the punt and the load together must weigh to displace that weight of water and what the load alone must weigh. Note that the bases of trapezoid $ABCD$ are 9 ft. and 12 ft.

6. What is the total volume of the punt in ex. 5? How heavy a load would make it sink completely?

FORMULAS AS EQUATIONS

In finding areas and volumes you have employed many formulas. Thus, to find the area of a rectangle you used the formula $A = bh$, and to find the volume of a rectangular solid you used $V = lwh$. Each of these formulas is also an *equation* and, like other equations, it contains both known and unknown numbers. Thus, in the equation $A = bh$, the area (A) is the quantity that you wish to find; hence A is the *unknown number*. Similarly, b and h are both known numbers, since the base and the height are given when you find the area.

In the equation or formula $A = lw$, we sometimes say that A is expressed *in terms of l and w.*

Exercises

1. In the formula for the area of a triangle, $A = \frac{1}{2}bh$, does A represent a known or an unknown quantity? Are b and h known or unknown quantities? Using this formula, find the area of a triangle having a base of 8 ft. and a height of 6 ft.

2. In finding the volume of a rectangular solid by the formula $V = lwh$, which letter stands for the unknown number? Which letters stand for known numbers? Using this formula, find the volume of a box 4 ft. long, 3 ft. wide, and 2 ft. high.

3. In the formula $C = 2\pi r$, which letter represents the unknown number? Which letter is the known number? Is π a known or an unknown number? Using this formula, find the circumference of a circle whose radius is 14 ft., letting $\pi = 3\frac{1}{7}$.

Tell what each of these formulas shows you how to find; then tell which letters represent known numbers and which unknown:

4. $A = \pi r^2$ **6.** $V = \frac{1}{3}Bh$ **8.** $V = \frac{4}{3}\pi r^3$

5. $C = \pi D$ **7.** $V = \pi r^2 h$ **9.** $A = \frac{1}{2}h(a + b)$

10. Tell what the formula $D = .09\,S^2$ shows you how to find (see page 16). In this formula is S known or unknown?

HOW WIDE SHOULD EACH PLOT BE?

The Palmer School is having a vegetable-growing contest. During the summer each pupil is to cultivate a plot containing 120 sq. ft. If Louise selects a plot 20 ft. long, how wide should she make her plot so that it will contain 120 sq. ft.?

To find out, consider the formula $A = lw$ as an equation. By studying the above problem, you find that in this case the *unknown number* is the width (w), while the *known numbers* are the area (A) and the length (l). You find the width thus:

Start with $A = lw$

Substituting 120 for A and 20 for l in this equation, you get

$$120 = 20w$$

To *solve* for w (which means to find the value of w), divide both sides of the equation by 20, which gives

$$6 = w$$

Thus you find that the plot should be 6 ft. wide.

Check the work by multiplying 20 by 6, which gives 120 as the area.

Exercises

In solving the following problems, start with the equation $A = lw$ and substitute the known numbers in it. Then solve the equation for the unknown number, as you did above. Check the work:

1. In the above contest John selects a plot 15 ft. long. How wide should he make his plot so that it will contain 120 sq. ft.?

2. Agnes chooses a narrow strip of land 40 ft. long. How wide should this strip be to contain 120 sq. ft.?

3. Warren selects a plot 16 ft. long. How wide should he make his plot in order that it will contain exactly 120 sq. ft.?

4. Alfred selects a plot of land that is 10 ft. wide. How long should this plot be to contain 120 sq. ft.? If Alfred finds out by substituting in the equation $A = lw$, what letter will represent the unknown quantity?

MAKING NEW FORMULAS FROM OLD ONES

In exercises 1 to 3 on page 75, you found the *width* of each plot by solving an equation. In such cases it would be shorter to use a formula for finding the *width* (w) of a rectangle when its *area* (A) and *length* (l) are known. Such a formula may be made as shown below. Remember that w is the unknown number.

Start with $A = lw$

To solve for w, you must get w by itself on one side of the equation.

Divide both sides by l: $\frac{A}{l} = w$

Turn the equation around: $w = \frac{A}{l}$

Thus you get a new formula, $w = \frac{A}{l}$, which states that *the width of a rectangle equals the area divided by the length.*

Exercises

1. Using the new formula above, find the width of a rectangular garden having an area of 180 sq. ft. and a length of 20 ft.

Substituting 180 for A and 20 for l in the formula above, you get $w = \frac{180}{20}$, or 9. Hence the width is 9 ft. Check by multiplying 20 by 9.

Using the formula above, find the widths of rectangular gardens having the following areas and lengths. Check the work:

2. $A = 340$ sq. ft.; $l = 20$ ft. **4.** $A = 325$ sq. ft.; $l = 20$ ft.

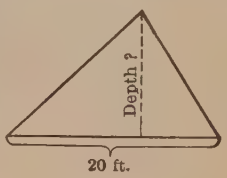
3. $A = 200$ sq. ft.; $l = 16$ ft. **5.** $A = 391$ sq. yd.; $l = 23$ yd.

6. Starting with $A = lw$, make a formula for finding the length of a rectangle when you know the area and the width.

To make a new formula from an old one, start with the old formula and solve it, as an equation, for the letter whose value you wish to find, regarding the other letters as known numbers.

FRANK MAKES A TRIANGULAR GARDEN

In the vegetable-growing contest (page 75), Frank is making his plot in the shape of a triangle. If he makes the base of the triangle 20 ft. long, what depth (or height) shall he make the triangle so that it will contain 120 sq. ft.?



To work this problem, start with the formula for the area of a triangle, as shown below. Remember that h is the unknown number.

The formula for the area is: $A = \frac{bh}{2}$

Substituting 120 for A and 20 for b in the formula, you get

$$120 = \frac{20\,h}{2}$$

Multiply by 2: $240 = 20\,h$

Divide by 20: $12 = h$

Hence the height of the triangle should be 12 ft.
Check the work by showing that $\frac{1}{2} \times 20 \times 12 = 120$.

Exercises

1. Doris is making a triangular garden 54 sq. ft. in area. Its base is 12 ft. long. Find the depth of the triangle, as above.

Find the depths of these triangular plots, as in ex. 1, *and check:*

2. $b = 4$ ft., $A = 10$ sq. ft.

3. $b = 5$ ft., $A = 45$ sq. ft.

4. $b = 18$ ft., $A = 72$ sq. ft.

5. $b = 15$ ft., $A = 45$ sq. ft.

6. $b = 22$ ft., $A = 110$ sq. ft.

7. $b = 15$ ft., $A = 97\frac{1}{2}$ sq. ft.

8. Find the base of a triangle whose area is 750 sq. ft. and whose height is 30 ft.; whose area is 360 sq. ft. and height, 20 ft.; whose area is 5000 sq. ft. and height, 50 ft.

SOME NEW FORMULAS FOR TRIANGLES

If you have to do many problems like examples 1 to 7 on page 77, it is easier to make a new formula for finding the *height* of a triangle when the *area* and the *base* are known. You can do this as follows:

The formula for the area is:	$A = \frac{bh}{2}$
Multiply by 2:	$2A = bh$
Divide by b:	$\frac{2a}{b} = h$
Turn the formula around:	$h = \frac{2A}{b}$

Thus you get a new formula, $h = \frac{2A}{b}$, which states that *the height of a triangle equals twice the area divided by the base.*

Exercises

1. Using the new formula above, find the height of a triangle whose area is 2700 sq. ft. and whose base is 90 ft.

Substituting in $h = \frac{2A}{b}$, you get $h = \frac{5400}{90}$, or 60.

Using the new formula, find the heights of these triangles:

2. $b = 40$ ft.; $A = 800$ sq. ft.

3. $b = 18$ in.; $A = 108$ sq. in.

4. $b = 7$ in.; $A = 17\frac{1}{2}$ sq. in.

5. $b = 9$ yd.; $A = 40\frac{1}{2}$ sq. yd.

6. Make a formula for finding the *base* of a triangle when the *area* and the *height* are known. Using this formula, find the base of a triangle whose area is 195 sq. ft. and height, 15 ft.

Work and check the following, using the proper formulas:

7. $b = 18, h = 24$. Find A.

8. $b = 12, h = 16$. Find A.

9. $b = 7\frac{1}{2}, h = 10$. Find A.

10. $A = 340, b = 8$. Find h.

11. $A = 380, h = 19$. Find b.

12. $A = 26.6, h = 7.6$. Find b.

DETERMINING THE DIMENSIONS OF A BIN

Mr. Day is building a bin to hold his coal for the winter. It is to be 8 ft. long and 7 ft. wide and high enough to contain 280 cu. ft. How high should the bin be?

You must find a formula for the height (h) of a rectangular solid when the volume (V), the length (l), and the width (w) are known.

Start with the formula: $V = lwh$

Divide by lw: $\frac{V}{lw} = h$

Turn the formula around: $h = \frac{V}{lw}$

Substitute: $h = \frac{280}{8 \times 7}$, or 5

Hence the bin should be 5 ft. high. *Check.* $8 \times 7 \times 5 = 280$.

Exercises

1. How high should a coal bin be to hold 5 T. of coal if it is 7 ft. long and 4 ft. wide and 1 T. occupies 35 cu. ft.?

2. From $V = lwh$, make a formula for finding l when V, w, and h are known; for finding w when V, l, and h are known.

Select the proper formula; then work and check the following:

3. Find V when $l = 6$, $w = 6$, and $h = 6$.

4. Find l when $V = 114$, $w = 2$, and $h = 3$.

5. Find w when $V = 920$, $l = 23$, and $h = 8$.

6. Mr. Blake made a tank 4 ft. long and 2 ft. high to hold 120 gal. of oil. If $7\frac{1}{2}$ gal. = 1 cu. ft., how wide was the tank?

Tell what should be put in place of the dots:

7. The height of a bin equals the .... divided by the product of the .... and the .....

8. To solve for D in $C = \pi D$, ... both sides by .....

FINDING THE DIAMETER OF A TREE

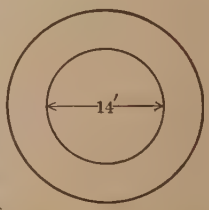
1. When Helen was in California, she saw a very large tree. She could not cut through the tree to find its diameter; but she measured the circumference (C) with a string and found that it was 198 in. Then she derived a formula for the diameter (D) from the formula $C = \pi D$. What formula did Helen make? If $\pi = 3\frac{1}{7}$, what was the diameter of the tree? How can you check the work?

2. Helen measured another tree whose circumference was 88 in. Using Helen's formula for D, find the diameter.

3. Find the diameter of a tree whose circumference is 77 in.; 132 in.; 154 in.; 11 ft. Check each result.

4. State in words the rule for finding the diameter of a tree when the circumference is known.

5. The circular stone tower of an old castle has an outside circumference of 72 ft. Find the diameter of the tower; the radius.



6. A tunnel leads to a circular room in the tower (ex. 5). This room is 14 ft. in diameter. Find its radius. If the difference between the radius of the tower and the radius of the room shows the thickness of the wall of the tower, how thick is the wall?

7. The formula for the volume of a cylinder is $V = \pi r^2 h$. Make a formula for finding h when V and r are known.

8. The radius of a cylindrical gas tank is 35 ft. Using the new formula that you found in ex. 7, find the height of the tank if it contains 115,500 cu. ft. of gas; 142,450 cu. ft.; 238,700 cu. ft.; 82,775 cu. ft. Check the work.

IMPROVEMENT TEST No. 4

Test 4 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
1183	8469	7774	7235	7832
327	7712	2886	558	625
6917	7876	432	3635	8488
9865	8635	3902	2671	6237
2752	631	7489	9783	816
928	7698	4164	2455	2998
1414	5402	410	9596	925
2849	4710	6967	3479	5190
4792	9854	959	932	3875

Test 4 B — Subtraction (Time 3 min.)

1.	2.	3.	4.
904260	467225	846643	778803
128436	123764	269454	289650
5.	**6.**	**7.**	**8.**
731091	750931	451833	570000
586134	164066	282355	348762
9.	**10.**	**11.**	**12.**
351616	828319	825921	976742
218287	471730	586225	678045

Test 4 C — Multiplication (Time 3 min. after copying)

1.	2.	3.	4.	5.
489	648	901	947	735
764	158	437	908	265

Test 4 D — Division (Time 3 min. after copying)

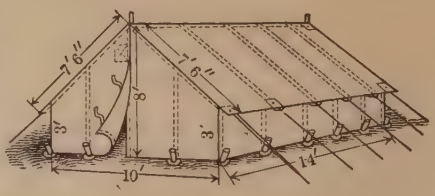
If there is a remainder, find result to the nearest tenth:

1. $23\overline{)14375}$ **2.** $92\overline{)65236}$

3. $78\overline{)32692}$ **4.** $49\overline{)28861}$

MAKING A TENT WATERPROOF

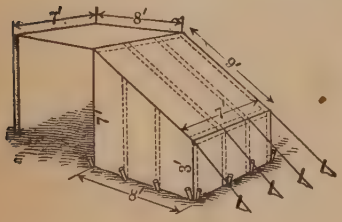
1. Mr. Bruce is going camping. He buys a tent of the shape and dimensions shown at the right. What is the total area in square feet of the outer surface of this tent?



The front of the tent consists of two trapezoids. Is the back the same?

2. Mr. Bruce (ex. 1) applies waterproof liquid to his tent. If 1 qt. of the liquid covers 25 sq. ft., how many quarts of it does he need? At 60¢ a quart, how much does it cost?

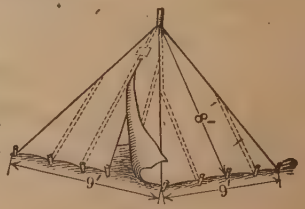
The waterproof liquid is sold only in whole quarts or gallons.



3. Mr. Lee has an automobile tent like the one shown here. How many square feet must he cover with waterproof liquid to make the tent waterproof? Find how many quarts he will need (see ex. 2), and the cost at 60¢ a quart.

4. How much would Mr. Lee (ex. 3) save or lose by buying the waterproofing in gallon cans at $2 a gallon?

5. Bob's tent is shaped like a pyramid. Each of its faces has a base of 9 ft. and a height of 8 ft. How many quart cans of waterproofing would he need to cover it? how many gallon cans?



6. Using the prices in ex. 3 and 4, find how much Bob would save or lose if he bought the waterproofing by the gallon.

CHAPTER V

PERCENTAGE

THREE TYPES OF PROBLEMS IN PERCENTAGE

Last year you used per cents in three types of problems:

1. A bronze statue weighing 50 lb. is 88% copper. How many pounds of copper are there in the statue?

Here you *find a given per cent of a number.*

To find 88% of 50 lb., first change 88% to .88. Then multiply 50 by .88, which gives 44.00, or 44. There are 44 lb. of copper in the statue.

Check the work by going over it again.

2. Mr. Orr earned \$40 last week. He saved \$3. What per cent of his earnings did he save?

Here you *find what per cent one number is of another.*

First find what fractional part \$3 is of \$40. \$3 is $\frac{3}{40}$ of \$40. Changing $\frac{3}{40}$ to a two-place decimal, you get $.07\frac{1}{2}$. Since per cent means hundredths, $.07\frac{1}{2} = 7\frac{1}{2}\%$.

$$40\overline{)3.00} \qquad .07\tfrac{1}{2} = 7\tfrac{1}{2}\%$$
$$\underline{2\,80}$$
$$20$$

Mr. Orr saved $7\frac{1}{2}\%$ of his earnings.

Check by taking $7\frac{1}{2}\%$ of \$40 to see if it equals \$3.

3. Jack receives \$9 for selling a radio. If this is 15% of the price of the radio, find the price of the radio.

Here you *find a number when a per cent of it is given.*

15% of the price = \$9.00
Then 1% of the price = \$9.00 ÷ 15, or \$.60
and 100% of the price = 100×\$.60, or \$60.00

Since 100% of the price is the entire price, the radio costs \$60.

To check, see if 15% of \$60 equals \$9.

A BOYS' SWIMMING CLUB

Check all answers as shown on page 83.

1. Of the 250 boys in the Drake School, 72% are in the Swimming Club. How many of the boys are in this club?

2. Of the boys in the club, 15% are on the swimming team. How many boys are on the team?

3. Last year the team won 15 of the 25 swimming meets they were in. What per cent of the meets did they win?

4. This year the team has 30 meets. How many must they win in order to win the same per cent as last year?

5. The captain has won 172 races in 6 yr. This is 86% of the races he has been in. How many races has he been in?

Other Problems

1. Jim sells brushes and receives as pay 30% of his sales. Find the amount of his sales on a day when his pay is $6.90.

2. An iceberg rises 45 ft. above the surface of the sea. This is 10% of its height. How high is the iceberg?

3. Of the 20 pupils in Joe's class, 17 had perfect attendance during May. What per cent of the class was that?

4. Edna gives 45% of all she earns to her mother. How much does she give her mother in a week when she earns $18.60?

Tell what numbers should be put in place of the dots:

5. 4% of $1700 is . . .
6. 22% of $7.50 is . . .
7. 45% of $8.80 is . . .
8. 16% of $2550 is . . .
9. 85% of $3800 is . . .
10. $24 is . . .% of $80
11. $17 is . . .% of $25
12. 14% of . . . is $28
13. 16% of . . . is $96
14. 30% of . . . is $36
15. 35% of . . . is 105
16. 10% of . . . is 565
17. $9 is . . .% of $40
18. $7 is . . .% of $200

BUYING AT A BARGAIN SALE

Gould's Store had a bargain sale at which the prices of all the goods were reduced. Such a reduction in price is called **discount.** Gould's advertised all hats at 20% discount, all coats at 15% discount, and all dresses at 25% discount.

1. Jane bought a coat that had cost $65. At 15% off, how much was her discount? How much did she pay for the coat?

2. Mrs. Brown bought a dress regularly priced at $24.80. At 25% off, how much discount did she receive? How much did she pay for the dress?

3. Mrs. Kennedy bought two hats that had been marked $3.75 and $4.25 before the sale. What was the amount of discount on each? How much did she save in all by buying the hats at the sale?

4. Mary bought for $4.50 a dress that was marked $6. How much discount did she get? What per cent of the regular price was the discount?

5. Mr. Blake has 450 pairs of shoes that are marked $7.50. He plans to reduce the price to $6 a pair. What per cent discount should he advertise?

6. If you want to offer for $21.60 a coat marked $24, what rate of discount should you advertise?

Using the regular price and the per cent of discount given below, find the amount of discount and the sale price:

7. Stove, $38, 30% off

8. Watch, $12, 10% off

9. Dress, $14, 20% off

10. Canoe, $45, $33\frac{1}{3}$% off

11. Radio set, $50.75, 25% off

12. Lawn mower, $17.85, 10% off

13. Fountain pen, $7.50, 20% off

14. Tennis racket, $5.75, 30% off

15. Vacuum cleaner, $59.60, 25% off

ESTIMATING RESULTS IN PERCENTAGE

Review these *fractional equivalents of per cents:*

$50\% = \frac{1}{2}$	$33\frac{1}{3}\% = \frac{1}{3}$	$12\frac{1}{2}\% = \frac{1}{8}$	$20\% = \frac{1}{5}$
$25\% = \frac{1}{4}$	$66\frac{2}{3}\% = \frac{2}{3}$	$37\frac{1}{2}\% = \frac{3}{8}$	$40\% = \frac{2}{5}$
$75\% = \frac{3}{4}$	$16\frac{2}{3}\% = \frac{1}{6}$	$62\frac{1}{2}\% = \frac{5}{8}$	$60\% = \frac{3}{5}$
$10\% = \frac{1}{10}$	$83\frac{1}{3}\% = \frac{5}{6}$	$87\frac{1}{2}\% = \frac{7}{8}$	$80\% = \frac{4}{5}$

These fractional equivalents are often useful in estimating results. Suppose you are told that 73% of the 200 children in a school were promoted. Since 73% is almost 75% and $75\% = \frac{3}{4}$, you know that almost $\frac{3}{4}$ of the children, or about 150 children, were promoted.

Exercises

Give the approximate fractional equivalent of each:

Thus, 34%, which is a little over $33\frac{1}{3}\%$, is a little over $\frac{1}{3}$.

1. 34%	61%	36%	31%	79%	39%	19%	23%
2. 76%	52%	58%	88%	12%	17%	66%	84%

3. In the Sutton School 16% of the 180 pupils are over 14 yr. of age. About what fractional part of 180 is that? Estimate roughly how many of the 180 pupils are more than 14 yr. old.

4. Only 41% of the 15,000 qualified voters of Bridgewater came to the polls last November. About how many of the voters of Bridgewater came to the polls?

5. Of the 3200 pupils in a high school, 88% were born in America. Estimate how many were born in America. Then find out exactly how many were born here.

Estimate the approximate values; then find the exact values:

6. 52% of 700	**9.** 19% of 455	**12.** 12% of 728
7. 65% of 660	**10.** 23% of 820	**13.** 27% of 940
8. 84% of 240	**11.** 32% of 432	**14.** 81% of 250

WHAT A PROFIT OF 9.7% MEANS

Last year the total sales of the Ives Car Company were \$1,955,000, and 9.7% of these sales was profit. How much was the company's profit last year?

First change 9.7% to a decimal. To do this, write 9.7% as $\frac{9.7}{100}$. Then divide 9.7 by 100 by moving the decimal point two places to the left. This gives .097. Hence 9.7% = .097. To find the company's profit, multiply \$1,955,000 by .097. What is the company's profit?

Check the multiplication by going over it again or by casting out 9's.

To change a per cent to a decimal, drop the sign % and move the decimal point two places to the left.

Thus, 27.3% = .273; 6.45% = .0645; 0.3% = .003.

Finding Profits

1. Mr. Rose averages a profit of 13.5% on his sales. Express 13.5% as a decimal. What is Mr. Rose's profit when he sells \$460 worth of goods? \$380 worth? \$500 worth?

Express these per cents as decimals:

2. 14.6% **4.** 17.4% **6.** 83.25% **8.** 8.2% **10.** 0.4%

3. 36.1% **5.** 99.3% **7.** 62.33% **9.** 3.5% **11.** 0.18%

Find these profits correct to the nearest cent:

12. 15% of \$850 **16.** 3.4% of \$730 **20.** 0.6% of \$1450

13. 1.5% of \$850 **17.** 2.5% of \$775 **21.** 0.8% of \$2050

14. 15.5% of \$850 **18.** 6.3% of \$862 **22.** 50.8% of \$1800

15. 1.55% of \$850 **19.** 5.7% of \$104 **23.** 4.25% of \$6600

24. A candy factory sold \$31,000,000 worth of candy in a year at 4.1% profit. How much profit was that?

25. The United States produces 580,000,000 tons of coal yearly, of which West Virginia produces 15.7%. How many tons of coal does West Virginia produce in a year?

FINDING PER CENTS TO THE NEAREST TENTH

Arthur borrowed \$120 for 1 yr. from Mr. Gordon and paid him \$3 interest. What rate of interest did he pay?

The rate was $\frac{3}{120}$. Changing $\frac{3}{120}$ to a decimal, you get .025. To change .025 to a per cent, move the decimal point two places to the right and annex the sign %, just as you do in changing .46 to 46%. You find that .025 = 2.5%.

Hence Arthur paid interest at the rate of 2.5%.

Check the work by finding 2.5% of \$120 to see if you get \$3.

$$
\begin{array}{r}
.025 = 2.5\% \\
120\overline{)3.000} \\
\underline{2\ 40} \\
600 \\
\underline{600}
\end{array}
$$

To change a decimal to a per cent, move the decimal point two places to the right and annex the sign %.

Thus, .153 = 15.3%; .1825 = 18.25%; and .007 = 0.7%.

Exercises

Change these decimals to per cents:

1. .157	**4.** .755	**7.** .067	**10.** .9675	**13.** .0875
2. .432	**5.** .125	**8.** .095	**11.** .4933	**14.** .0909
3. .625	**6.** .048	**9.** .003	**12.** .6667	**15.** .0625

16. Ed sold an automobile for \$480 and received \$12 commission. What per cent of the selling price was that?

17. The Red Lion team won 18 out of 26 games. Find, to the nearest tenth of 1%, what per cent of the games they won.

Changing $\frac{18}{26}$ to a four-place decimal, you get .6923, or 69.23%. Correct to the nearest tenth of 1%, 69.23% is 69.2%.

Hence the team won 69.2% of the games.

Find per cents in ex. 18 *to* 22 *to the nearest tenth of* 1%.

18. Louise spends \$18 of her salary of \$28 for room and board. What per cent of it does she spend this way?

Find what per cent the first number is of the second:

19. 12, 160	**20.** 43, 75	**21.** 85, 400	**22.** 87, 350

COMPUTING THE STANDING OF BASEBALL CLUBS

Here are the actual records of the number of games won and lost up to a certain date by the teams of the National League. The standing of the teams is found by computing, to the nearest tenth of 1%, what per cent each team has won of all the games it has played.

Team	*Won*	*Lost*	*P. C.*	*P. C.*
Pittsburgh	54	39	.581	58.1
Cincinnati	56	43		
St. Louis	52	44		
Chicago	50	46	↑	↑
Brooklyn	49	49		
New York	46	48	*A*	*B*
Boston	38	57		
Philadelphia	37	56		

1. The Pittsburghs have won 54 games and lost 39 games; hence they have played 93 games in all. To learn their standing, find what per cent 54 is of 93. Carry the result to four decimal places. This gives .5806, which is .581 correct to thousandths, or 58.1% to the nearest tenth of 1%.

```
.5806
93)54.0000
   46 5
   ----
    7 50
    7 44
    ----
     600
     558
     ---
      42
```

The newspapers usually report the standing of the clubs as shown in Column *A* at the top of the page. This method is used only in baseball records and is never found in other statistical reports. The letters *P. C.* above a column mean *per cent;* hence the numbers under *P. C.* in Column *A* should represent per cents. But the rating .581 does not represent .581%, since the Pittsburghs have won 58.1%, and not .581%, of their games. Hence, to be technically correct, the ratings should be written as shown in Column *B*. In Column *A*, where the ratings are reported as three-place decimals like .581, it would be correct if the heading were "Rating" or "Standing" instead of *P. C.*

2. Find, to the nearest tenth of 1%, what per cent of all its games each of the other teams above has won.

3. What would the ratings of the Bostons and of the Philadelphias be if they were found only to the nearest whole per cent? Could you then tell which of these teams is ahead? Then why are the ratings computed to the nearest tenth of 1%?

WHICH TEAM WAS AHEAD?

When the Chicagos had won 50 games and lost 48, the Brooklyns had won 51 games and lost 49. Compute the standing of each team to find which team was ahead.

If the standings of both teams are computed to the nearest tenth of 1%, you find that each team had won 51.0% of its games. Hence apparently the teams were tied. If, however, the computations are carried to another decimal place, you find that the Chicagos had won 51.02% and the Brooklyns, 51.00%. Hence the Chicagos were ahead. When two teams stand the same to the nearest tenth of 1%, it is necessary, in order to determine which team is ahead, to find results to the nearest hundredth of 1%.

Exercises

Find ratings to the nearest tenth of 1%. *If two ratings then seem equal, find them to the nearest hundredth of* 1%:

1. AMERICAN LEAGUE

Team	*Won*	*Lost*	*P. C.*
New York	64	34	
Cleveland	56	44	
Philadelphia	51	47	
Detroit	52	48	
Washington	48	46	
Chicago	50	49	
St. Louis	41	57	
Boston	30	67	

2. AMERICAN LEAGUE

Team	*Won*	*Lost*	*P. C.*
New York	65	34	
Cleveland	57	44	
Philadelphia	51	48	
Detroit	52	49	
Chicago	51	49	
Washington	48	47	
St. Louis	41	58	
Boston	31	67	

3. NATIONAL LEAGUE

Team	*Won*	*Lost*	*P. C.*
Pittsburgh	62	46	
St. Louis	65	50	
Cincinnati	65	51	
New York	58	54	
Chicago	59	55	
Brooklyn	55	62	
Boston	47	67	
Philadelphia	42	68	

4. NATIONAL LEAGUE

Team	*Won*	*Lost*	*P. C.*
Pittsburgh	65	47	
St. Louis	69	50	
Cincinnati	68	51	
Chicago	61	57	
New York	58	58	
Brooklyn	57	64	
Boston	47	70	
Philadelphia	43	71	

FINDING BATTING AVERAGES

1. When Bell of St. Louis had been at bat (*A. B.*) 386 times, he had made 136 hits (*H.*). What was his batting average?

You must find what per cent his number of hits was of the number of times he had been at bat. Since 136 is 35.2% of 386, Bell's average was 35.2%.

2. Find the batting averages of these players, to the nearest tenth of 1%. If this gives two players a tie, find their averages to the nearest hundredth of 1%:

Player	*A. B.*	*H.*	*P. C.*
Brown	389	133	
Donohue	71	24	
Cuyler	379	126	
Roush	349	116	
Williams	196	65	
Walker	374	122	
Gibson	46	15	

Player	*A. B.*	*H.*	*P. C.*
Goslin	345	121	
McNulty	37	13	
Collins	330	115	
Welch	89	30	
Bennett	78	26	
Simmons	382	123	
Miller	255	82	

3. When the members of the Cincinnati National Team had been at bat 3910 times, they had made a total of 1023 hits. What was the team's batting average?

4. Find the batting average of each team:

Team	*A. B.*	*H.*	*P. C.*
Cincinnati	3510	1023	
Pittsburgh	3481	1005	
Philadelphia	3490	1000	
Chicago	3625	1034	

Team	*A. B.*	*H.*	*P. C.*
St. Louis	3599	1021	
New York	3520	1000	
Boston	3548	1006	
Brooklyn	3579	968	

5. The leading National League pitcher recently was Meadows of Pittsburgh. He had pitched 18 games and had won 14. What per cent of the games pitched had he won?

6. A pitcher had won 11 games out of all he had pitched, and had lost 4. What per cent of his games had he won?

THE EXPENSES OF A GREAT CITY

One year New York spent $21,792,122 for fire protection. What per cent, to the nearest tenth of 1%, was this expense of the city's total expenditure of $399,618,885?

To work this problem, you may find what per cent 21,792,122 is of 399,618,885, as in *A*. The answer is 5.5%.

A.

$$399618885\overline{)21792122.0000}\quad .0545 = 5.5\%$$

19980944 25
1811177 750
1598475 540
212702 2100
199809 4425
12892 7675

B.

$$39962\overline{)2179.0000}\quad .0545 = 5.5\%$$

1998 10
180 900
159 848
21 0520
19 9810
1 0710

A quicker way is first to express 399,618,885 and 21,792,122 in *round numbers* as 399,620,000 and 21,790,000. Then the expenses for fire protection are $\frac{21,790,000}{399,620,000}$, or $\frac{2179}{39962}$, of the total expenditures. Changing this fraction to a per cent, as in *B*, you again get 5.5% as answer.

In round numbers 399,618,885 becomes 399,620,000 because the thousands' digit is more than 5, and 21,792,122 becomes 21,790,000 because the thousands' digit is less than 5.

Exercises

Express figures in round numbers to the nearest thousand or ten thousand. Find answers to the nearest tenth of 1%.

1. One year the city of New York spent $94,758,925 for education out of a total expenditure of $399,618,885. What per cent of the total were the expenses for education?

2. In the same year the city of New York spent $34,421,939 for public health. What per cent of the total was that?

3. Find what per cent of the total expenses each of these expenditures was: $16,236,971 for state taxes; $24,392,784 for hospitals and homes.

FACTS ABOUT POPULATION

Express figures in round numbers to the nearest thousand or ten thousand. Compute per cents to the nearest tenth of 1%.

1. At the 1920 census, the United States had a total population of 105,710,620. The number of people living in cities was 54,304,603. What per cent of the entire population was this?

2. The population of Mississippi was 1,790,618, of which 240,121 people lived in cities and 1,550,497 in the country. What per cent of the inhabitants of Mississippi lived in cities? What per cent lived in the country?

3. In the state of Texas, which had a population of 4,663,228, there were 1,512,689 persons living in centers of 2500 or more inhabitants. What per cent of the population of the state lived in these centers? If the rest of the population lived in the country, what per cent lived in the country?

4. New Jersey had 3,155,900 inhabitants, of whom 2,474,936 lived in cities and 680,964 lived in rural districts. What per cent of the total population lived in cities? What per cent lived in rural districts?

5. New York State had 10,385,227 inhabitants, of whom 1,795,387 lived in country districts. What per cent lived in the country? What per cent lived in cities?

6. Rhode Island had a total population of 604,397, with 589,180 people living in cities. What per cent of the population of this state lived in cities?

7. Which of the states in ex. 2 to 6 had a large per cent of their inhabitants living in cities? Which state had the largest per cent living in country districts?

8. In 1910 the United States had a rural population of 49,806,146 and an urban population of 42,166,120. Was a greater per cent of the total population living in cities in 1910 or in 1920 (see ex. 1)?

PER CENTS SMALLER THAN 1%

1. For collecting a bill for \$320, Mr. Nash charges $\frac{1}{2}$ of 1% of the amount of the bill. How much does he charge?

To find $\frac{1}{2}$ of 1% of \$320, first find 1% of \$320, which is \$3.20. Then find $\frac{1}{2}$ of \$3.20. The charge is \$1.60.

$\frac{1}{2}$ of 1% is sometimes written $\frac{1}{2}$%; but the form $\frac{1}{2}$ of 1% is much easier to interpret.

1% of \$320 = \$3.20

$\frac{1}{2}$ of 1% of \$320 = \$1.60

2. A bank charges Mr. Blake 0.1% for collecting \$600 for him. How much does the bank get?

Change 0.1% to a decimal by moving the decimal point two places to the left. This gives .001. Multiplying \$600 by .001 gives \$.60. Hence the bank gets \$.60.

In business 0.1% is usually written as $\frac{1}{10}$ of 1%. In science, however, the form 0.1% is more often used.

\$ 600
.001
\$.600

Exercises

1. Each workman in the Fulton Factory contributes $\frac{1}{5}$ of 1% of his wages to a sick fund. If Mr. Case receives \$180 a month in wages, how much does he give to the fund?

Find the following:

2. $\frac{1}{2}$ of 1% of \$800

3. $\frac{1}{4}$ of 1% of \$400

4. $\frac{3}{4}$ of 1% of \$840

5. $\frac{1}{10}$ of 1% of \$90

6. 0.5% of \$840

7. 0.6% of \$750

8. 0.2% of \$255

9. 0.1% of \$700

10. 0.1% of \$720

11. 0.2% of \$445

12. 0.4% of \$245

13. 0.3% of \$110

14. When the United States had 105,710,620 inhabitants, the District of Columbia had 437,571. What per cent of the country's population had the District of Columbia?

Express the figures in round numbers to the nearest thousand and find the answer to the nearest tenth of 1%.

15. Which is greater, $\frac{1}{2}$ of 1% of \$200 or 50% of \$200? $\frac{1}{2}$ of 1% of \$1000 or 0.5% of \$1000?

PER CENTS LARGER THAN 100%

1. The whole of a number is 100% of it because 100% = 1.00, or 1. Thus, 100% of the pupils in your class is the same as 1 × the number of pupils, or all the pupils.

2. Similarly, 200% = 2.00, or 2; hence 200% of a number is 2 × the number. 300% of a number is 3 × the number.

3. To find 135% of \$1900, change 135% to 1.35. Then multiply \$1900 by 1.35, which gives \$2565. Why should the answer be more than \$1900?

Exercises

Express the following as decimals:

1. 400%	**3.** 500%	**5.** 209%	**7.** 780%	**9.** 201%
2. 354%	**4.** 115%	**6.** 162%	**8.** 468%	**10.** 1200%

Express these as per cents, using the rule on page 88:

11. 2.84	**13.** 1.75	**15.** 18.08	**17.** 4.12	**19.** 3.
12. 3.05	**14.** 4.62	**16.** 10.05	**18.** 1.93	**20.** 15.

21. Last summer there were 380 boys at Camp Sloan. Now the enrollment is 130% as great. Find the present enrollment.

22. Mr. Mack's income this year was 3 times as great as last year. Express this in percentage language.

Find the following and check the work:

23. 125% of \$868	**27.** 128% of \$495	**31.** 1200% of \$890
24. 300% of \$550	**28.** 302% of \$166	**32.** 2000% of \$330
25. 812% of \$127	**29.** 115% of \$980	**33.** 1525% of \$180
26. 401% of \$336	**30.** 785% of \$288	**34.** 1850% of \$425

35. Mr. Foster earns \$50 a week. Mr. Burns earns 150% as much. How much does Mr. Burns earn?

36. Five years ago Mr. Amson had \$4500 in the savings bank. He now has 350% of that amount. How much has he now?

INCREASE AND DECREASE

1. Five years ago Mr. Gay's land was worth \$2400. Its value has since increased 150%. What per cent of the former value is the present value? How much is the land worth now?

The former value is 100% of itself. The land is now worth 150% more, or 250% of its former value. What is 250% of \$2400?

2. During the World War an imported dye increased 300% in price. What per cent was the later price of the original price? If the dye cost \$2 a pound before the war, what was its cost during the war?

3. In May eggs cost 25% less than in January. What per cent was the May price of the January price?

Tell what numbers should be put in place of the dots:

4. 150% more than \$500 = 250% of \$500, or \$1250.

5. 100% more than \$250 = . . .% of \$250, or . . . .

6. 40% less than \$2000 = . . .% of \$2000, or . . . .

7. \$30 increased by 20% of itself = 120% of \$30, or . . . .

8. \$20 increased by 80% of itself = . . .% of \$20, or . . . .

9. In 5 yr. a boy's weekly salary was raised from \$8 to \$28. The increase was what per cent of the original salary?

The amount of increase was \$28 − \$8, or \$20. You must find what per cent \$20 is of \$8. \$20 ÷ \$8 = 2.50, or 250%. Hence the boy's salary was increased 250%.

$$8\overline{)20.00}$$
$$2.50 = 250\%$$

10. In one year a factory increased the value of its daily output from \$2600 to \$3120. Find the per cent of increase.

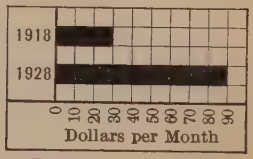
Below are the profits of stores for two successive years. Find, to the nearest tenth of 1%, the per cent of increase or decrease:

11. \$5600; \$7000
12. \$3680; \$3450
13. \$8350; \$9698
14. \$7800; \$7098
15. \$1200; \$2580
16. \$6336; \$7128
17. \$3640; \$3093
18. \$5600; \$6652
19. \$4500; \$3987

SHOWING INCREASE AND DECREASE BY GRAPHS

1. On the preceding page you compared the amounts of increase with the original amounts by means of per cents. You can make a similar comparison by means of bar graphs.

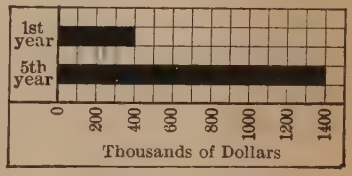
2. This graph shows the increase in the rent of Mrs. Carr's house from 1918 to 1928. The rent was \$30 a month in 1918. What was it in 1928?



RENT OF A HOUSE

The difference in length between the 1918 bar and the 1928 bar shows the amount of increase. Find the amount of increase. Also compute the per cent of increase.

3. This graph shows a store's profits during its first and fifth years of business. Find the amount of increase in the profits. Also compute the per cent of increase. Show that the bar for the fifth year should be 350% of, or $3\frac{1}{2}$ times, the bar for the first year. Count the squares in each bar to see whether this relation is true.



PROFITS OF A STORE

Make bar graphs to show the following facts. Then find the amount and the per cent of increase or decrease:

4. Farm wages in the United States averaged \$30 a month in 1915 and \$48 a month in 1925.

5. The retail price of milk in the United States averaged 7¢ a quart in 1900 and 14¢ a quart in 1925.

6. The average cost of kerosene in this country was 74¢ a gallon in 1865 and 10¢ a gallon in 1925.

7. There were 450 public day schools in the city of New York in 1900 and 660 in 1926. Let each square represent 100 schools.

SELLING BOOKS ON COMMISSION

Roy Barr sold books last year for the Star Publishing Company. He received as pay 35% of the total amount of his sales. The amount he received was his **commission** and 35% was his **rate of commission.**

1. Roy sold $280 worth of books in January. How much did he receive as his commission?

2. After Roy's commission was deducted from $280, how much was left for the company? This amount was the company's **net proceeds.**

3. Roy's sales amounted to $320 in February and to $400 in March. Find his commission and also the company's net proceeds for each month.

These are Roy's sales for the rest of the year. Find his commission and the company's proceeds for each month:

4. April, $340
5. May, $415
6. June, $375
7. July, $293
8. August, $465
9. September, $190
10. October, $245
11. November, $462
12. December, $510

13. Find Roy's total sales last year; his total earnings.

14. This year Roy has been selling books in a bookstore at a regular salary of $27 a week. If he sells $15,600 worth in a year, what per cent of his sales does his yearly salary amount to?

15. The other sales clerk in the bookstore where Roy works gets $30 a week. He sold $24,000 worth last year. What per cent of his sales did he get?

The first number is the amount of sales, the second number is the commission. Find the rate of commission:

16. $800; $160
17. $930; $310
18. $460; $207
19. $400; $90
20. $340; $85
21. $550; $99
22. $920; $276
23. $800; $122
24. $740; $259

COMPUTING PROFITS ON THE COST

Mr. Moore charges \$25 for a dress which costs him \$15. This does not mean that he makes \$10 profit. \$7 of the \$10 goes for **overhead** or **operating expenses** like rent, heat, and wages, and the profit is only \$3. The selling price must include not only the cost and the profit, but also the expenses.

Selling Price = Cost + Expenses + Profit

Exercises

1. Mrs. Howe paid \$450 for Christmas cards which she sold for \$750. Her overhead expenses were \$120. How much was her profit? What per cent of the cost was the profit?

To find the profit, add the cost and the expenses and subtract their sum from the total sales.

2. Mr. Jones bought a piece of land for \$12,000. After paying \$2100 for taxes, he sold it for \$13,500. How much did he lose? Find the per cent of his loss on the cost.

Find the amount of profit and, to the nearest tenth of 1%, the per cent of profit on the cost:

	Cost	Expenses	Sales		Cost	Expenses	Sales
3.	\$2400	\$700	\$3700	**7.**	\$10,500	\$2675	\$14,780
4.	\$2575	\$439	\$3664	**8.**	\$25,600	\$7500	\$35,370
5.	\$1200	\$405	\$1929	**9.**	\$45,775	\$15,600	\$70,575
6.	\$3500	\$650	\$5075	**10.**	\$65,000	\$17,000	\$97,000

Note. The difference between the cost and the selling price is sometimes called the *gross profit*, while the *gross profit* less the *expenses* is called the *net profit*. The *gross profit* is not all profit, because it includes the operating expenses. Prominent business associations recommend that the term *gross profit* be discontinued, since retail shopkeepers often misunderstand it, thinking that gross profits are real profits. The same associations recommend the use of the word *margin* in place of *gross profit*. In this book the word *profit* means *actual* or *net* profit, which is found as explained at the top of this page.

COMPUTING PROFITS ON THE SELLING PRICE

On page 99 you found the per cent of profit *on the cost.* Nowadays, business men often figure their profits *on the selling price.* Since commissions, discounts, and certain taxes are computed on the selling price, or sales, it is simpler to compute the profits also on the sales. Moreover, a merchant always knows how much he receives for an article, but it is often hard to separate its cost from the operating expenses.

Always state whether the per cent of profit or loss is computed on the cost or on the selling price.

Exercises

1. Mr. Gregg buys coats at \$30 each and sells them at \$50 each. His expenses average \$15 per coat. Hence his profit averages \$5 per coat. Show that his profit is $16\frac{2}{3}\%$ of the cost and only 10% of the selling price.

In ex. 2 *to* 11, *find all per cents to the nearest tenth of* 1%.

2. Last year the Drew Hat Shop spent \$2360 for goods. The expenses were \$960 and the sales, \$4150. What was the profit? the per cent of profit on the sales?

3. Last year Mr. Thompson spent \$2,320,000 for goods and \$1,600,000 for overhead expenses. His sales totaled \$4,480,000. Find the per cent of his profit on the sales.

Find the profit and the per cent of profit on the sales. Also find the per cent of profit on the cost:

	Cost	Expenses	Sales		Cost	Expenses	Sales
4.	\$2050	\$820	\$3280	**8.**	\$27,670	\$9900	\$45,000
5.	\$1483	\$800	\$2500	**9.**	\$33,792	\$19,200	\$57,600
6.	\$3575	\$1100	\$5500	**10.**	\$47,858	\$20,217	\$87,900
7.	\$4152	\$1938	\$6800	**11.**	\$39,040	\$21,960	\$73,200

12. In the above examples, is the per cent of profit greater when computed on the cost or on the selling price?

COMPUTING EXPENSES ON THE SELLING PRICE

You have seen that profits are often figured on the selling price. Overhead expenses are also frequently computed as a per cent of the selling price, because it is easier to compute them that way than to find what per cent they are of the cost. A business man can tell his expenses and his total sales for the past year from his books. Therefore he can easily find what per cent his expenses were of the sales. This shows him about what per cent to allow for expenses during the present year.

If Mr. Day's sales last year were \$75,000 and his expenses \$22,500, his expenses were $\frac{22,500}{75,000}$, or 30%, of his sales. What per cent of his sales should he allow for expenses this year?

By computing profits and expenses on the sales, a merchant knows how much of each dollar taken in is profit and how much of it will be used to cover expenses and cost.

Exercises

1. A man buys goods for \$400. His overhead expenses are \$160 and his profit is \$80. Find the selling price. What per cent of the selling price goes for profit? for overhead? for cost?

These numbers show the total sales and the total expenses for a year. Find what per cent the expenses are of the sales:

	Sales	Expenses		Sales	Expenses
2.	\$87,500	\$21,000	**4.**	\$981,000	\$327,000
3.	\$225,000	\$60,750	**5.**	\$754,500	\$226,350

6. In ex. 4 to 11 on page 100, find what per cent the expenses are of the sales.

7. Mr. Dorr's sales amounted to \$51,200. The cost of materials was \$20,480 and the overhead, \$12,800. What per cent of his sales was the overhead? the profit? the cost? How much of every \$1 he received went for each of these three items?

DETERMINING THE SELLING PRICE

Mr. Thomas runs a clothing store and is determining the price at which he should sell a coat that cost him \$18. By keeping careful yearly accounts of his sales, costs, overhead expenses, and profits, he has found that his overhead is usually 28% of his sales and that he needs 12% profit on all sales to run his business successfully. At what price should he sell the coat in order to cover his overhead and profit as well as the original cost of the coat?

The overhead and the profit total 40% of the selling price; hence the cost is 60% of the selling price.

Since 60% of the selling price = \$18

1% of the selling price = \$18 ÷ 60, or \$.30

and 100% of the selling price = 100×\$.30, or \$30.00

Hence the coat that cost \$18 should be marked \$30.

Exercises

1. Allowing 10% of his sales for profit and 35% for overhead, at what price should a dealer sell a hat that cost \$3.30?

The per cents of overhead and profit in ex. 2 to 9 are computed on the selling price. Find the selling price for each:

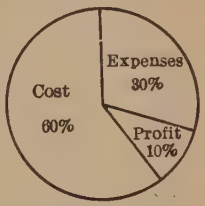
	Cost	Overhead	Profit		Cost	Overhead	Profit
2.	\$14	18%	12%	**6.**	\$567	19%	11%
3.	\$64	22%	14%	**7.**	\$424	34%	13%
4.	\$24	25%	15%	**8.**	\$279	22%	16%
5.	\$29	32%	10%	**9.**	\$357	37%	12%

10. Mr. Ward keeps a clothing store. He buys fur coats at \$240 each. His expenses are 28% of his sales and his profits are 8% of his sales. At what price shall he mark each coat?

11. The Brent Company sells vacuum cleaners. 36¢ out of every \$1 it earns goes for overhead. If the company wants 8% profit on the selling price, what should be the price of a vacuum cleaner for which the company paid \$35?

THE SALES DOLLAR

Each dollar received from sales is sometimes called "a sales dollar." Business men often analyze their sales dollar to see how it is used. For example, if a furniture dealer finds that 60% of his sales represent costs, 30% of his sales are expenses, and 10% of his sales are profits, then the figure at the right shows how his sales dollar is divided.

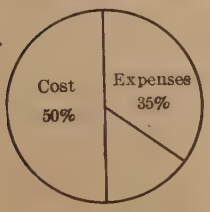


THE SALES DOLLAR

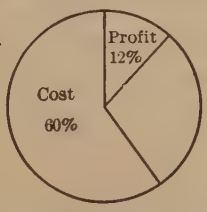
Exercises

In each sales dollar, tell how the blank part should be labeled:

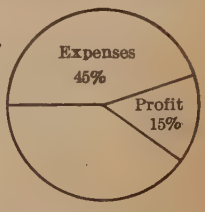
1.



2.



3.



4. Mr. Todd's expenses average $33\frac{1}{3}\%$ of his sales; his profits are $16\frac{2}{3}\%$ of his sales. How much of a sale of \$42.36 is profit? How much covers expenses? How is the rest used?

5. Draw a picture of the sales dollar for ex. 4.

6. Last year the sales of one store were \$129,500 and the expenses, \$41,440. The sales of another store were \$78,400 and the expenses, \$29,792. Find out which store was managed more economically.

Find the amount of profit. Then tell what per cent of the S. P. *is profit, what per cent is expenses, and what per cent is cost:*

	Cost	Expenses	S. P.		Cost	Expenses	S. P.
7.	\$300	\$60	\$400	**10.**	\$176	\$198	\$440
8.	\$490	\$49	\$700	**11.**	\$216	\$162	\$450
9.	\$510	\$90	\$900	**12.**	\$750	\$500	\$1500

TRADE DISCOUNT

You buy groceries and clothing from a *retail merchant*, and he buys them from a *wholesale merchant*. The wholesale merchant sends the retailer a catalogue describing his goods and giving their *list prices*.

The prices of goods change frequently. Instead of printing a new catalogue whenever a price changes, the wholesaler sends the retailer a *discount sheet* stating what discounts are allowed on the list prices. Such discount is called **trade discount**; it is found in the same way as the discount at a sale. The list price less the discount is called the **net price.**

Exercises

1. Mr. Parker orders goods listed at \$300. He gets a trade discount of 24%. What is the net price of the goods?

First find the discount, which is 24% of \$300, or \$72. Then find the net price, which is \$300 − \$72, or \$228.

2. Mr. Robins buys bookcases listed at \$45.60. He receives a 15% discount. What is the net price?

Find the discount and the net price when the list price and the rate of discount are as follows:

3. \$840, 20%	**8.** \$765, 13%	**13.** \$78.79, 15%
4. \$922, 60%	**9.** \$285, 15%	**14.** \$45.60, $12\frac{1}{2}$%
5. \$314, 15%	**10.** \$540, $16\frac{2}{3}$%	**15.** \$346.70, 35%
6. \$360, 25%	**11.** \$496, $12\frac{1}{2}$%	**16.** \$432.95, 25%
7. \$276, 40%	**12.** \$624, $33\frac{1}{3}$%	**17.** \$256.81, $16\frac{2}{3}$%

18. Green and Company are offering a discount of 30% on baseball suits listed at \$6.50. Jones and Company have similar suits marked \$7.25, and offer 40% discount. Which company sells the suits cheaper? how much cheaper?

19. Which costs less, an 18-dollar clock at 15% discount or a 22-dollar clock at 25% discount? how much less?

CHAIN DISCOUNTS

Mr. White, a retail merchant, bought goods listed at \$1200. The trade discount was 15%, and Mr. White received an extra discount of 10% because the order was large. Two discounts like this on the same bill may be spoken of as a discount of 15%, 10%. They are called **successive** or **chain discounts.**

To find how much Mr. White paid for the goods, you can use either method *A* or method *B*, as explained below:

A. Find 15% of \$1200 and subtract it from \$1200, leaving \$1020. Then take 10% of \$1020 and subtract it from \$1020, leaving \$918 as the net price.

B. If you deduct a discount of 15% from \$1200, you have 85% of \$1200 left. Hence take 85% of \$1200, which gives \$1020. Since the next discount is 10%, the net price is 90% of \$1020, or \$918.

A.

	\$1200
Less 15%	180
	\$1020
Less 10%	102
	\$ 918

B.

\$1200	\$1020
.85	.90
\$1020.00	\$918.00

Both of the above methods give \$918 as the net price, but method *B* is sometimes quicker.

Exercises

1. If Mr. White (above) received an extra discount of 2% for prompt payment, you would find 2% of \$918 and subtract it from \$918. What would be the net price then?

Find the net prices by both of the above methods:

2. \$450, less 20%, 5%

3. \$376, less 25%, $2\frac{1}{2}$%

4. \$980, less 30%, 2%

5. \$675, less 20%, 3%

6. \$45,000, less 60%, 6%, 5%

7. \$77,500, less 15%, 10%, 2%

8. \$90,000, less $33\frac{1}{3}$%, 20%, $2\frac{1}{2}$%

9. \$125,000, less 20%, 30%, 5%

10. A dealer can buy shoes listed at \$1280 with a chain discount of 20%, 5% or with a single discount of 25%. Which would give him the lower net price? Which discount seems larger?

THE ORDER OF DISCOUNTS

1. Mr. Monroe, a furrier, was buying furs whose list price was \$1500. Mr. Green offered him a discount of 25%, 10% on the furs, while Mr. Fenton offered a discount of 10%, 25%. Mr. Monroe thought that Mr. Green was offering the better discount because he gave the larger discount on the full amount. Find the net price in each case and see if Mr. Monroe was right.

2. By using method *B* on page 105, you can easily see why the order of discounts in ex. 1 above did not affect the net price. For Mr. Green's furs, the net price is $.90\times.75\times\$1500$; for Mr. Fenton's furs, the price is $.75\times.90\times\$1500$. Since the same numbers are multiplied together in each case, the results are the same. Prove this by doing the multiplication for each.

Find each net price, taking the discounts in the order given. Then change the order of the discounts, taking the last one first. Are the results alike in both cases?

3. \$240; 20%, 5%

4. \$620; 30%, 2%

5. \$430; 5%, $2\frac{1}{2}$%

6. \$700; 35%, 5%

7. \$900; 45%, 10%

8. \$9268; 25%, 2%

9. \$5105; 20%, 10%

10. \$1406; 5%, 5%, 2%

11. \$8275; 10%, 5%, 2%

12. \$7332; 75%, 5%, 3%

13. On goods listed at \$800, which is best, a discount of 15%, 10%, 5% or a discount of 5%, 15%, 10% or a discount of 10%, 15%, 5% or a discount of 5%, 10%, 15%? Find the net price in each case and compare the results.

14. Is a chain discount of 15%, 10%, 5% better than a single discount of 30%, which equals the sum of the chain discounts? Is the same thing true of any chain discount when compared with a single discount equal to the sum of the chain discounts?

The order in which successive discounts are deducted does not make any difference in the net price.

USING EQUATIONS IN PERCENTAGE

Percentage problems can be solved by equations, as shown in the examples below:

1. For selling a house, Mr. Ray was paid a commission of 5% of the amount for which he sold the house. If his commission was \$350, what amount did he receive for the house?

Let x = number of dollars received for house

Then $5\%\ x$, or $\frac{5}{100}x$ = Mr. Ray's commission, in dollars

The equation is $\frac{5}{100}x = 350$

Multiply by 100: $5x = 35{,}000$

Divide by 5: $x = 7000$

Hence the house was sold for \$7000. *Check.* 5% of \$7000 = \$350.

2. Mr. Cox finds that it costs 15% more to build a house now than it did 5 yr. ago. If it costs \$9200 to build a certain house now, how much would the same house have cost 5 yr. ago?

Let x = cost, in dollars, 5 yr. ago

The equation is $x + \frac{15}{100}x = 9200$

Since x is the same as $\frac{100}{100}x$, the equation may be written as

$$\frac{100}{100}x + \frac{15}{100}x = 9200$$

Add the x's: $\frac{115}{100}x = 9200$

Multiply by 100: $115x = 920{,}000$

Complete the solution and check the work.

Exercises

1. Joe received a commission of 15% for selling a boat. If his commission was \$9, for what price did he sell the boat?

Solve the following and check the results:

2. $\frac{25}{100}x = 16$

3. $\frac{62}{100}x = 31$

4. $\frac{35}{100}p = 42$

5. $\frac{95}{100}r = 76$

6. $\frac{16}{100}c = 24$

7. $\frac{1}{100}s = 72$

8. $\frac{28}{100}p = 35$

9. $\frac{8}{100}x = 19$

10. $x + \frac{15}{100}x = 92$

11. $x + \frac{10}{100}x = 77$

12. $x + \frac{25}{100}x = 20$

13. $p + \frac{45}{100}p = 87$

SOLVING PROBLEMS BY EQUATIONS

Solve these problems by equations and check the work:

1. Tom sold books at a commission of 40%. If he received $14 as his commission, find the amount of his sales.

2. John made a radio set and sold it for $75. This was 25% more than it cost him. How much did it cost him?

3. Mary sold some candy for 60¢ a pound, thus gaining 20% of the cost. How much per pound did the candy cost Mary?

4. A town's population has increased 28% in 5 yr. If the population is now 12,160, what was it 5 yr. ago?

5. If I buy a desk for $320, at what price shall I sell it so that my expenses and profit together equal 36% of the selling price?

Let x = selling price, in dollars. If expenses and profit are 36% of the selling price, the cost is 64% of the selling price. Hence $\frac{64}{100}x = 320$.

6. Mr. Bailey has a hardware store. At what price should he sell saws that cost him $1.50 each if he allows 30% of the selling price for expenses and 10% of the selling price for profit?

7. Mr. Dort receives a weekly salary of $25 besides a commission of 5% on all goods he sells. Last week his salary and commission together amounted to $60. Find the amount of his sales.

8. Last year Mr. Dort's sales were 10% greater than the year before. If his sales amounted to $37,400 last year, what were his sales the year before?

Solve the following and check the work:

9. $\frac{2}{5}x = \$5.40$

10. $\frac{3}{4}x = \$18.60$

11. $\frac{9}{100}x = \$2.88$

12. $\frac{98}{100}x = \$1000$

13. $\$1850 = \frac{125}{100}x$

14. $\$1806 = \frac{6}{100}x$

15. $\$1225 = \frac{98}{100}x$

16. $\$97.50 = \frac{5}{100}x$

17. $\$12.96 = p + \frac{8}{100}p$

18. $\$66.00 = x + \frac{32}{100}x$

19. $\$15{,}000 = x + \frac{10}{100}x$

20. $\$180.60 = x + \frac{1}{5}x$

MORE WORK WITH EQUATIONS

Mary bought a tennis racket for \$3.76, which was 6% less than the regular price. What was the regular price?

Let p = regular price, in dollars

Then 6% p, or $\frac{6}{100}p$ = discount, in dollars

Since Regular Price − Discount = Net Price, you have this equation:

$$p - \tfrac{6}{100}p = 3.76$$

Subtract the p's: $\tfrac{94}{100}p = 3.76$

Multiply by 100: $94p = 376$

Divide by 94: $p = 4$

The regular price of the racket is \$4. Check the work.

Exercises

Solve these problems by equations and check the work.

1. At a 15% discount sale Helen bought goods for a dress for \$9.20. What was the regular price of the material?

2. Miss Vail receives 6% discount on a bill. If the net amount of the bill is \$42.30, find the original amount of the bill.

3. The sales in Mr. Robert's store were 8% less last year than the year before. If last year's sales amounted to \$69,000, what was the amount of the sales the year before?

4. Mabel has some money in the savings bank, on which she receives 4% interest. If she received \$12 in interest last year, what amount did she have in the bank?

Solve the following and check the work:

5. $x - \frac{1}{4}x = \$120$

6. $x - \frac{50}{100}x = \$390$

7. $x - \frac{75}{100}x = \$720$

8. $p - \frac{24}{100}p = \$380$

9. $p - \frac{18}{100}p = \$410$

10. $x - \frac{1}{5}x = \$11.12$

11. $x - \frac{17}{100}x = \$58.10$

12. $x - \frac{2}{100}x = \$151.41$

13. $c - \frac{10}{100}c = \$1334.70$

14. $m - \frac{1}{3}m = \$2610.40$

COMPARING THE EQUATION WITH OUR FORMER METHOD

A real estate dealer sold a house for $18,400, which was 15% more than he paid for it. How much did it cost him?

Old Method — Analysis	*New Method — Equation*
1. 100% of cost = whole cost 15% of cost = increase	1. Let c = cost Then $\frac{15}{100}c$ = increase
2. 100% of cost +15% of cost = $18,400	2. $c + \frac{15}{100}c = 18{,}400$
3. 115% of cost = $18,400	3. $\frac{115}{100}c = 18{,}400$
4. 1% of cost = $\frac{\$18{,}400}{115}$	4. $115\,c = 1{,}840{,}000$
5. 100% of cost = $100 \times \frac{\$18{,}400}{115}$	5. $c = 16{,}000$
6. Hence cost = $16,000	6. Hence cost = $16,000

Studying the above solutions, you notice these points:

1. In Steps 1 and 2 the methods are much alike; but the equation, which expresses the facts in shorthand, is quicker.

2. In Steps 3 to 6 of the old method, you must think out every step in terms of the original problem. But in using equations, you think of the problem only until the equation is formed (Step 2) and then mechanically solve for the unknown number.

3. In using equations in percentage, it is best to write each per cent as a fraction with 100 as the denominator.

Exercises

Solve these problems by both the old and the new method:

1. Mr. Low's expenses and profit are 35% of his sales. At what price should he mark a coat that cost him $13?

2. A dealer is allowing his customers a discount of 15% on the marked price of fur coats. At what price shall he mark a coat for which he wants to get $340?

IMPROVEMENT TEST No. 5

Test 5 A — Addition (Time 4 min.)

1. $7\frac{2}{3}$, $2\frac{1}{2}$, $3\frac{2}{3}$, $5\frac{5}{6}$

2. $3\frac{3}{4}$, $2\frac{1}{2}$, $1\frac{3}{8}$, $5\frac{5}{16}$

3. $3\frac{1}{3}$, $2\frac{2}{5}$, $5\frac{1}{3}$, $6\frac{1}{10}$

4. $9\frac{1}{4}$, $4\frac{1}{2}$, $3\frac{1}{5}$, $1\frac{3}{10}$

5. $6\frac{1}{6}$, $5\frac{1}{2}$, $3\frac{3}{4}$, $8\frac{5}{12}$

Test 5 B — Subtraction (Time 4 min.)

1. $6\frac{1}{3} - 3\frac{2}{3}$

2. $9\frac{5}{8} - 1\frac{7}{16}$

3. $5 - 4\frac{9}{10}$

4. $8\frac{3}{4} - 5\frac{2}{3}$

5. $7\frac{4}{5} - 6\frac{7}{10}$

6. $4\frac{1}{2} - 2\frac{5}{6}$

7. $3 - 1\frac{3}{4}$

8. $2\frac{5}{6} - \frac{1}{4}$

9. $9\frac{3}{8} - 2\frac{2}{3}$

10. $8\frac{3}{8} - 6\frac{1}{2}$

11. $1\frac{1}{2} - 1\frac{2}{5}$

12. $6\frac{2}{3} - 2\frac{3}{5}$

Test 5 C — Multiplication (Time 4 min. after copying)

1. $625 \times 31\frac{1}{4}$

2. $48\frac{2}{3} \times 75$

3. $488 \times 18\frac{3}{8}$

4. $14 \times \frac{2}{3}$

5. $\frac{8}{15} \times \frac{9}{10}$

6. $1\frac{1}{3} \times \frac{9}{16}$

7. $\frac{5}{12}$ of 64

8. $\frac{4}{15} \times 3\frac{3}{4}$

9. $7\frac{4}{5} \times 3\frac{5}{6}$

Test 5 D — Division (Time 4 min. after copying)

1. $5\frac{1}{5} \div \frac{4}{5}$

2. $\frac{5}{8} \div \frac{2}{5}$

3. $28 \div \frac{7}{8}$

4. $9\frac{1}{3} \div \frac{1}{2}$

5. $\frac{1}{2} \div 1\frac{1}{3}$

6. $8\frac{1}{4} \div 3\frac{3}{4}$

7. $7\frac{1}{4} \div \frac{5}{8}$

8. $\frac{3}{16} \div 1\frac{1}{5}$

9. $16\frac{1}{2} \div 3$

10. $6\frac{2}{3} \div 15$

11. $\frac{9}{10} \div 6\frac{3}{4}$

12. $17\frac{1}{4} \div 9$

REVIEW

Solve these problems by equations and check the work.

1. The population of Waterford is 38% less than it was 5 yr. ago because the Novelty Factory has left that city. If the population is now 27,900, what was it 5 yr. ago?

2. The school enrollment in the Newton city schools is 45% more than it was 10 yr. ago. The enrollment is now 7685 pupils. What was it 10 yr. ago?

3. If the net price of an article is to be \$400 and the trade discount is 20%, what should the list price be?

4. The attendance report for the Warren School showed that 96% of the pupils were present last Monday. If 16 pupils were absent that day, what is the total enrollment of the school?

5. The Economy Dry Goods Store pays \$1.10 per yard for silk. If its overhead is 35% of the selling price and its profit is 10% of the selling price, at what price per yard should the silk be sold in order to cover both overhead and profit?

The equation may be formed as in ex. 5, page 108, or by using the fact that *cost + expenses + profit = selling price*, as shown below. If x = selling price, then $\frac{35}{100}x$ = expenses, and $\frac{10}{100}x$ = profit.

The equation is $1.10 + \frac{35}{100}x + \frac{10}{100}x = x$

Add the x's: $1.10 + \frac{45}{100}x = x$

Subtract $\frac{45}{100}x$: $1.10 = \frac{55}{100}x$

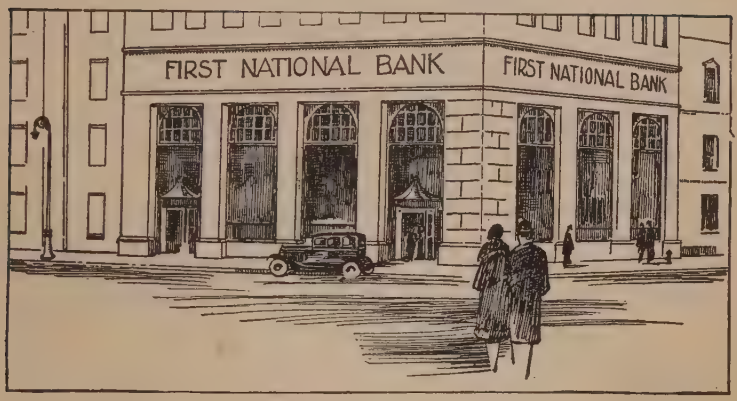
Complete the solution and check the work.

6. A dealer pays \$1.50 each for hats. At what price should he sell them if he allows 33% of the selling price for overhead and 7% of the selling price for profit?

7. At what price shall a dealer sell furniture that cost him \$256 if he pays his salesman a commission of 8% of the selling price, if his overhead is 20% of the selling price, and if he wishes to make a profit of 8% of the selling price?

CHAPTER VI

COMMERCIAL BANKS



THE SERVICES RENDERED BY BANKS

Commercial banks are so important in everyday life that the majority of the people in this country make more or less use of them. A commercial bank will lend money to a reliable business man and to other responsible persons.

In order to get money to lend, a commercial bank offers certain services to attract people to put their money in the bank. These services to depositors are as follows:

(1) It offers them a safe place to keep their money.

(2) It permits them to pay out any of the money they have in the bank by a written and signed order called a **check**.

(3) It cashes the checks that they receive from other persons.

(4) It sometimes pays a low rate of interest on an account if a large amount of money is kept on deposit.

In general, a bank makes no charge to its depositors for the above services unless the account is very small.

DEPOSITING MONEY IN A BANK

When John Lee deposits money in the Empire Bank, where he has an account, he fills out a deposit slip, as shown at the right. This slip tells how much money Mr. Lee puts in the bank and how much of it is in bills, in coin, and in checks. The deposit slip also includes certain information about each check deposited. Thus, if a check is drawn on a local bank, the depositor usually writes the name of the bank, as "Franklin Bank." If a check is drawn on a bank in another city, the depositor writes only the name of the city, as "St. Louis."

Deposited by
John Lee
in the
EMPIRE BANK
New York, N. Y.
March 20, 19—

Bills	144	00
Coin	15	85
Checks		
Franklin Bank	37	50
St. Louis	14	25
	211	60

WRITING CHECKS

If John Lee owes $220.30 to Roy Page of Detroit, he writes the check shown below and mails it to Mr. Page. This check is an order on the Empire Bank of New York, where Mr. Lee keeps his money, to pay $220.30 to Mr. Page. Mr. Lee mails this check to Mr. Page in Detroit. The further travels of this check are described on pages 115 and 116.

New York, Nov. 2, 19— No. 4267

THE EMPIRE BANK

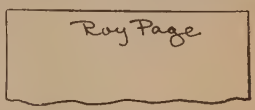
Pay to the order of Roy Page $220 30/100

Two hundred twenty and 30/100 **Dollars**

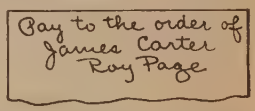
John Lee

INDORSING A CHECK

When Roy Page of Detroit receives the check mentioned on page 114, he takes it to the Union Bank in Detroit, in which he keeps his money, and deposits it to the credit of his account. Before depositing it, however, he makes out a deposit slip (see page 114) and also indorses the check by writing his name across the back of the left end of it, as shown above. This is called *an indorsement in blank.* By indorsing the check Mr. Page acknowledges the payment of it through the bank.



Instead of depositing the check in the bank, Mr. Page may ask Mr. Carter to cash it for him. In this case, Mr. Page may indorse the check in blank, as above, thereby making it payable to Mr. Carter or to any one else; or he may indorse it *in full,* as shown at the right, thereby making the check payable only to Mr. Carter or his order. An indorsement in full is safer than a blank indorsement in case the check is lost.



Exercises

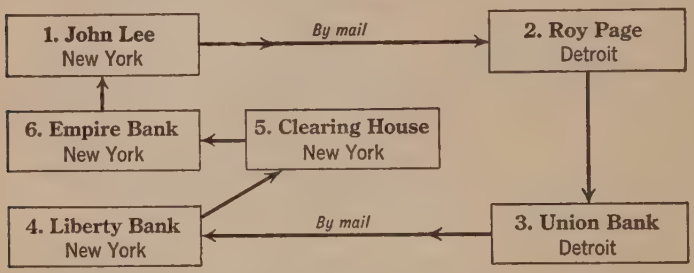
1. In the second indorsement shown above, suppose that Mr. Carter, after receiving the check from Mr. Page, wishes to turn it over to Alice Brown. Write the indorsement in full that Mr. Carter will put on the check.

2. Mr. Adams deposited the following in the Citizens Bank of Cleveland: $25 in bills; $15.83 in coin; a check on a Denver bank for $17.45; and a check on the Union Bank of Cleveland for $29.50. Make out the deposit slip.

3. John Gray wrote a check on the Union Bank for $165.98 to the order of Paul Lynn. Mr. Lynn sent it to George Bond. Make out the check and indorse it in full for Paul Lynn.

THE TRAVELS OF A CHECK

The check at the bottom of page 114 is written because John Lee of New York owes $220.30 to Roy Page of Detroit. Its whole history is one of travel, as is shown below.



Step 1. John Lee has a checking account in the Empire Bank of New York. He writes a check for $220.30 on this bank and mails it to Roy Page in Detroit.

Step 2. The check arrives at Mr. Page's office in Detroit.

Step 3. Mr. Page indorses the check and deposits it in the Union Bank of Detroit, where $220.30 is added to his account.

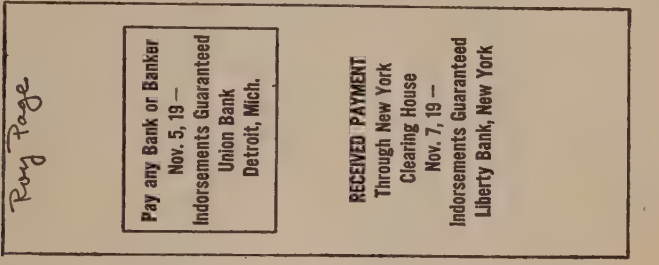
Step 4. Most banks in large cities, like New York, Chicago, San Francisco, Detroit, and Philadelphia, do business with one bank in each of the other large cities. The Union Bank indorses Mr. Lee's check and mails it to the Liberty Bank in New York, with which it does business.

Step 5. The check arrives at the Liberty Bank in New York, and this bank sends it by messenger the next morning to the New York Clearing House, where it is handed over to a messenger from the Empire Bank. Thus Mr. Lee's check gets back to the Empire Bank, on which it was drawn.

Step 6. The Empire Bank deducts $220.30 from Mr. Lee's account. At the end of the month the bank returns the canceled check to him. If he is wise, he will keep this check as a receipt.

TRACING THE TRAVELS OF A CHECK

When Mr. Lee's check (see page 114) is returned to him, he finds across the back of it the three indorsements shown below. These record the check's travels since the day he first wrote it.



1. Mr. Page indorsed the check at the Union Bank just before depositing it. Why did he wait until then to indorse it?

2. Which indorsement shows that the Union Bank of Detroit sent the check to another bank? Does this indorsement show where it was sent? What statement on the back of the check shows that the Union Bank sent the check to the Liberty Bank?

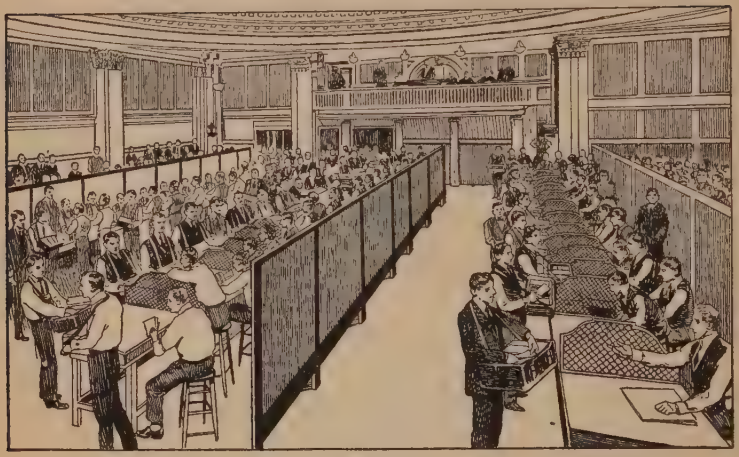
3. On what day and where does the Liberty Bank turn the check over to the Empire Bank?

4. Ask some one who has a checking account to show you some checks that have come back from the bank. Study these checks and their indorsements and try to diagram their travels.

5. Rose Mason sends a check for $54.20 on the Copley Bank of Boston to Ada Jones of Denver. Mrs. Jones deposits it in the Commercial Bank of Denver, which sends it to the National Bank of Boston. This bank takes it to the Boston Clearing House. Where does it go then? Diagram its travels.

6. Guy Adams sends Fred Towns of Chicago a check for $3.50 drawn on the National Bank of St. Louis. Mr. Towns has his account in the Central Bank of Chicago, which does business with the Columbia Bank of St. Louis. Diagram the check's travels.

THE WORK OF THE CLEARING HOUSE



Every large city in the United States has a clearing house organized and maintained by the banks of that city. At each clearing house messengers from the banks of the city meet daily to exchange the checks which those banks have against one another and to balance the difference.

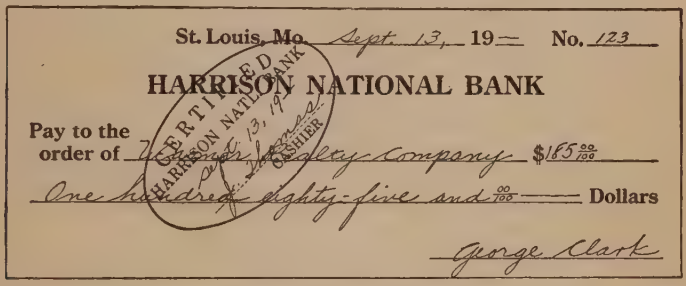
The above picture shows the interior of the New York Clearing House while the banks are exchanging checks. At each desk is seated a man representing some New York bank. In front of each desk stands a messenger from another bank, delivering checks which his bank holds against the bank represented at that desk. These messengers move from desk to desk until they have delivered to the proper banks all the checks in the leather cases that they are carrying.

The total amount of the checks that change hands in a clearing house is called the **clearings.** In New York the clearings are often over $1,000,000,000 a day. The actual exchange of all the checks representing this huge amount is made in the Clearing House within 15 minutes.

CERTIFIED CHECKS

The Warner Realty Company of Chicago sends George Clark of St. Louis a bill for $185 and asks for a **certified check** in payment. This means that the company wants Mr. Clark's bank to guarantee that he has enough money to pay the check.

Mr. Clark makes out a check for $185. The cashier at his bank stamps it " Certified," as shown below.


St. Louis, Mo. Sept. 13, 19— No. 123
HARRISON NATIONAL BANK
CERTIFIED
HARRISON NATIONAL BANK
Sept. 13, 19—
CASHIER
Pay to the order of Warner Realty Company $185 00/100
One hundred eighty-five and 00/100 Dollars
George Clark

The bank deducts $185 at once from Mr. Clark's account and holds it until the check comes in for payment.

Exercises

1. The Warner Realty Company, to which Mr. Clark sends the above certified check, has an account with the Capitol Bank of Chicago, which does business with the Traders Bank of St. Louis. Make a diagram showing the travels of the check. St. Louis has a clearing house.

2. Mr. Clark has $389.50 in the bank before the above check is certified. What is his balance after it is certified?

3. After the check is certified, Mr. Clark deposits $31 in bills, $42.20 in coin, and a check for $45. Make out the deposit slip. What is Mr. Clark's bank balance after these transactions?

4. If Mr. Clark then presents a check for $400 to his bank to be certified, will the cashier certify it? Why not? If the check were for $69, would the bank certify it?

PAYING A BILL BY A BANK DRAFT

Alan Curt, who runs a small shop in Seattle, must pay in advance for $112.44 worth of goods which he orders from James Robb of Chicago. He therefore buys a **bank draft** for this amount from The Mechanics Bank of Seattle.

Seattle, Wash., Sept. 2, 19— No. 1275

THE MECHANICS BANK

Pay to the order of Alan Curt ——— $112 44/100

One hundred twelve and 44/100 ——— Dollars

To THE FIRST NATIONAL BANK
Chicago, Ill.

Henry Macy
Cashier

In this draft the Seattle bank orders The First National Bank of Chicago to pay $112.44. Mr. Curt has the draft made out payable to himself, rather than to Mr. Robb, so that his name will appear on it, thus showing that he sent the draft.

Mr. Curt indorses the draft as follows: "Pay to the order of James Robb. Alan Curt," and mails it to Mr. Robb.

When Mr. Robb receives the draft, he deposits it in his bank in Chicago, which sends it to the Chicago Clearing House. Here it is turned over to The First National Bank of Chicago. The draft is finally returned to The Mechanics Bank of Seattle, which keeps it.

A bank draft is really a check of one bank on another bank.

Exercises

1. In the bank draft above, which bank is ordered to pay Mr. Curt? Where is its name printed on the draft? On a check, where is the name of the bank that is ordered to pay?

2. In the above draft, which bank orders the payment of the money? Who orders payment on a check?

3. Why does Mr. Curt send a bank draft instead of a check?

COLLECTING A BILL BY A SIGHT DRAFT

You have learned that banks provide a convenient means of paying bills. Banks also help in the *collection* of bills.

If the Chelsea Coal Company of Pittsburgh sends Charles Moss of Baltimore a bill for $450 and he fails to pay it, the company may make out this **sight draft:**

$450 00/100 Pittsburgh, Pa., Jan. 31, 19—

At sight pay to the order of

The Hamilton National Bank, Pittsburgh

Four hundred fifty and 00/100 Dollars

Value received and charge to the account of

To Charles Moss
Baltimore, Md.

Chelsea Coal Company
per B. P. S.

The Chelsea Coal Company leaves this draft with The Hamilton Bank, where it has an account. This bank indorses the draft and sends it to a bank in Baltimore. A representative of the Baltimore bank presents the draft to Mr. Moss. If he owes the money and wants to keep his business reputation, Mr. Moss will pay the $450 at once. The Baltimore bank will then send the money to The Hamilton Bank of Pittsburgh, which will credit it to the Chelsea Coal Company's account after deducting about $\frac{1}{10}$ of 1% of the $450 as a collection fee.

Exercises

1. In a sight draft, the name of the one who is ordered to pay is written in the lower left-hand corner. Is this true of a bank draft? of a check?

2. Is Mr. Moss more likely to pay his debt after receiving a sight draft from a bank in his own city than after receiving a letter from Pittsburgh from the Chelsea Coal Company? Why?

3. How does The Hamilton Bank indorse the draft before sending it to Baltimore?

MAKING OUT BANK DRAFTS AND SIGHT DRAFTS

Use the drafts on pages 120 *and* 121 *as models.*

1. To pay George Wood of Chicago $274.50, Morris Wells bought a bank draft from the Pacific Bank of Los Angeles, which does business with the American Bank of Chicago. Write the draft in favor of Mr. Wells and indorse it in full for him.

2. The Pacific Bank charged a fee of $\frac{1}{10}$ of 1% for the draft. What fee did Mr. Wells have to pay? How much did he pay in all for the draft?

3. Mr. Wood deposited the draft in the La Salle Bank of Chicago. Where would this bank send it? Trace the travels of the draft, using a diagram.

4. Write a bank draft for $160.30 of a bank in St. Louis on a bank in Philadelphia. Let it be made out to the order of Ruth Cook and indorsed to the order of Jane Downes.

5. Philip Galt of St. Paul owed Max Thorp of New Orleans $685. As Mr. Galt did not pay the money when it was due, Mr. Thorp made out a sight draft payable to the order of the Colonial Bank of New Orleans. Write the draft.

6. In ex. 5, describe what happened from the time the Colonial Bank got the draft until Mr. Thorp got the money.

7. If the Colonial Bank charged Mr. Thorp $\frac{1}{10}$ of 1% for collecting the $685, how much of the $685 did he receive?

8. What is the purpose of a sight draft? of a bank draft?

9. You owe Albert Meek of Portland, Maine, $75.30 and you have no checking account. What kind of draft will you send him? Write the draft and indorse it.

10. Thomas Stone of Little Rock, Ark., has failed to pay $813.50 which he owes you. What kind of draft will you use to collect the money? Write the draft, using the name of some bank in your city. If your bank's fee is $\frac{1}{10}$ of 1%, how much will you receive in all?

SENDING MONEY BY A POSTAL MONEY ORDER

Another safe way of sending money, besides the check and the bank draft, is the **postal money order.** Mr. Moore buys a money order for $50 from a post office in his home town and sends it to Mr. Lent. Mr. Lent presents it to a post office in his town and receives $50.

At the right are the fees for postal money orders. The fee for Mr. Moore's order for $50 is 18¢. Hence the order costs $50.18 in all.

Amount	*Fee*
$0.01 to $2.50	5¢
2.51 to 5.00	7¢
5.01 to 10.00	10¢
10.01 to 20.00	12¢
20.01 to 40.00	15¢
40.01 to 60.00	18¢
60.01 to 80.00	20¢
80.01 to 100.00	22¢

Exercises

How much, including the amount of the order and the fee, would you pay for each of these postal money orders?

1. $2.00	**5.** $80.00	**9.** $1.20	**13.** $20.02
2. $18.00	**6.** $80.20	**10.** $2.80	**14.** $33.80
3. $25.00	**7.** $75.00	**11.** $4.75	**15.** $99.50
4. $62.00	**8.** $11.50	**12.** $5.12	**16.** $100.00

17. No money order is issued for more than $100. To send $150 by postal money order, you must buy two orders, one for $100 and one for $50. How much would these orders cost you?

18. What postal money orders must you buy, and what is the total fee, to send $120.12? to send $135.50? $240.35? $76.60?

19. Mr. Ford buys postal money orders for $100, $74.30, $65.48, $170, $4.02, and $160.21. How much could he save on them by having a checking account?

20. Money can also be sent by express money order and by telegraph. Find out what you can about these methods.

COMPUTING INTEREST BY THE 6% METHOD

Before learning about borrowing money from banks, you must review a very important method of computing interest, known as the *6%, or 60-day, method*, which you learned last year. The rule for finding interest by this method is as follows:

To find the interest at 6% *for* 60 *da., move the decimal point in the principal two places to the left; to find the interest for* 6 *da., move the decimal point three places to the left.*

1. Find the interest on \$400 at 6% for 72 da.

Change 72 da. to 60 da. + 6 da. + 6 da. 60 da. is \$4.00; for 6 da. it is \$.40; for 72 da. it is \$4.00 + \$.40 + \$.40, as shown at the right.

Always change the time into parts of, or multiples of, 60 da. or 6 da. Thus, 30 da. = $\frac{1}{2}$ of 60 da.; 120 da. = 2×60 da.; 2 da. = $\frac{1}{3}$ of 6 da.; 54 da. = 60 da. − 6 da., etc.

The interest on \$400 for

\$4.00 Int. for **60** da.
.40 Int. for **6** da.
.40 Int. for **6** da.
\$4.80 Int. for **72** da.

2. Find the interest on \$240 for 90 da. at 7%; at 5%.

The interest for 90 da. at 6% is \$3.60. The interest for 90 da. at 1% is $\frac{1}{6}$ of \$3.60, or \$.60. Hence the interest for 90 da. at 7% is \$3.60 + \$.60, or \$4.20.

Likewise, the interest on \$240 for 90 da. at 5% is \$3.60 − \$.60, or \$3.

\$2.40
1.20
\$3.60 Int. at **6%**
.60 Int. at **1%**
\$4.20 Int. at **7%**

Exercises

By the 6% *method, find the interest at* 6%, 7%, *and* 5% *on:*

1. \$200 for 60 da.
2. \$350 for 36 da.
3. \$800 for 90 da.
4. \$480 for 54 da.
5. \$900 for 80 da.
6. \$125 for 60 da.
7. \$950 for 12 da.
8. \$785 for 42 da.
9. \$370 for 96 da.
10. \$250 for 75 da.
11. \$500 for 66 da.
12. \$100 for 30 da.
13. \$280 for 90 da.
14. \$360 for 72 da.
15. \$120 for 78 da.
16. \$450 for 120 da.
17. \$150 for 152 da.
18. \$2400 for 93 da.

FINDING THE EXACT DAYS BETWEEN TWO DATES

Suppose you wish to borrow money from a bank from July 9 until Sept. 1. To find the interest, you must first count the exact number of days from July 9 to Sept. 1, thus:

From July 9 to July 31 there are	22 da.
In the whole month of August there are	31 da.
Sept. 1, the last day of the loan, counts as . . .	1 da.
Hence from July 9 to Sept. 1 there are	54 da.

In finding the exact number of days between two dates, count the last day, but not the first day.

In computing interest on loans, most banks count the exact number of days between dates and use 360 da. as a year.

Exercises

Find the exact number of days from:

1. Apr. 6 to Oct. 3
2. Aug. 1 to Dec. 18
3. Apr. 9 to Aug. 30
4. Mar. 3 to Aug. 18
5. Mar. 15 to Aug. 8
6. Mar. 28 to May 2
7. Mar. 2 to July 3
8. July 5 to Dec. 17
9. June 3 to Aug. 15
10. Aug. 4 to Sept. 16
11. Sept. 15 to Nov. 5
12. May 11 to Sept. 12

13. Mr. Lunt borrowed \$90 at 6% from May 1 to June 30. For how many days did he borrow it? What was his interest?

14. Mrs. Case borrowed \$1000 from Mar. 1 to May 12. How many days did she keep the money? Find the interest at 6%.

Counting exact days between dates and letting 360 *da.* = 1 *yr., find the interest at* 6%. *Use the* 6% *method:*

15. \$200; June 1 to Aug. 15
16. \$525; May 12 to Aug. 4
17. \$460; Apr. 10 to June 3
18. \$100; Mar. 4 to Apr. 3
19. \$650; Oct. 2 to Dec. 7
20. \$500; Sept. 16 to Dec. 15

A TABLE FOR FINDING SIMPLE INTEREST

Most banks find interest by using a table like that below.

Table for Finding Simple Interest at 6%

Based on 360 days per year

Days	$100	$200	$300	$400	$500	$600	$700	$800	$900	$1000
1	.017	.033	.05	.067	.083	.10	.117	.133	.15	.167
2	.033	.067	.10	.133	.167	.20	.233	.267	.30	.333
3	.050	.100	.15	.200	.250	.30	.350	.400	.45	.500
4	.067	.133	.20	.267	.333	.40	.467	.533	.60	.667
5	.083	.167	.25	.333	.417	.50	.583	.667	.75	.833
6	.100	.200	.30	.400	.500	.60	.700	.800	.90	1.000
7	.117	.233	.35	.467	.583	.70	.817	.933	1.05	1.167
8	.133	.267	.40	.533	.667	.80	.933	1.067	1.20	1.333
9	.150	.300	.45	.600	.750	.90	1.050	1.200	1.35	1.500
10	.167	.333	.50	.667	.833	1.00	1.167	1.333	1.50	1.667
20	.333	.667	1.00	1.333	1.667	2.00	2.333	2.667	3.00	3.333
30	.500	1.000	1.50	2.000	2.500	3.00	3.500	4.000	4.50	5.000
40	.667	1.333	2.00	2.667	3.333	4.00	4.667	5.333	6.00	6.667
50	.833	1.667	2.50	3.333	4.167	5.00	5.833	6.667	7.50	8.333
60	1.000	2.000	3.00	4.000	5.000	6.00	7.000	8.000	9.00	10.000
70	1.167	2.333	3.50	4.667	5.833	7.00	8.167	9.333	10.50	11.667
80	1.333	2.667	4.00	5.333	6.667	8.00	9.333	10.667	12.00	13.333
90	1.500	3.000	4.50	6.000	7.500	9.00	10.500	12.000	13.50	15.000
100	1.667	3.333	5.00	6.667	8.333	10.00	11.667	13.333	15.00	16.667

1. At 6%, find the interest for 50 da. on $200; on $20.

Find 50 da. in the left-hand column of the table, and lay a ruler or the edge of a sheet of paper under the 50 and horizontally across the table. The number just above the ruler in the column marked $200 is 1.667. Hence the interest on $200 for 50 da. at 6% is $1.667, or $1.67.

Since $20 is $\frac{1}{10}$ of $200, the interest on $20 is $\frac{1}{10}$ of $1.667, which is obtained by moving the decimal point in $1.667 one place to the left. This gives $.1667, or $.17, as the interest on $20 for 50 da. at 6%.

2. Find the interest on $220 for 50 da. at 6%.

Since $220 = $200 + $20, add the interest on $200 for 50 da. and the interest on $20 for 50 da., as found in ex 1. This gives $1.667 + $.1667, or $1.8337. Keep the extra decimal places in $1.667 and $.1667 until you get the sum, $1.8337. Then express the result as $1.83.

USING THE INTEREST TABLE

All loans below are at 6% simple interest.

1. Find from the interest table the interest on a 10-day loan of \$400; of \$600; of \$800; of \$1000. Do you have to change the position of your guiding paper in this example?

2. Find the interest on \$600 for 20 da.; on \$50 for 20 da.; on \$650 for 20 da.

3. Tell how to find the interest on \$730 for 30 da.

4. Find the interest for 40 da. on \$300; on \$30; on \$3; on \$333; on \$3000.

Using the table, find the interest, to the nearest cent, on:

5. \$600 for 6 da. **8.** \$50 for 30 da. **11.** \$330 for 60 da.

6. \$200 for 8 da. **9.** \$80 for 40 da. **12.** \$580 for 40 da.

7. \$900 for 7 da. **10.** \$90 for 80 da. **13.** \$140 for 20 da.

14. Using the table, find the interest on \$840 for 53 da.

Think of \$840 as \$800 + \$40, and of 53 da. as 50 da. + 3 da.

First find the interest for 50 da. on \$800 and on \$40, as shown at the right. Then find the interest for 3 da. on \$800 and on \$40.

Int. for	50 da. on	\$800 =	\$6.667
Int. for	50 da. on	40 =	.333
Int. for	3 da. on	800 =	.40
Int. for	3 da. on	40 =	.02
			\$7.420

Add these four results, which gives \$7.42 as the total interest.

15. Find the interest for 25 da. on \$700; on \$500; on \$900.

16. Find the interest on \$200 for 34 da.; for 93 da.

Using the table, find the interest, to the nearest cent, on:

17. \$550 for 48 da. **21.** \$200 for 166 da. **25.** \$325 for 90 da.

18. \$620 for 54 da. **22.** \$810 for 110 da. **26.** \$675 for 60 da.

19. \$380 for 42 da. **23.** \$950 for 170 da. **27.** \$148 for 50 da.

20. \$470 for 33 da. **24.** \$830 for 275 da. **28.** \$442 for 90 da.

29. Check your answers in ex. 25 to 28 by the 6% method.

BORROWING FROM A BANK

Fred Shaw is known by The Colonial Bank to be **reliable.** Therefore he is able to borrow from this bank $350 at 6% by writing this **promissory note:**

$350 00/100 Denver, Colo., Mar. 4, 19~

Three months after date I promise to pay

to the order of The Colonial Bank

Three hundred fifty and 00/100 ~~~~ Dollars

Value received

No. 148 Due June 4, 19~ Fred Shaw

Interest is not mentioned in this note because The Colonial Bank collects the interest *in advance.* Interest paid to a bank in advance is called **bank discount.**

Since the money is borrowed on Mar. 4 for 3 mo., it is due on June 4. It is exactly 92 da. from Mar. 4 to June 4. By the interest table you find that the interest on $350 for 92 da. at 6% is $5.37. The bank *discounts the note* by deducting this $5.37 in advance from the $350. Mr. Shaw receives the difference, $344.63, which is called the **proceeds.**

Exercises

1. How much does Mr. Shaw pay the bank on June 4?

2. Make out a note, dated Aug. 1, to any bank, promising to pay $90 in 60 da. Find the bank discount at 6%. How much do you get on Aug. 1? How much do you pay in 60 da.?

At 6%, find the bank discount and the proceeds on these loans:

3. $600 from Apr. 2 to May 2

4. $120 from Mar. 8 to Apr. 9

5. $180 from Aug. 15 to Nov. 15

6. $550 from Mar. 22 to July 23

7. $1400 for 60 da.

8. $1600 for 30 da.

9. $1250 for 30 da.

10. $2500 for 90 da.

11. Write the notes in ex. 7 to 10, supplying names, etc.

GIVING SECURITY FOR A NOTE

A man whose financial responsibility is not well known must give security to a bank for the money he borrows or else have some one in good standing indorse his note. Stocks and bonds are often given as security, or **collateral,** for a note.

$1500.00 Los Angeles, Aug. 16, 19—

On demand FOR VALUE RECEIVED I

promise to pay to The First National Bank or order, at said Bank, fifteen hundred and no/100 DOLLARS, with interest at the rate of 6% per annum, payable quarterly on the first days of January, April, July, and October, having deposited with said Bank as collateral security for the payment of this note the following property:

2 California Railroad $1000 - 5% bonds due 1944

having a present market value of $2020 * * * * I hereby give to said Bank full power to sell the whole or any part of said collaterals, on the non-performance of any of the promises herein contained.

* * * * * * * *

John Sherman

The above **collateral note** for $1500 is secured by bonds worth $2020. Because bonds change in value, banks usually lend only 80% or less of the value of the collateral.

This note is payable *on demand* and is called a **demand note.** If the bank should need the money or lose confidence in John Sherman, it could demand payment at once. Otherwise he can keep the money until he is ready to repay it. Most demand notes are secured by collateral or by indorsement. Since the date of payment of the demand note is not known, the interest is not collected in advance. It is usually paid quarterly, interest being charged for the exact number of days in the quarter.

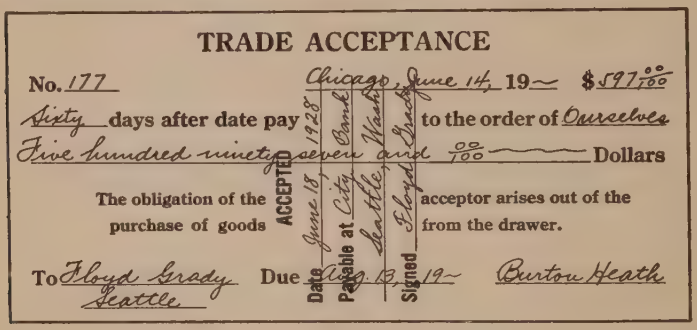
1. Tell on what dates interest is due on the above note. Why is the interest not paid in advance?

2. How many days are there from Aug. 16 to Oct. 1? At 6%, how much interest is paid on Oct. 1?

3. How much interest is due Jan. 1, 1929? If the $1500 is repaid Mar. 4, 1929, how much interest is due then?

DISCOUNTING A TRADE ACCEPTANCE

Burton Heath sells Floyd Grady $597 worth of goods and sends this draft, or **trade acceptance,** with the shipment:


TRADE ACCEPTANCE

No. 177 Chicago June 14, 19— $597.00

Sixty days after date pay to the order of Ourselves

Five hundred ninety seven and 00/100 Dollars

The obligation of the purchase of goods ACCEPTED June 18 1928 acceptor arises out of the from the drawer.

Payable at City Bank Seattle, Wash.

To Floyd Grady Seattle Due Aug 13, 19— Signed Floyd Grady Burton Heath

When "Accepted" is written across the face of the acceptance, it becomes a promissory note, which Mr. Grady promises to pay in 60 da.; but, unlike other notes, it draws no interest. After acceptance, Mr. Grady returns the paper to Mr. Heath.

If Mr. Heath needs the money on June 22, which is 52 da. before Mr. Grady has agreed to pay it, he may take the acceptance to a bank and *discount* it, receiving $597 less the bank discount. The bank discount on $597 for 52 da. at 6% is $5.17; hence Mr. Heath receives $597 − $5.17, or $591.83. The bank now holds the acceptance until Aug. 13, when Mr. Grady pays $597 to the bank.

Exercises

1. On Mar. 11 George Wills of Cincinnati sent Arthur Moon of Buffalo a trade acceptance for $384, payable in 90 da. Write out the acceptance and accept it for Mr. Moon. When is the acceptance due? If Mr. Wills sells it to a bank on Mar. 21, how much does he get for it if it is discounted at 6%?

2. Write a trade acceptance for $214, drawn on Glenn Owen, dated Sept. 5, and due in 30 da. Accept it for Mr. Owen. If on Sept. 15 you discount the acceptance at a bank at 5%, how much do you get for it? How much does Mr. Owen pay the bank?

DISCOUNTING AN INTEREST-BEARING NOTE

Owen Judd borrowed money from his friend, David Ross. He gave his friend this promissory note:

$800 $\frac{00}{100}$ Nashville, Tenn., Mar. 21, 19—

Ninety days after date I promise to pay

to the order of David Ross

Eight hundred and $\frac{00}{100}$ Dollars

with interest at 6%

Due June 19, 19— Owen Judd

This note states that the rate of interest is 6%. When the note is due, Mr. Ross will get the \$800 principal + \$12 interest, or \$812 in all. Why is interest mentioned on this note, but not on the note to a bank shown on page 128?

Suppose that on Apr. 26 Mr. Ross needs the \$800 very much. Mr. Judd does not have to pay the note until June 19. Therefore Mr. Ross indorses the note and sells it to a bank on Apr. 26. The total value of the note, including interest, will be \$812 on June 19; but the bank does not give Mr. Ross that amount. Since the bank is really lending Mr. Ross this \$812 from Apr. 26 to June 19 (54 da. in all), it deducts in advance the bank discount on \$812 for 54 da. at 6%, which is \$7.31. Hence Mr. Ross gets \$812 − \$7.31, or \$804.69, for the note. On June 19, when the note is due, Mr. Judd will pay \$812 to the bank. June 19 is the **day of maturity** of this note.

In figuring its discount, the bank finds the exact number of days to the day of maturity; but it counts 360 da. as 1 yr.

To find the bank discount on an interest-bearing note, first find the amount (interest + principal) to be paid on the note when it is due; then compute the bank discount on this amount, at the bank's rate, from the day on which the note is discounted until the day on which it is due.

DISCOUNTING NOTES AND TRADE ACCEPTANCES

Write notes bearing interest at 6% for the following, supplying names. Find the total amount due on each on the day of maturity, counting exact days between dates:

	Amount of Loan	Date of Loan	Date Due		Amount of Loan	Date of Loan	Date Due
1.	$480	June 1	Aug. 6	**5.**	$1200	Mar. 1	June 29
2.	$520	Apr. 4	Apr. 14	**6.**	$1600	May 2	Oct. 29
3.	$600	Mar. 18	June 17	**7.**	$2000	Apr. 21	May 11
4.	$850	Mar. 24	June 22	**8.**	$3600	Mar. 20	Dec. 31

9. The note in ex. 1 was sold to a bank on July 1. How many days did the bank hold it? Find the bank discount on it at 7%. How much did the bank pay the man who sold it? How much did the bank receive on Aug. 6?

10. Find the proceeds on the note in ex. 6 if it was discounted by a bank on July 1 at 6%. How much did the bank receive for it at maturity?

11. If the notes in ex. 3, 4, 5, 7, and 8 are each discounted on May 1 at 6%, find the proceeds on each.

12. John Miller sends Hugh White a trade acceptance for $450 on May 6, payable in 90 da. On what date is it due?

13. If Mr. Miller sold the trade acceptance (ex. 12) on June 20 to a bank which discounted it at 6%, how much did he get? How much did Mr. White pay the bank on Aug. 4?

14. Find the date of the day of maturity if it comes 90 da. after Mar. 2; 60 da. after Apr. 5; 30 da. after Oct. 15; 90 da. after May 1; 60 da. after Sept. 3.

15. Name two sources from which a business man may borrow money and five ways in which he can send money to Denver. Name two commercial forms which help him collect the bill for a shipment to Seattle, and tell the difference between them.

IMPROVEMENT TEST No. 6

Test 6 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
251	916	697	626	188
949	274	875	274	335
654	835	604	695	204
833	312	192	863	813
864	379	178	239	345
676	138	790	137	176
802	565	821	967	422
520	384	919	594	898
694	207	415	357	845
956	943	368	556	194
282	579	847	773	840
727	730	624	459	236

Test 6 B — Subtraction (Time 3 min.)

1. 6730724	2. 8914468	3. 5713531
1253365	1188887	1557244
4. 9758006	5. 6823257	6. 8572751
2773464	3457397	3112857
7. 6009948	8. 4207862	9. 9925161
1880469	2745565	6387342

Test 6 C — Multiplication (Time 3 min. after copying)

1.	2.	3.	4.	5.
954	829	706	583	673
285	139	975	781	624

Test 6 D — Division (Time 3 min. after copying)

1. $65\overline{)33735}$ 2. $27\overline{)18333}$ 3. $83\overline{)29631}$

4. $59\overline{)51035}$ 5. $47\overline{)36848}$

REVIEW

1. Paul Barron buys from Albert Wills of New York 60 dictionaries @ \$1.89 and 25 encyclopedias @ \$5.92, with discounts of $12\frac{1}{2}\%$, 5%, and an extra discount of 2% if he pays the bill within 10 da. Mr. Barron pays at once by a check drawn on the Merchants Bank of Baltimore. Write the check.

2. Mr. Wills has an account in the Grant Bank of New York, which does business with the Colonial Bank of Baltimore. Diagram the travels of the check in ex. 1 until it reaches the Baltimore Clearing House. What happens to it there?

3. Suppose that Mr. Barron buys a bank draft at the Merchants Bank of Baltimore, drawn on the Central Bank of New York, to pay the bill in ex. 1. Make out the draft, indorse it for Mr. Barron, and diagram its travels.

4. Louis Marsh of Denver owes Mr. Barron \$356, which is long past due. By what kind of draft can Mr. Barron try to collect the money? Write the draft and describe its travels.

5. On Oct. 3 Mr. Barron borrows \$535 at 6% from the Merchants Bank of Baltimore, giving a demand note for it. Write the note. Mr. Barron repays the money on Nov. 8. How much does he pay the bank on that date?

6. In what ways is it a convenience to Mr. Barron to have a checking account in the Merchants Bank?

Solve these equations and check your results:

7. $5x + 25 = 35$

8. $5x - 4x = 40$

9. $4x + 8x = 96$

10. $6x + 22 = 34$

11. $3x - 80 = 55$

12. $\frac{1}{3}x + 10 = 40$

13. $\frac{1}{5}x - 12 = 13$

14. $\frac{2}{3}x = 30$

15. $\frac{9}{10}x = 108$

16. $1\frac{3}{4}x = 350$

17. $2\frac{2}{3}x = 400$

18. $3\frac{1}{2}x = 87\frac{1}{2}$

19. $\frac{75}{100}x = 90$

20. $\frac{125}{100}x = 75$

21. $\frac{1}{2}x + 18 = 53$

22. $4x + \frac{1}{2}x = 81$

23. $x + \frac{1}{2}x = 270$

24. $x - \frac{1}{4}x = 1500$

25. $x + \frac{20}{100}x = 450$

26. $x + \frac{9}{100}x = 436$

27. $x - \frac{15}{100}x = 442$

CHAPTER VII

THRIFT AND COMPOUND INTEREST

KEEPING MONEY IN A SAVINGS BANK

Last year you learned that by putting your money in a savings bank, you can make it earn interest for you.

Most savings banks pay interest semiannually (twice a year) or quarterly (four times a year). If this interest is left in the bank, it also draws interest. Interest paid on the principal and also on the interest left in the bank is called **compound interest.** Interest paid only on the original principal is **simple interest.** The principal plus the interest is called the **amount.**

On Jan. 1, 1929, Leo puts $400 in a bank which pays 4% interest compounded semiannually. Hence he gets $\frac{1}{2}$ of 4%, or 2%, interest every half year. On July 1, 1929 his interest is 2% of $400, or $8. On Jan. 1, 1930 his interest is 2% of $408, or $8.16. How much in all has he in the bank on Jan. 1, 1930?

$400.00 Prin., Jan. 1, 1929
8.00 Int., July 1, 1929
$408.00 Prin., July 1, 1929
8.16 Int., Jan. 1, 1930
$416.16 Prin., Jan. 1, 1930

Exercises

1. What is Leo's interest on July 1, 1930? on Jan. 1, 1931?

Compute the interest due July 1 not on $416.16, but on $416, since, *in computing interest, savings banks disregard cents in the principal.*

2. Miss Adams invests $800 at 6% compounded semiannually. What rate of interest is that every 6 mo.? Find her interest and her new principal for the first 8 half years.

3. About how long did it take to work ex. 2? A shorter way to work such problems is to use a table like that on page 136.

A TABLE FOR FINDING COMPOUND INTEREST

Table Showing Amount of $1 at Compound Interest

Years or Periods	1%	2%	3%	4%	5%	6%
1	1.01000	1.02000	1.03000	1.04000	1.05000	1.06000
2	1.02010	1.04040	1.06090	1.08160	1.10250	1.12360
3	1.03030	1.06121	1.09273	1.12486	1.15763	1.19102
4	1.04060	1.08243	1.12551	1.16986	1.21551	1.26248
5	1.05101	1.10408	1.15927	1.21665	1.27628	1.33823
6	1.06152	1.12616	1.19405	1.26532	1.34010	1.41852
7	1.07214	1.14869	1.22987	1.31593	1.40710	1.50363
8	1.08286	1.17166	1.26677	1.36857	1.47746	1.59385
9	1.09369	1.19509	1.30477	1.42331	1.55133	1.68948
10	1.10462	1.21899	1.34392	1.48024	1.62889	1.79085
11	1.11567	1.24337	1.38423	1.53945	1.71033	1.89830
12	1.12683	1.26824	1.42576	1.60103	1.79586	2.01220
13	1.13809	1.29361	1.46853	1.66507	1.88565	2.13293
14	1.14947	1.31948	1.51259	1.73168	1.97993	2.26090
15	1.16097	1.34587	1.55797	1.80094	2.07893	2.39656
16	1.17258	1.37279	1.60471	1.87298	2.18287	2.54035
17	1.18430	1.40024	1.65285	1.94790	2.29202	2.69277
18	1.19615	1.42825	1.70243	2.02582	2.40662	2.85434
20	1.22019	1.48595	1.80611	2.19112	2.65330	3.20714
24	1.26973	1.60844	2.03279	2.56330	3.22510	4.04893
28	1.32129	1.74102	2.28793	2.99870	3.92013	5.11169
35	1.41660	1.99989	2.81386	3.94609	5.51602	7.68609

Using the Table

1. Find the amount of $1 in 2 yr. at 4% compounded yearly.

Hold a ruler horizontally under the 2 in the column marked "Years or Periods." The number just above the ruler in the 4% column is 1.08160 Hence the amount of $1 in 2 yr. at 4% is $1.08.

2. With interest compounded annually, find the amount to which $1 grows in 2 yr. at 5%; in 5 yr. at 6%; in 20 yr. at 4%; in 15 yr. at 3%; in 35 yr. at 6%; in 18 yr. at 5%, 4%, 3%, and 6%.

3. Find the amount of $50 in 7 yr. at 4% compounded yearly.

Find the amount of $1 for 7 yr. and multiply the result by 50.

4. Find the amount of $125 in 10 yr. at 4% compounded annually; of $250 in 18 yr. at 5% compounded annually.

COMPOUNDING INTEREST SEMIANNUALLY

Edna has \$200 in the bank at 4% interest compounded semiannually. To what amount will the \$200 grow in 2 yr.?

Since the yearly rate is 4%, the rate for each half year (or for each interest period) is 2%. In 2 yr. there are 4 half years, or 4 interest periods. Hence in the table on page 136, hold a ruler under 4 in the column marked "Years or Periods" and find the number just above the ruler in the 2% column. This gives \$1.08243 as the amount of \$1. Multiplying \$1.08243 by 200, you get \$216.49 as Edna's amount in 2 yr. Why do you look in the 2% column rather than in the 4% column?

Exercises

1. Peter puts \$50 in a bank at 4% interest compounded semiannually. How long is each interest period? How many interest periods are there in 3 yr.? What is the rate for a period? Find the amount to which the \$50 will grow in 3 yr.

Find the amounts, by the table, at 6% compounded semiannually:

	Principal	Time		Principal	Time		Principal	Time
2.	\$400	2 yr.	**7.**	\$550	6 yr.	**12.**	\$385	10 yr.
3.	\$300	3 yr.	**8.**	\$275	5 yr.	**13.**	\$270	12 yr.
4.	\$910	1 yr.	**9.**	\$900	8 yr.	**14.**	\$480	14 yr.
5.	\$650	4 yr.	**10.**	\$157	9 yr.	**15.**	\$868	14 yr.
6.	\$380	7 yr.	**11.**	\$750	3 yr.	**16.**	\$452	$17\frac{1}{2}$ yr.

17. Mrs. Dean invested \$480 at 4%, compounded twice a year. How much did she have after 2 yr.? after 4 yr.? 6 yr.? 8 yr.? 9 yr.? 10 yr.? How much of each amount was interest?

18. Lucy puts \$100 in a savings bank at 4% compounded quarterly. How long is each interest period? How many such periods are there in 5 yr.? What is the rate of interest for a period? What amount has Lucy in the bank after 5 yr.?

19. Find the amount to which \$760 invested at 4% compound interest, payable quarterly, will grow in 6 yr.

WHAT A MAN LOSES BY HOARDING HIS MONEY

Use the interest table on page 136 *for these problems.*

1. Many people hide or bury their money for safe keeping. Can you tell a safer and wiser thing to do with it?

2. The peasants of one country have about $400,000,000 in gold hidden on their farms. To about what amount would this sum grow in a bank in 10 yr. at 4% compounded yearly? How much interest could it thus earn for the peasants in 10 yr.?

3. One peasant kept $100 buried for $17\frac{1}{2}$ yr. Show that if he had invested it at 4% compounded semiannually, it would have grown to almost twice its original value in $17\frac{1}{2}$ yr.

4. Another man had $600 hidden in a box. His neighbor had $450 in a safe. Show that each sum would practically double in $17\frac{1}{2}$ yr. if invested at 4% compounded semiannually.

5. If the peasant in ex. 3 had invested his $100 at 5% compounded annually, to what amount would it have grown after 14 yr.? Compare this amount with the original principal. Would the amount have been even greater if interest had been compounded semiannually?

6. Suppose that the peasant in ex. 3 had found a good investment paying 6% interest compounded every 6 mo. Show that his money would have more than doubled in 12 yr.

With interest compounded semiannually, money doubles in $17\frac{1}{2}$ yr. at 4%, in 14 yr. at 5%, and in 12 yr. at 6%.

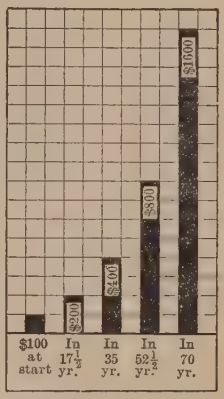
7. Jack is now $10\frac{1}{2}$ yr. old. If he puts $100 in a bank at 4%, compounded twice a year, to how much will it amount when he is 28 yr. old? Since this amount will double again in $17\frac{1}{2}$ yr., how much will it be when he is $45\frac{1}{2}$ yr. old?

8. Lena has $50 in gold in her jewel box. How much interest does she lose on it in 12 yr. by not putting it in a savings bank at 4% compounded semiannually?

HOW MONEY GROWS AT COMPOUND INTEREST

Albert wins \$100 for an essay when he is 14 yr. old. He puts the money in a bank at 4% interest compounded semiannually. In $17\frac{1}{2}$ yr. the investment will double and he will have \$200 instead of \$100. In another $17\frac{1}{2}$ yr. this \$200 will also double and he will have \$400. If he leaves this \$400 in the bank for $17\frac{1}{2}$ yr., it will grow to \$800. How many more years will it take for this \$800 to double?

In $17\frac{1}{2}$ years \$100 grows to \$200.
In 35 years \$100 grows to \$400.
In $52\frac{1}{2}$ years \$100 grows to \$800.
In 70 years \$100 grows to \$1600.
In $87\frac{1}{2}$ years \$100 grows to \$3200.



How Money Grows at Compound Interest

Exercises

1. Continue the table above to show the amount to which \$100 grows in 105 yr.; in $122\frac{1}{2}$ yr.; in 140 yr.; in $157\frac{1}{2}$ yr.

2. Show that if a man had invested \$140 in 1789 and if it had been compounded semiannually at 4%, it would have grown to \$35,840 by 1929.

3. John has been spending 5¢ a day for candy. At the age of 13 he saves this money for one year (365 da.) and invests it at 4% compounded semiannually. How much will this money amount to when John is $30\frac{1}{2}$ yr. old? 48 yr. old? $65\frac{1}{2}$ yr. old?

4. Abe spent \$15 for a toy train when he was 11 yr. old. By the time he was 46 yr. old to how much would this money have grown in a savings bank at 4% compounded semiannually?

At 4% interest compounded semiannually, show to what amount the following will grow in 35 yr.; in 70 yr.; in 175 yr.:

5. \$1 **6.** \$5 **7.** \$20 **8.** \$35 **9.** \$200 **10.** \$250

LETTING YOUR MONEY WORK ITS HARDEST

Have you a 10-dollar bill which you are keeping in your toy bank? If so, you are letting it waste energy with which it could earn money for you.

Tim had two 10-dollar bills. He lent Bill No. 1 at 6% simple interest and invested Bill No. 2 at 6% interest compounded semiannually.

1. How much simple interest did Bill No. 1 earn in 1 yr.? in 2 yr.? in 3 yr.? in 5 yr.?

2. From the table on page 136, show that at 6% compounded semiannually Bill No. 2 earned $3.44 in 5 yr. How much more did this 10-dollar bill earn in 5 yr. than Bill No. 1?

3. Show that Bill No. 2 was worth more than twice its original value in 12 yr. How much more does the $10 at compound interest earn in 12 yr. than the $10 at simple interest?

4. Tim's brother kept his money at home in a box. By not having his money earn all it could, how much did he lose in 12 yr. on one 10-dollar bill? on ten 10-dollar bills?

Assume that Tim's money could have earned interest at the rate of 6%, compounded semiannually.

5. On July 2, 1828, Dirk Van Dorn deposited $15 in a savings bank of New York. On July 1, 1928, when his great-grandson inherited the account, the $15, at an average of 5% interest compounded annually, had grown to $1972.52. How much of this amount was interest? How much less would the $15 have earned in the same period at 5% simple interest?

6. How much would Dirk have lost for his heirs if he had kept the $15 at home instead of investing it as he did?

7. Show that $1000 at 6% compounded semiannually earns $43.92 more in 5 yr. than $1000 at 6% simple interest.

8. In 7 yr., how much more will $1500 earn at 4% interest compounded quarterly than at 4% simple interest?

HELPING YOUR SAVINGS TO GROW

You have seen that money left in the bank grows quickly. It grows even more quickly if you add to it regularly.

Table Showing Amount if $1 Per Year Is Deposited

This table gives the total amount (with compound interest) accumulated *at the end of each year* if $1 is deposited *at the beginning of each year*.

Years or Periods	2%	$2\frac{1}{2}$%	3%	4%	5%	6%
1	1.0200	1.0250	1.0300	1.0400	1.0500	1.0600
2	2.0604	2.0756	2.0909	2.1216	2.1525	2.1836
3	3.1216	3.1525	3.1836	3.2465	3.3101	3.3746
4	4.2040	4.2563	4.3091	4.4163	4.5256	4.6371
5	5.3081	5.3877	5.4684	5.6330	5.8019	5.9753
6	6.4343	6.5474	6.6625	6.8993	7.1420	7.3938
7	7.5830	7.7361	7.8923	8.2142	8.5491	8.8975
8	8.7546	8.9545	9.1591	9.5828	10.0266	10.4913
9	9.9497	10.2034	10.4639	11.0061	11.5779	12.1808
10	11.1687	11.4835	11.8078	12.4864	13.2068	13.9716
18	21.8405	22.9460	24.1169	26.6712	29.5390	32.7600
20	24.7833	26.1833	27.6765	30.9692	34.7193	38.9927
30	41.3794	45.0003	49.0027	58.3283	69.7608	83.8017
34	48.9945	53.9282	59.4621	72.6522	89.3203	110.4348
40	61.6100	69.0876	77.6633	98.8265	126.8398	164.0477
50	86.2710	99.9215	116.1808	158.7738	219.8154	307.7561

Studying the Table

1. If you deposit $1 in a bank at the beginning of each year at 4% compounded annually, the interest period is 1 yr. At the end of 1 yr. the principal and interest total $1.04. Where is this shown in the above table?

2. If, at the beginning of the second year, you add $1 to the $1.04, making $2.04, your interest for this year is 4% of $2.04, or $.0816, and your amount at the end of the second year is $2.04 + $.0816, or $2.1216. Where is this shown in the table?

3. If you deposit $1 at the beginning of each year at 4% compounded yearly, find from the table your amount at the end of 3 yr.; of 4 yr.; of 5 yr.; of 6 yr.; of 7 yr.; of 30 yr.

WATCHING YOUR SAVINGS GROW

1. If at the beginning of each year you put \$25 in a bank that pays 4% interest compounded annually, what amount will you have at the end of 5 yr.? at the end of 20 yr.? of 30 yr.?

For each period, find from the table on page 141 the amount if \$1 is deposited every year. Then multiply the result by 25.

2. The day John Allen was born his father began to invest \$100 a year at 6% compounded annually, to build up a fund for John's college education. If John went to college when he was 18 yr. old, how large was the fund by that time?

Find the amounts if the following sums are deposited yearly with interest compounded annually. Use the table on page 141.

	Deposit	Rate	Time		Deposit	Rate	Time
3.	\$25	5%	4 yr.	**6.**	\$100	5%	9 yr.
4.	\$50	4%	6 yr.	**7.**	\$250	6%	30 yr.
5.	\$60	5%	7 yr.	**8.**	\$275	6%	40 yr.

DEPOSITING MONEY EVERY HALF YEAR

1. Will Davis deposited \$10 every half year at 4% compounded semiannually. Hence there were two interest periods each year and he received 2% each period. Find, from the table on page 141, how much he had at the end of 1 yr.

2. How many interest periods were there in 2 yr.? Find the amount of Will's deposits at the end of 2 yr.; at the end of 3 yr.; 4 yr.; 5 yr.; 10 yr.; 15 yr.; 20 yr.; 25 yr.

3. How much did Will's amount grow from the end of the 5th to the end of the 10th year? from the end of the 10th to the end of the 15th? from the 15th to the 20th? the 20th to the 25th? In which 5-year period was its growth greatest?

4. Clara said that Will's money grew almost twice as fast from the 20th to the 25th year as it did from the 5th to the 10th year. Was she right?

DEPOSITING MONEY EVERY HALF YEAR

1. If Elsie deposits \$10 at the beginning of each half year at 4% compounded semiannually, she has \$111.69 in 5 yr., \$247.83 in 10 yr., \$1163.33 in 30 yr., \$1529.78 in 35 yr., \$1976.47 in 40 yr., and \$2521.00 in 45 yr. How much do her savings grow from the 30th to the 35th year? from the 35th to the 40th year? from the 40th to the 45th year? The growth for each of these 5-year periods is about how many times as great as that from the 5th to the 10th year?

At compound interest savings grow faster and faster the longer they are left in the bank. During each 5 years the gain is much greater than during the preceding 5 years.

2. How much has Rose after 5 yr. if she deposits \$15 every 6 mo. at 5% interest compounded semiannually?

In ex. 2 to 11, use the table on page 141.

3. How much does Rose (ex. 2) deposit in 5 yr.? How much of her total amount at the end of 5 yr. is interest?

4. Bert's grandfather has smoked three 15-cent cigars a day for over 50 yr. How much could he have saved in a year (365 da.) by not smoking? If he had deposited this amount once a year for 50 yr., to what sum would it have grown in 50 yr. at 4% compounded annually?

5. Anna spends 10¢ a day for candy. How much does she spend for candy in 6 mo. of 30 da. each? If she deposited this sum every 6 mo. at 5% compounded semiannually, how much would she have 10 yr. after the first deposit?

Find the amounts if these sums are deposited every 6 mo. and if the interest is compounded semiannually:

	Deposit	Rate	Time		Deposit	Rate	Time
6.	\$20	6%	4 yr.	**9.**	\$100	5%	17 yr.
7.	\$85	5%	5 yr.	**10.**	\$400	6%	10 yr.
8.	\$70	4%	15 yr.	**11.**	\$550	4%	20 yr.

GETTING THE ADVANTAGES OF COMPOUND INTEREST

1. On Jan. 2, 1922, Mr. Smith loaned Mr. Burt $1000 for 6 yr. Mr. Burt paid 6% simple interest on the $1000 at the end of every year. As soon as Mr. Smith received the $60, he reinvested it at 6% interest compounded annually. On Jan. 2, 1928, at the end of 6 yr., Mr. Burt paid back the $1000, and also the last year's interest. How much had Mr. Smith then, including the money he had reinvested and the interest on it?

On Jan. 2, 1923, at the beginning of the 2d year, Mr. Smith invested the first $60 at 6% compounded annually. He also invested $60 at the beginning of the 3d, 4th, 5th, and 6th years, making in all 5 yearly deposits. Using the table on page 141, show that yearly deposits of $60 for 5 yr. at 6% amount to $358.52. To find how much Mr. Smith had on Jan. 2, 1928, add to this $358.52 the $1000 that Mr. Burt paid back together with Mr. Burt's last interest payment of $60.

2. If on Jan. 2, 1922 Mr. Smith had invested the $1000 at 6% compounded annually instead of lending it to Mr. Burt, how much would he have had in 6 yr.? (See table, page 136.) Compare this with the amount he had in ex. 1 after 6 yr.

3. When Ella was $12\frac{1}{2}$ yr. old she received a $100 Liberty bond, which paid interest each 6 mo. at 4%. She put each interest payment, except the last, in a savings bank at 4% compounded semiannually. How much had Ella in the bank at the end of $17\frac{1}{2}$ yr. from the time she received the bond?

Ella deposited $2 each period for 34 periods; this assumes that the first interest payment was due 6 mo. after she received the bond.

4. When Ella was 30 yr. old (after $17\frac{1}{2}$ yr.) she sold the bond for $100 and received the last $2 interest. How much had she in all, including the amount in the savings bank?

5. If Ella had sold the bond for $100 when she was $12\frac{1}{2}$ yr. old and had put the $100 in the bank at 4% compounded semiannually, how much would she have had after $17\frac{1}{2}$ yr.?

If money is invested at simple interest, the benefits of compound interest may be obtained by reinvesting the interest

IMPROVEMENT TEST No. 7

Test 7 A — Reduction (Time 3 min.)

Change to exact decimals:

1. 99%	**2.** 3%	**3.** 200%	**4.** 0.1%	**5.** $\frac{1}{2}$%
6. 10%	**7.** 9%	**8.** 106%	**9.** 5.5%	**10.** $\frac{1}{4}$ of 1%

Change to common fractions in lowest terms:

11. 60%	**12.** 1%	**13.** $12\frac{1}{2}$%	**14.** 16%	**15.** 95%

Change to per cents:

16. $\frac{5}{6}$	**17.** 9	**18.** .08	**19.** .215	**20.** .092
21. $\frac{9}{10}$	**22.** .5	**23.** $.06\frac{1}{4}$	**24.** .006	**25.** 2.25

Test 7 B — Finding Per Cents (Time 4 min. after copying)

Find answers correct to the nearest cent:

1. 5% of \$34.50	**2.** $\frac{1}{5}$% of \$590
3. 135% of \$150	**4.** 82.7% of \$400
5. $6\frac{1}{4}$% of \$3470	**6.** 200% of \$550.95
7. $66\frac{2}{3}$% of \$175.11	**8.** $\frac{1}{10}$ of 1% of \$1258

Test 7 C — Comparing Numbers (Time 4 min. after copying)

What numbers should be put in place of the dots?

1. 69 is . . .% of 1150	**2.** 32 is . . .% of 6400
3. 203 is . . .% of 725	**4.** 8856 is . . .% of 7380
5. 1560 is . . .% of 2340	**6.** 6806 is . . .% of 3320

Test 7 D — Per Cent of Increase (Time 3 min. after copying)

What per cent greater is the second number than the first?

1. 3140; 3611 **2.** 5862; 11,724 **3.** 5600; 21,566

NOTE. To score Test 7 A, find what per cent of the examples are right. If 88% are right, the score is 9; if 84% are right, the score is 8, etc.

DAVID SAVES $1 A MONTH

David Carter saves $1 a month. He can put this money in a savings bank at 4%, payable quarterly, or in a building and loan association at 6%, payable quarterly. The table below tells how his money would grow at each rate. After 5 yr. he would have $66.4974 at 4% or $70.0647 at 6%.

AMOUNT ACCUMULATED AT THE END OF EACH YEAR IF $1 IS DEPOSITED AT THE BEGINNING OF EACH MONTH. INTEREST COMPOUNDED QUARTERLY

YEARS	4%	6%	YEARS	4%	6%
1	12.2624	12.3954	9	130.0922	143.2462
2	25.0227	25.5515	10	147.6368	164.4317
3	38.3012	39.5149	12	184.8923	210.7826
4	52.1187	54.3351	14	225.2346	262.9966
5	66.4974	70.0647	16	268.9195	321.8151
6	81.4599	86.7596	18	316.2240	388.0739
7	97.0299	104.4789	20	367.4480	462.7139
8	113.2321	123.2855	25	514.8538	693.2732

1. David puts $1 a month in a building and loan association at 6%. His brother puts $1 a month in a bank at 4%. How much more will David have than his brother after 2 yr.? after 5 yr.? 8 yr.? 12 yr.? 20 yr.? 25 yr.?

2. How much does David deposit in 1 yr.? in 2 yr.? in 4 yr.? in 6 yr.? How much of his amount in 6 yr. is interest?

3. Jenny Blair, who is 13 yr. old, earns money by telling stories to children in her neighborhood. How much will she have when she is 18 yr. old if she invests $12 of her earnings every month at 6% compounded quarterly?

The amount for $12 is 12 times the amount for $1.

4. For 10 yr. Ruth has invested $4 at the beginning of each month at 4% compounded quarterly. Doris has invested $48 at the end of each year at 4% compounded annually. Doris now has $576.29. How much more has Ruth than Doris?

CHAPTER VIII

INSTALLMENT BUYING

FINDING THE RATE OF INTEREST

1. John borrowed \$120 from Mr. Fox. He paid it back in 1 yr. with \$18 interest. What rate of interest did he pay?

Since John paid \$18 interest on \$120 for 1 yr., find what per cent \$18 is of \$120. Since \$18 is $\frac{18}{120}$, or 15%, of \$120, John paid 15% interest.

2. Alice borrowed \$80 from Mr. Wood. She repaid it in 3 mo. with \$5 interest. What rate of interest did she pay?

If the interest is \$5 for 3 mo., this is at the rate of \$20 interest for 1 yr. Since \$20 is 25% of \$80, Alice paid interest at the rate of 25% yearly.

To find the yearly rate of interest, first find the amount of interest that would be charged if the loan were for 1 year; then find what per cent this amount is of the principal.

Unless otherwise specified, the rate of interest charged in borrowing money always means the rate for one year.

Exercises

1. John Carter borrowed \$150 from Paul Wirt, who charged him \$4.50 a month interest. How much interest did he pay in a year? What yearly rate was he paying?

2. Mr. Mace paid \$18 quarterly interest on a note for \$1200. Find the annual interest; the rate of interest.

Find the yearly rate of interest on the following:

	Principal	Time	Interest		Principal	Time	Interest
3.	\$300	1 yr.	\$30	**7.**	\$1500	3 mo.	\$30
4.	\$450	1 yr.	\$27	**8.**	\$2000	1 mo.	\$80
5.	\$350	3 mo.	\$35	**9.**	\$1750	2 yr.	\$245
6.	\$125	6 mo.	\$15	**10.**	\$3600	3 yr.	\$540

BUYING ON THE INSTALLMENT PLAN

1. The Miller family want to buy some furniture. It will cost \$88 if they pay all cash or \$98 if they pay \$18 cash and the rest in installments of \$16 a month.

How many months will it take the Miller family to pay for the furniture if they buy it on the installment plan?

2. If the Millers already have \$18 and can save \$16 a month, how long would it take them, to the nearest half month, to save enough to pay cash for the furniture? How much sooner can the family have the use of the furniture if they buy it on the installment plan?

3. Some people find it easier to pay a small amount weekly on furniture which they already have than to save up enough money in advance to pay all cash for the furniture. Why?

4. When a store sells on the installment plan, it has the extra expenses of investigating its customer's reputation, of collecting the weekly or monthly payments, and of taking the risk that some customer may fail to pay all his installments. Is the extra \$10 which is charged on the installment plan in ex. 1 all extra profit to the company? Is this extra \$10 an extra cost to the customer who buys on installments?

5. A farmer can buy a double harness by paying \$54.75 cash or by paying \$5 down and \$5 a month for 11 mo. How much more is the installment price than the cash price?

6. An electric washing machine is sold for \$20 down and payments of \$7 a month for 1 yr. A discount of $12\frac{1}{2}\%$ is offered for cash payment. How much can Mrs. Thomas save by paying cash when she buys the machine?

7. A piano can be purchased for \$420 cash or on the installment plan with the following payments: \$10 down and \$12 each month for 38 mo. Find the difference between the cash price and the installment price.

RATE OF INTEREST IN INSTALLMENT BUYING

Mrs. Ames wants some new dining-room furniture, the cash price of which is \$120. She cannot spare \$120 in cash and she cannot borrow it from a bank, because she has nothing to offer as security. The dealer, however, will sell her the furniture for \$135 and let her pay \$30 down and \$15 a month for the next 7 mo. Thus the dealer, by letting Mrs. Ames retain the use of part of the price of the furniture, is really lending her money. Find the yearly rate of interest on this loan.

First find how much the dealer really lends Mrs. Ames. If Mrs. Ames buys the furniture for \$135 on Feb. 1 and pays \$30 down, she owes \$135 − \$30, or \$105, during February. On Mar. 1 she pays another \$15 and thus reduces the loan during March to \$90. On Apr. 1 she pays another \$15; therefore she owes only \$75 during April. How much does she owe during May? during June? during July? during August?

Feb.	\$105 for 1 mo.
Mar.	90 for 1 mo.
Apr.	75 for 1 mo.
May	60 for 1 mo.
June	45 for 1 mo.
July	30 for 1 mo.
Aug.	15 for 1 mo.
Total	\$420 for 1 mo.

The table above shows the amounts which Mrs. Ames owes for periods of 1 mo. each. Adding these amounts, you find that she owes in all the equivalent of \$420 for 1 mo.

Since Mrs. Ames pays \$135 for furniture whose cash price is \$120, she pays \$15 more than the cash price. This \$15 may be considered as interest on the money she owes. Hence she pays \$15 interest on a total loan of \$420 for 1 mo. To find the rate of interest, you must first find the amount of interest that would be charged for 1 yr., as shown on page 147. Interest of \$15 a month on \$420 is the same as interest of \$180 a year on \$420. Hence the rate of interest which Mrs. Ames pays on the \$420 is $\frac{180}{420}$, or about 43%. Is 43% a high yearly rate of interest to pay on a loan?

INTEREST IN INSTALLMENT BUYING

1. The cash price of a phonograph is \$118. Charles buys it for \$145 on the installment plan, paying \$25 down and \$15 monthly for 8 mo. Show, by a table like that on page 149, that the dealer lends him the equivalent of \$540 for 1 mo. How much interest does Charles pay for 1 mo.? Find the equivalent yearly interest and the yearly rate of interest.

Find, to the nearest whole per cent, the rate of interest charged under the installment plan on the following:

2. *Electric Washing Machine*

Cash Price	\$75
Installment Price	\$85
\$5 cash, \$10 monthly	

3. *Great Authors Series*

Cash Price	\$18
Installment Price	\$20
Terms, \$2 down and \$3 a month	

4. *Higrade Gas Stove*

Cash Price	\$44
Installment Price	\$50
\$10 cash, \$5 a month	

5. *Athletes' Dumb-bells*

Cash Price	\$25
Installment Price	\$30
Terms, \$15 down, \$3 weekly for 5 weeks	

6. Mrs. Roe can buy a fur coat on the installment plan for \$150 on these terms: \$15 down and \$15 a month for 9 mo. If she pays all cash, she gets a discount of 10% from the price of \$150. What is the cash price? Find, to the nearest whole per cent, what annual rate of interest Mrs. Roe pays if she buys the coat on the installment plan.

7. At the Star Store a vacuum cleaner can be purchased for \$65 cash, or for \$12.50 down and \$5 a month for 12 mo. At Green's Department Store the same kind of vacuum cleaner can be purchased for \$65 cash or for \$12 down and \$10 a month for 6 mo. The installment plan is equivalent to paying what rate of interest at the Star Store? at Green's Store? Which store makes the better offer?

PROBLEMS

1. Bertha Foster bought a rebuilt typewriter for $50, paying $5 down and $5 a month. After 2 mo. she did not need the typewriter; therefore she stopped her monthly payments and the company took back the typewriter. How much did it cost her to have the typewriter for 2 mo.?

2. Bertha (ex. 1) could have rented a similar typewriter at $2 a month. How much cheaper would this have been than buying it on the installment plan and failing to pay after 2 mo.? Is it wise to buy on the installment plan if you are not sure that you will make all the payments?

3. If Bertha had used the typewriter for 3 yr., how much cheaper would buying it on the installment plan have been than renting it (see ex. 1 and 2)? At the end of the 3 yr. who would have owned the typewriter in each case?

4. Clyde Smith found that he could buy a radio set for $150 cash or for $175 on the following terms: $25 down and $25 a month for the next 6 mo. If he bought the set on the installment plan, what yearly rate of interest would he have to pay?

5. Clyde's uncle offered to lend him $125 for 5 mo. at 6%. If Clyde borrows this amount from his uncle to buy the radio set for cash, how much cheaper will this be for Clyde than buying on the installment plan?

Clyde would have to borrow only $125 from his uncle, because even on the installment plan he would have to pay $25 down.

6. On Jan. 1 Harvey Mason borrowed $100 from the Bell Loan Company, agreeing to pay it back at the rate of $10 monthly for 10 mo., beginning Feb. 1. The company charged him $6 interest and $2 for investigating his reputation. So far as Harvey was concerned, this was equivalent to a total interest charge of $8. What rate of interest did Harvey really pay?

As on the installment plan, he owed $100 for 1 mo., $90 for 1 mo., etc.

THE COST OF INSTALLMENT BUYING

1. How do the rates of interest on installment buying, as found on pages 150 and 151, compare with the rates usually charged for loans by banks?

2. Does a bank that lends money to a well-known man on good security take as great a risk as a furniture store that sells to comparative strangers on the installment plan? Does the bank or the store have more work in investigating its customers, in collecting the payments, and in bookkeeping?

3. Many states have laws that allow interest rates of from 1% to 3% a month on small loans (loans of not over $300). To what yearly rates are these rates equal? Which rates found on pages 150 and 151 are within the limits permitted for small loans? Which rates are higher?

Advantages and Disadvantages of Installment Buying

You have seen that the installment plan is often a help to the purchaser in enabling him to have the use of articles sooner than if he had to pay cash. It also forces him to save regularly in order to make regular payments. These advantages are so apparent that about $\frac{1}{7}$ of the retail buying in the United States is now done on the installment plan.

If, however, a person can pay cash, it is much cheaper to do so, because it costs the merchant more to sell on the installment plan. The customer should consider installment buying as the equivalent of a small loan and see what yearly rate of interest he will have to pay. If the rate is too high, he will be wise to wait until he has saved enough to pay cash for the article. The rate is usually higher for luxuries, like jewelry, than it is for necessities, like household goods. While the customer has to pay a high rate of interest in installment buying, the merchant does not make so high a rate on his money. Why not?

BUYING HOMES ON THE INSTALLMENT PLAN

Houses are often purchased on the installment plan. One way to do this is to borrow money to buy the house and then to pay back each month a certain part of the amount borrowed.

Suppose that Mr. Cox, in order to buy a house, borrows \$6000 from an insurance company, agreeing to pay the \$6000 back at the rate of \$42 a month and also to pay interest each month on the amount he still owes, at an annual rate of 6%.

At the end of the first month Mr. Cox pays \$42 plus interest on \$6000 for 1 mo. at 6%, which is \$30. Hence he pays \$42 + \$30, or \$72, the first month. Since only \$42 of this payment is used to reduce the principal, he then owes \$6000 − \$42, or \$5958. The second month he again pays \$42 plus interest on \$5958 for 1 mo. at 6%, which is \$29.79. Hence he pays \$71.79 the second month. The principal is then \$5958 − \$42, or \$5916.

Each month's interest is computed by the 6% method (page 124). Hence the interest on \$5958 is \$59.58 for 2 mo., or \$29.79 for 1 mo.

As the principal decreases \$42 each month, the interest and the total payment also decrease, as shown below:

	Principal	Payment on Principal	Interest on Principal	Total Payment
For 1st month	\$6000	\$42	\$30.00	\$72.00
For 2d month	5958	42	29.79	71.79
For 3d month	5916	42	29.58	71.58
For 4th month	5874	42	29.37	71.37

Exercises

1. Continue the above table for the next 12 months.

2. How much will Mr. Cox pay back on the principal in 4 yr.? What will be the principal after the 48th payment? What will be the interest on the 49th payment?

3. Find the principal after the 138th payment. Then make a table like that above, showing the payments after the 138th month. How long does it take to pay the entire debt?

OWN YOUR OWN HOME, PAY LIKE RENT

Here is another way to buy a house on monthly payments:

Mr. Howe sells a house to Mr. Lane for $9000. Mr. Lane pays $3000 cash and gives Mr. Howe a **mortgage** for the other $6000, agreeing to pay it off in monthly installments of $90. Part of this $90 is interest on the $6000 at 6% and the rest goes toward reducing the mortgage. Hence, instead of paying rent, Mr. Lane is using the $90 a month to buy his own house.

A mortgage gives Mr. Howe the right to sell the house if Mr. Lane does not make his payments.

The first month's interest on $6000 is $30. Hence, of the first payment of $90, $30 is interest and the other $60 is used to reduce the debt. Mr. Lane then owes $6000 − $60, or $5940.

The second month Mr. Lane again pays $90, out of which $29.70 is deducted as interest for 1 mo. on $5940, leaving $60.30 to be applied for reducing the principal. The principal then becomes $5940 − $60.30, or $5879.70.

The table below shows how this plan of payment works.

	Principal	Total Payment	Interest on Principal	Reduction on Principal
1st month	$6000.00	$90	$30.00	$60.00
2d month	5940.00	90	29.70	60.30
3d month	5879.70	90	29.40	60.60
4th month	5819.10	90	29.10	60.90

BUYING HOUSES BY MONTHLY PAYMENTS

1. As the principal in the table on page 154 is reduced each month, how does the amount of interest change? Does the reduction on the principal each month increase or decrease? Is there any change in Mr. Lane's total payment each month? How does the plan on page 154 differ from that on page 153?

2. The interest for the third month, in the table on page 154, is \$29.40. On what amount is this interest computed? Show how to find this interest by the 6% method.

3. The interest for the fourth month is \$29.10. On what amount is this computed? Show how it is computed.

4. Copy the table given on page 154 and continue it for 20 mo. more. Compute all interest to the nearest cent.

5. What will the principal be at the end of 2 yr. (at the beginning of the 25th month)? Is the principal being reduced more or less quickly now than during the first year?

6. Mr. Adams bought a house for \$12,000. He paid \$4000 cash; the other \$8000 he paid by monthly payments of \$75, which included interest at 6%. Make a table like the one on page 154, showing the first 12 payments.

7. Alice Jones bought a bungalow for \$2000. She paid \$500 down and the balance like rent. Her monthly payment, including interest at 6%, was \$25. Make a table like the one on page 154, showing the first 12 payments.

8. Mr. Todd bought a house for \$5000, paying \$2000 down. The monthly payment, including interest at 6%, was \$60. Make a table like that on page 154 for the first 6 payments.

9. In ex. 8, suppose that each monthly payment of \$60 was applied entirely to reducing the principal and that each month's interest was paid in addition to the \$60, as on page 153. Make a table showing the first 6 payments.

HOW LONG WILL IT TAKE TO PAY THE MORTGAGE?

When Mr. Lane buys his house (see page 154), he wants to know how many months it will take him to pay off the $6000 mortgage. To find out exactly by computing the balance due after every payment of $90 would take several hours; but a rough estimate can be made in the following way:

The table on page 154 shows that the first reduction of the principal is $60. Since the interest becomes smaller each month and the reduction of the principal larger, the last payment of $90 is almost entirely a reduction of the principal. Hence, for a rough estimate, you may consider all of this last $90 as a reduction of the principal. Then the first monthly reduction is $60 and the last is $90. Adding these amounts and dividing the result by 2, you get $75 as the average monthly reduction. Dividing the total mortgage of $6000 by $75, you get 80 as the approximate number of months needed to pay off the debt.

This result is close enough for a rough estimate. An exact computation shows that Mr. Lane must make 81 full payments and also an 82d payment of $26.70 to pay off the $6000 mortgage and the interest on it.

To estimate roughly how many months it will take to pay off a mortgage, divide the total mortgage by the average amount deducted from the principal each month.

Exercises

In the following, consider any fraction of a month in the result as another whole month.

1. On page 155, find the approximate number of months required to pay off the entire debt in ex. 6; in ex. 7; in ex. 8.

Find the approximate number of months required to pay off the mortgage in each of the following:

	Price of House	Cash Payment	Amount of Mortgage	Total Monthly Payment	Rate of Interest
2.	$3000	$1000	$2000	$25	6%
3.	$9000	$4500	$4500	$85	6%
4.	$12,000	$6000	$6000	$100	6%
5.	$15,000	$5000	$10,000	$150	6%

BUILDING AND LOAN ASSOCIATIONS

In order to build a house, a person may borrow money from a building and loan association. Instead of repaying the loan in monthly installments, he *buys shares* in the association by making monthly payments, which are equivalent to monthly deposits in a savings bank. When these shares *mature* (become worth $100 or $200 each), they are given back to the association and thus the loan is paid off in a lump sum. A person may also buy shares in an association as a form of investment, without borrowing money to build a house. In many associations, shares earn about 6% compound interest.

There are about 13,000 building and loan associations in this country, with nearly 12,000,000 members (shareholders). About one third of all the families in the United States are interested in these associations, either as investors or as borrowers.

These associations are also called *savings and loan associations, homestead associations, building associations,* and *coöperative banks.*

Buying Shares in a Building and Loan Association

1. Mr. Hunt buys 10 *shares* in a building and loan association. They will *mature,* or be worth $200 each, in about 139 mo. He pays for them at the rate of $1 a month on each share. How much does Mr. Hunt pay on the 10 shares in 1 mo.? in 1 yr.? in 5 yr.? in 10 yr.? in 139 mo.?

2. In 139 mo. Mr. Hunt's deposits, with compound interest, equal $200 per share, and the company then pays him this amount. How much does he receive in all? How much of this total amount represents deposits? How much is interest?

3. Some associations sell $100 shares at 50¢ a month. Such shares mature, or become worth $100 each, in about 139 months. If you buy 20 such shares, how much in all do you pay for them in 139 months? How much are they worth at maturity?

$100 shares, maturing in 80 mo., are also sold at $1 a month each.

BORROWING FROM BUILDING AND LOAN ASSOCIATIONS

Mr. Trent built a house with the help of the Franklin Building and Loan Association. The house cost $5000. Mr. Trent furnished $1800 and the association lent him $3200, with the property as security. On this loan he paid 7% interest, or $18.67 a month. He also bought 32 $100 shares in the Franklin Association, on which he paid 50¢ a share monthly. Hence, in all, he paid the association $34.67 each month.

$18.67	Monthly Interest
16.00	Payment on Shares
$34.67	Monthly Payment

In about 139 mo. the 32 shares were worth $100 each, and Mr. Trent turned them over to the association to pay his debt. Thus, by paying only $34.67 a month, he canceled his loan and the house was entirely his.

Associations in New York, Ohio, and some other states use a system of repaying the loans similar to that outlined on page 154.

Exercises

1. Why was Mr. Trent's plan wiser than renting a house?

2. Mr. Dane borrows $6600 from the Lincoln Building Association. At 7%, how much interest must he pay monthly? How many $200 shares must he buy to cover the debt?

3. If Mr. Dane (ex. 2) pays $1 a month per share, how much, including interest on his loan, must he pay to the association in 1 mo.? in the 139 mo. required for the shares to mature? This is how much more than the amount he borrowed?

4. Who gets the shares in ex. 3 at the end of 139 mo.?

5. Mr. Lee borrows $2400 from the Economy Building Association at 7% interest. To repay the debt he buys $100 shares at 50¢ a month per share. How many shares does he buy? How much does he pay the association each month, including payments on the shares and interest?

BUILDING YOUR OWN HOME

1. Mr. Gray wishes to build a house. He therefore borrows $4200 from the Prudence Savings and Loan Association. Find the monthly interest on this loan at 6%.

2. Mr. Gray (ex. 1) buys $100 shares for $4200, on which he pays $1 a month per share. What is his total payment, including interest on the loan, in 1 mo.? in 80 mo.?

3. In about 80 mo. Mr. Gray's shares mature and are used to pay off his debt. How much more than the $4200 loan has he paid in all? Then how much did the loan cost him?

4. If Mr. Gray had borrowed the $4200 for 80 mo. from a bank at 6%, how much interest would he have paid in the 80 mo. ($6\frac{2}{3}$ yr.)? How much more is this than the cost of borrowing the $4200 from a savings and loan association (ex. 3)?

The reason the total interest is more if Mr. Gray borrows from a bank is because he keeps the $4200 for the whole 80 mo. instead of repaying it in monthly installments. The shares that Mr. Gray buys (ex. 2 and 3) are the equivalent of repaying the association's loan in monthly payments.

5. Mr. Collins wishes to build a $10,000 house; but he has only $3500 in cash. He can borrow 50% of the value of the property from a bank or 65% to 70% of its value from a building and loan association. Where will it be better for him to borrow the money?

6. Mr. Davis borrows $9000 at 7% from a homestead association. To repay it, he buys shares of $100 each, at $1 a month per share. How much does he pay in all each month?

7. If the shares reach their full value in 80 mo., how much more than the loan does Mr. Davis (ex. 6) pay the association? Then what is the total cost of the loan?

A man who borrows from a building and loan association decreases his debt with each payment on his shares, which is much easier than repaying the entire loan at once. Likewise, the association's risk on the loan becomes less and less each month.

GENERAL REVIEW

1. A man in Galveston, who has no checking account, wishes to send $75 to some one in Detroit. Name two forms in which he can safely send the money.

2. Jane got $150 from her uncle when she was 14 yr. old. Would it be better for her to put this money in a savings bank or to open a checking account with it? Why?

3. What amount will you have in the bank at the end of 5 yr. if you deposit $30 every half year in a bank paying 4% interest compounded semiannually? Use the table on page 141.

4. Mrs. Foster can buy a gas range for $80 cash or on the installment plan by paying $5 down and $7.50 a month for 11 mo. How much more does Mrs. Foster pay if she buys the range on the installment plan than if she pays cash?

5. Mr. Jones received a legacy of $500. He kept this money in a safe in his home. Find out how much interest he lost in 10 yr. by not investing it at 6% interest compounded semiannually. In working this problem, do you use the interest table on page 136 or the one on page 141?

6. If interest is compounded annually, find the amount to which $450 will grow in 10 yr. at 6%; in 15 yr. at 4%; in 20 yr. at 5%. Use the table on page 136.

7. Tom Miller deposited $150 in a savings bank when he was 18 yr. old. If the bank pays interest at 4% compounded semiannually, how much will he have in the bank $17\frac{1}{2}$ yr. later?

8. Mr. Taylor borrowed $3500 from the Farmers Building and Loan Association at 7% interest. He also bought enough $100 shares at 50¢ a month per share to cancel his debt eventually. How many shares did he have to buy? How much does he pay the association each month, including the payments on the shares and the interest? What happens at the end of about 139 mo., when the shares mature?

CHAPTER IX

STOCKS AND BONDS

HOW A CORPORATION IS FORMED

A dramatic club needed \$400 its first year for costumes and other expenses. To raise the money, the members formed the Woodside Dramatic Company and sold **shares** in this *company* at \$1 a share. \$1 was the *par value* of these shares.

1. How many shares did the company have to sell, at \$1 a share, in order to raise \$400?

2. Albert bought 20 shares and Paul, 15 shares. How much did each have to pay for his shares? Albert and Paul and other people who owned shares were **shareholders** in the company.

3. The first year the company made a net profit of \$32, which was divided among the shareholders in the form of **dividends.** Since there were 400 shares, the dividend on each share was $\frac{1}{400}$ of \$32, or \$.08. Show that this was a dividend at the rate of 8% on each \$1 share.

4. If Mary Reed had 2 shares, find her dividend. Find the dividend on 5 shares; on 14 shares; 25 shares; 35 shares.

5. Because the profits were good, Marion wanted to buy some of Albert's shares. Albert charged her \$1.20 for each share, which was above the par value, or *above par.*

6. The third year the company made no profit and people were anxious to sell the shares they had. How would this affect the price of the shares? When \$1 shares are sold for less than \$1, they are said to be *below par.*

A company like the Woodside Dramatic Company is called a **corporation.** Shares in a corporation are often called **stocks.**

CORPORATIONS

A large business is usually owned and run in a similar way to the Woodside Dramatic Company, but on a larger and more complex scale. For example, a man starting an automobile factory is seldom either able to pay its entire cost alone or willing to take such a large risk without assistance from others. He therefore organizes a corporation, which is chartered by the state government. The total par value of the shares issued by a corporation is called its **capital stock.**

Most shares of stock have a par value of $100, although a few corporations issue shares with a par value of $50 or $25 or even $1. The shareholders, or stockholders, who own the stock, become owners of the corporation. Each stockholder receives a *stock certificate* showing the number of shares he owns. The stockholders elect a *board of directors* to manage the corporation. In this election, each stockholder has one vote for every share of stock he owns in the corporation.

Hence a large factory or a railroad is owned and run by the coöperative support of many people. The Pennsylvania Railroad is owned by about 150,000 stockholders, including residents of every state of the Union. The American Telephone and Telegraph Company is owned by over 400,000 people and is worth almost $2,000,000,000.

Explain why a very large business is owned by many people. What other large corporations do you know about?

The profits of a corporation, after a sum has been set aside for emergencies, are divided among the stockholders as **dividends.** If a corporation with a capital stock of $500,000 declares a dividend of $25,000 in a year, its rate of dividend is $25,000 ÷ $500,000, or 5%, *of the par value of the stock.* Hence a man who owns 2 of the $100 shares will receive 5% of $200, or $10, in dividends. How much would a man owning 5 of the $100 shares receive in dividends?

FINDING DIVIDENDS

All shares in the following have a par value of \$100.

1. A corporation has a capital stock of \$800,000. How many shares has it if each share has a par value of \$100?

2. Last year the corporation in ex. 1 distributed \$56,000 in dividends. What rate of dividend did it pay?

3. In ex. 2, how much was the dividend on 1 share of stock? How much was Mr. Ray's dividend if he owned 120 shares?

4. If the capital stock of a company consists of 5250 shares, find the total par value of its capital stock. How much did it pay in dividends in a year when the rate was 5%?

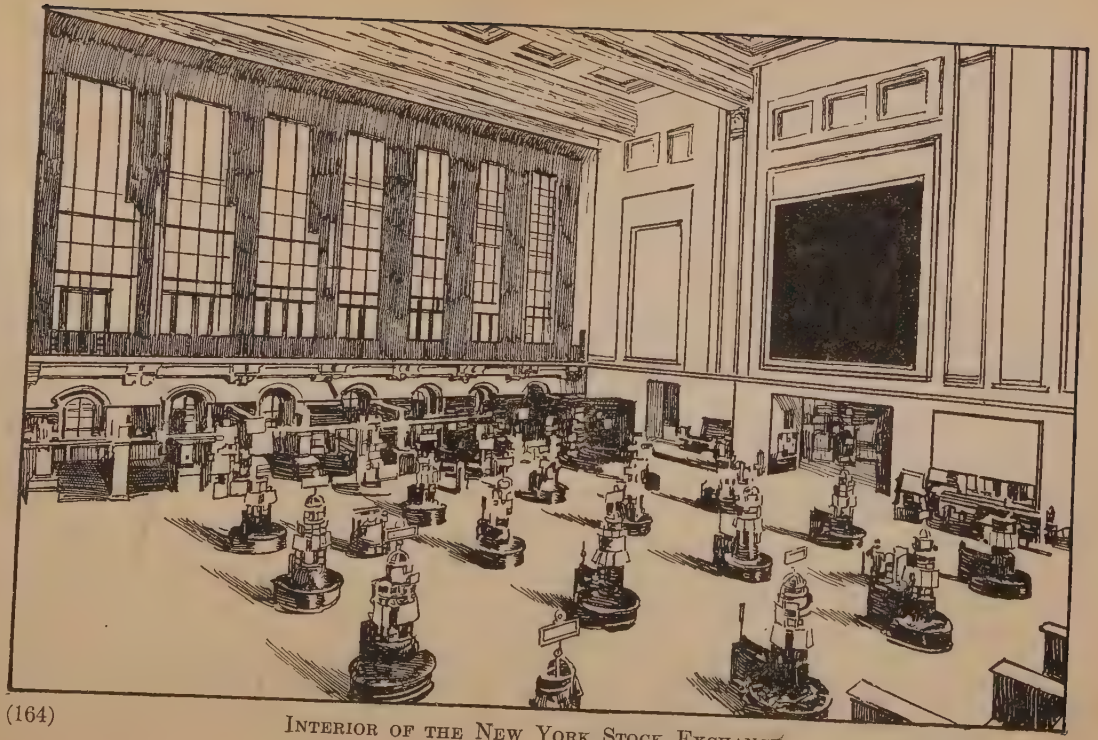
5. Mr. Toole bought 25 shares of stock in the Holsted Fruit Company at \$100 per share. His dividends for the first year were \$112.50. Find the per cent of his dividends.

Find the rate of dividend; then find the amount of dividend on 50 *shares of stock:*

	Capital Stock	Dividends		Capital Stock	Dividends
6.	\$250,000	\$10,000	**10.**	\$300,000	\$18,000
7.	\$585,000	\$40,950	**11.**	\$890,000	\$35,600
8.	\$780,000	\$42,900	**12.**	\$1,200,000	\$72,000
9.	\$950,000	\$47,500	**13.**	\$1,325,000	\$99,375

Preferred Stock

Some corporations have two kinds of stock, *preferred stock* and *common stock*. The preferred stock receives a fixed rate of dividend, like 6% or 7%, and the remaining profits, if any, are divided among the holders of the common stock. The dividends on the common stock may rise or fall with the prosperity of the business, while the dividends on the preferred stock continue at the guaranteed rate. The preferred stock of a successful and well-managed corporation is usually considered a very desirable investment.



(164) INTERIOR OF THE NEW YORK STOCK EXCHANGE

THE STOCK EXCHANGE

The picture on page 164 shows the interior of the New York Stock Exchange, where the stocks and bonds of over 1000 different corporations are bought and sold. Certain groups of stocks are sold at each trading post on the floor of the exchange. On the average, over 1,500,000 shares of stock are sold daily.

The New York Stock Exchange is a national market place, serving investors all over the country. Orders to buy and to sell stock on that exchange come in daily from practically all our larger cities. There are also smaller stock exchanges in Chicago, San Francisco, and other important cities.

A man who wishes to buy or sell stocks usually does so through a **broker** who is a member of a stock exchange. The broker buys the stock for his customer at the stock exchange and charges him a small fee, called **brokerage.**

The members of the New York Stock Exchange have branch offices in all our large cities. Each branch office is connected by private telegraph wire with the broker's New York office. It is thus possible for a man in Seattle, for instance, to give an order at a Seattle broker's office for the purchase of stocks on the New York Stock Exchange and to have a reply within a few minutes, stating that the order has been filled.

The price at which a stock sells is its **market value.** If a reliable corporation is paying a dividend of 6 % or more, the market value of a share of its stock is usually over $100, or *above par.* If the company is insecure and the dividend is low or if no dividend at all is being paid, the market value of each share will be under $100, or *below par.*

The prices at which important stocks are selling are printed each day in the newspapers. In large cities the newspapers usually devote several pages a day to information about stocks and bonds because they are so important in the business of the modern world. Look in one of the larger newspapers and see if you can find the stock news.

STUDYING STOCK QUOTATIONS

Here are some *stock quotations* from a newspaper. For each stock the total number of shares sold during the day is given, as well as the price per share at the opening of the exchange, the highest and lowest prices during the day, and the price at the close of the exchange. The number in parentheses after each stock shows its last rate of dividend; pf. means preferred stock. The par value of each share is $100.

STOCK QUOTATIONS

SALES	STOCK AND DIVIDEND RATE	OPEN	HIGH	LOW	CLOSE
2,800	Atlantic Telephone (9)	$167\frac{1}{2}$	$168\frac{3}{4}$	$167\frac{1}{2}$	$167\frac{5}{8}$
8,900	Brazil Rubber	$50\frac{1}{8}$	$51\frac{1}{4}$	$49\frac{1}{2}$	51
2,600	Goodwin Steel, pf. (7)	133	133	$132\frac{1}{2}$	$132\frac{5}{8}$
19,500	Marx Motors (4)	$63\frac{7}{8}$	$64\frac{3}{4}$	$63\frac{3}{8}$	$63\frac{1}{2}$
4,000	Pacific R. R. (5)	$88\frac{3}{8}$	$88\frac{3}{4}$	$87\frac{7}{8}$	$88\frac{1}{2}$
300	Standard Silk (8)	$146\frac{1}{2}$	148	$146\frac{1}{2}$	$147\frac{1}{4}$
4,600	Winthrop Canning	$14\frac{3}{4}$	15	$14\frac{1}{4}$	$14\frac{1}{4}$

Exercises

1. The first sales of Atlantic Telephone stock were at $167\frac{1}{2}$, or $167.50 per share. What were the highest and the lowest prices paid for this stock during the day?

2. What was the last rate of dividend on Atlantic Telephone stock? Why does this stock sell above par?

3. What other stocks are selling above par? Why are people willing to pay so much for these stocks?

4. The Winthrop Canning and Brazil Rubber stocks are not paying dividends. How has this affected their prices? Judging from their prices, which of these stocks is more likely to begin paying dividends in the near future?

5. Show that the stocks paying the lowest dividends are selling at the lowest prices.

BUYING AND SELLING STOCKS

A person wishing to buy or sell stocks listed on the stock exchange has to employ a stock broker to carry out the transaction for him. The broker charges a fee called **brokerage,** which is a certain amount per share and varies with the market price of the stock. The fees charged by brokers on the New York Stock Exchange are shown in the table at the right.

Market Price per Share	Broker's Fee per Share
\$10 to \$25	$12\frac{1}{2}$¢
\$25 to \$50	15¢
\$50 to \$75	$17\frac{1}{2}$¢
\$75 to \$100	20¢
\$100 to \$200	25¢

Exercises

1. Mr. Rees purchases 10 shares of Standard Silk stock at $146\frac{3}{4}$. How much does he pay in all?

The price per share is \$146.75. Hence 10 shares cost $10 \times \$146.75$, or \$1467.50. Since the price is between 100 and 200, the brokerage is 25¢ per share, or \$2.50 for 10 shares. Mr. Rees pays $\$1467.50 + \$2.50$, or \$1470.

2. Mr. Lake's broker sells 50 shares of Pacific R. R. stock for him at $88\frac{3}{8}$. How much does Mr. Lake receive?

$88\frac{3}{8}$ means $\$88.37\frac{1}{2}$. Hence 50 shares sell for $50 \times \$88.37\frac{1}{2}$, or \$4418.75. The brokerage is 20¢ per share, or \$10 in all. This is deducted from the amount paid to Mr. Lake by the broker. Hence Mr. Lake gets \$4408.75.

Tell how much it costs, including brokerage, to buy:

3. 100 shares of Marx Motors at $64\frac{3}{4}$.

4. 50 shares of Brazil Rubber at $50\frac{1}{4}$.

5. 15 shares of Goodwin Steel, preferred, at $132\frac{5}{8}$.

6. 200 shares of Central R. R., preferred, at $114\frac{7}{8}$.

If a broker sells the following shares of stock for you, tell what net amount he will send you after the sale:

7. 40 shares at $48\frac{5}{8}$

8. 20 shares at $146\frac{3}{4}$

9. 80 shares at $101\frac{1}{8}$

10. 100 shares at $79\frac{1}{4}$

11. 300 shares at $27\frac{5}{8}$

12. 600 shares at $169\frac{1}{2}$

SELLING STOCKS AT A PROFIT

Many persons buy real estate in a growing city and sell it several years later at a good profit. A person may also buy the stock of a well-managed corporation in the hope that, as the corporation develops, the price of its stock will increase. By selling the stock when it reaches a higher price, the owner makes a profit. Such a transaction is entirely legitimate; but it requires both good judgment and foresight in selecting the stock.

Some people, however, buy stocks with the hope of selling them a few days later at a higher price, relying on the daily fluctuations in the prices of stocks. Such people have not the patience to wait several years for a substantial growth in the corporation whose stocks they are purchasing. They want quick profits; hence they take a chance. Such buying and selling is *speculation* and in most cases it eventually results in heavy losses for the speculator.

Avoid speculation. It is dangerous and leads to disaster!

Exercises

1. Mr. Marvin bought 25 shares of Interstate R. R. stock at $98\frac{3}{4}$. Several years later, owing to prosperous railroad conditions, he was able to sell it at $133\frac{1}{2}$. What profit did he make on the sale?

He paid $25 \times \$98.75$ for the stock + \$5.00 brokerage. When he sold it he received $25 \times \$133.50 - \$6.25$ brokerage. Find Mr. Marvin's profit. Also show how the brokerage fee is computed in each case.

2. While Mr. Marvin held the stock, he received a total of \$21 in dividends on each share. Find his total dividends. What was his total profit, including his profit in ex. 1?

3. Mr. Roe bought 100 shares of oil stock at 33. Next day the value dropped and he sold the stock at $19\frac{1}{2}$. Find his total loss, including brokerage. Was Mr. Roe speculating?

SELLING STOCKS AT A PROFIT OR LOSS

Find the profit or the loss on the following, using the table on page 167 for the brokerage fees:

1. 600 shares bought at 74 and sold at 74.
2. 200 shares bought at $69\frac{1}{4}$ and sold at $83\frac{1}{2}$.
3. 500 shares bought at $86\frac{7}{8}$ and sold at $73\frac{1}{4}$.
4. 150 shares bought at $97\frac{1}{2}$ and sold at $112\frac{3}{4}$.
5. 100 shares bought at $112\frac{5}{8}$ and sold at 104.

BUYING STOCKS AS AN INVESTMENT

People with much money to invest often buy high-grade stocks because such investments may yield 6% interest or more, while a savings bank pays only 4% to $4\frac{1}{2}$%.

1. Mr. Forrest buys as an investment 1 share of Ralco 7% preferred stock at 112. What rate of income does Mr. Forrest receive on this investment?

Since dividends are paid on the par value of a stock, a 7% stock pays $7 a year dividend. Hence Mr. Forrest's investment of $112 yields $7 a year. By finding what per cent $7 is of $112, show that his money is earning $6\frac{1}{4}$%. The brokerage may here be disregarded since it is so small.

Find the rate of income on stocks purchased at these prices and paying dividends at the rates stated. Count $100 as the par value and disregard brokerage:

2. $80, 4%
3. $80, 5%
4. $80, 6%
5. $80, 7%
6. $60, 3%
7. $50, 5%
8. $96, 6%
9. $125, 5%
10. $180, 9%
11. $150, 12%
12. $125, 10%
13. $125, $7\frac{1}{2}$%

Find the rate of income, to the nearest tenth of 1%, on stocks whose prices and rates of dividend are as follows:

14. $104, 6%
15. $140, 7%
16. $82, 7%
17. $96, 4%
18. $90, 5%
19. $83, 5%
20. $85, 7%
21. $92, 6%

BONDS

If a city wishes to build a new school, it can borrow money by issuing *bonds*. A corporation may also issue bonds to raise money with which to extend its business.

A **bond** is a kind of promissory note in which a government or a corporation promises to pay a certain sum at a stated time. Bonds generally run from 5 yr. to 50 yr. The face value of a bond is usually $1000, although bonds in denominations of $500 and smaller amounts are also issued. Every bond has a fixed rate of interest, like 4% or 5%, payable semiannually. Bonds are often secured by a mortgage on the corporation's property. There is no such security for stocks.

If a railroad wants to borrow $2,000,000 to make improvements, it may issue 2000 bonds of $1000 each, due in 30 yr. and paying 5% interest. The interest is paid on the face value; hence the yearly interest in this case is $50, payable in semiannual installments of $25.

The interest on a bond is often paid by means of small **coupons** attached to the bond and dated 6 mo. apart. On a 5% bond for $1000, each coupon is like a check for $25. When a coupon is due, the owner cuts it off the bond and usually deposits it in his bank. The bank then collects it for him. In the case of a *registered bond*, the corporation has the owner's name and sends him the interest by check instead of paying it by coupon.

Bonds of the highest grade, like United States government bonds, may pay only 3% to 4%. A first-class railroad bond pays from 4% to 5%, while the bonds of a less dependable corporation may have to pay 6% or 7% to attract buyers.

Important bonds, like stocks, are sold on the stock exchange and their prices are given daily in the newspapers. A bond listed: "Newtown 4s, 1953" is a Newtown bond paying 4% interest and due in 1953. On the New York Stock Exchange about $15,000,000 worth of bonds are sold daily.

THE INTEREST ON BONDS

Give the rate of interest, the date due, and the amount of interest paid each 6 mo. on these $1000 bonds:

1. Minneapolis $3\frac{1}{2}$s, 1954
2. Penn. Steel 5s, 1963
3. Hudson R. R. 7s, 1940
4. Warren Silk 6s, 1936
5. United Sugar $6\frac{1}{2}$s, 1936
6. U. S. Liberty 1st 4s, 1947

7. Find the interest rate and the date due of some of your city's bonds. For what purpose were the bonds issued?

8. Mr. Field owns $8000 worth of the Colorado Electric Company's 5% coupon bonds. How much interest from these bonds does he receive every 6 mo.?

9. In 1929 an oil company sold $120,000,000 worth of 20-year 5% bonds. Find the yearly interest on these bonds. When will the company pay back the $120,000,000?

10. Rosedale County needs $250,000 to improve its roads. If it raises this money by 4% bonds, interest payable semi-annually, how much in all will the county pay out each 6 months in interest on these bonds?

11. In what way does the rate of interest on a bond help to show how safe the bond is?

12. A bond may run for 20 yr., 50 yr., or more. How does it differ in this respect from a promissory note?

13. Does a 6% preferred stock always pay a 6% dividend if the profits of the company are sufficient to cover it?

14. The common stock of the Akron Motor Company paid a 6% dividend last year. Does the company guarantee to pay the same rate of dividend each of the following years? Does common stock always pay the same rate of dividend?

15. Does a 6% bond always pay the same rate of interest each year? Is this rate of interest guaranteed?

16. Why are bonds usually a safer investment than stocks?

WHY THE PRICES OF BONDS FLUCTUATE

Since bond quotations show the price in dollars of a $100 bond, the price of a $1000 bond is 10 times the quotation. Thus, a $100 bond quoted at $102\frac{3}{4}$ costs $102.75 and a $1000 bond quoted at $102\frac{3}{4}$ costs $10 \times \$102.75$, or $1027.50. Similarly, a $1000 bond quoted at $87\frac{1}{2}$ costs $10 \times \$87.50$, or $875.

On the New York Stock Exchange, the broker's fee for purchasing $1000 bonds is usually $2 per bond. Hence for one $1000 bond quoted at $87\frac{1}{2}$, the price is $875 + $2, or $877.

The change, or *fluctuation*, in the market prices of bonds is due to these factors:

1. If the corporation that issued the bonds becomes less prosperous, the price of the bonds will drop *below par* because the public has less confidence in them.

2. A rise or fall in the rates of interest paid on other loans will make bonds, on which the interest rate is fixed, correspondingly less or more desirable. If money is scarce and many people wish to borrow, the prevailing rates of interest will be high and it will be easy to get 6% or 7% interest on a well-secured loan. A bond paying only 5% will then have to be offered at a price below par, possibly as low as 83, to attract customers. A 5% bond purchased at 83 yields an income of about 6%. On the other hand, when there are many people wishing to lend money and few wishing to borrow, the prevailing interest rates will be low and it will be necessary to pay a price above par for a high-grade 5% bond.

In general, the prices of bonds go down when prevailing interest rates are high and they go up when interest rates are low.

3. The price of a $1000 bond gets nearer and nearer to $1000 as the due date, or *date of maturity*, approaches. If you buy at 90 a good 4% bond which is due in 1940, the price of the bond will gradually rise to 100 as 1940 approaches. Can you explain why? Remember that a bond is a kind of promissory note.

BUYING BONDS

1. A \$1000 bond is quoted at $85\frac{1}{2}$. Find the cost, including brokerage, of 1 bond; of 5 bonds; of 15 bonds.

2. Find the cost of a \$1000 Utah Railway 5% bond quoted at $98\frac{1}{2}$. How much yearly interest does this bond pay?

Remember that interest is always computed on the par value of a bond.

Find the cost, including brokerage, of the following bonds. In each problem also find the total yearly interest:

3. One \$1000 Union Telephone 5s, 1946, at 103.

4. One \$1000 French Government 7s, 1949, at $105\frac{1}{2}$.

5. One \$1000 Robbins Chocolate $5\frac{1}{2}$s, 1940, at $102\frac{1}{2}$.

6. Five \$1000 U. S. Treasury $4\frac{1}{4}$s, 1947, at 107.

7. Five \$1000 Michigan Motors 6s, 1940, at $91\frac{1}{2}$.

8. Twenty \$1000 Gulf R. R. 4s, 1948, at $95\frac{1}{2}$.

9. Two \$1000 Columbia Trolley 4s, 1998, at $94\frac{5}{8}$.

10. Give several possible reasons why Columbia Trolley bonds (ex. 9) were selling at less than Union Telephone bonds (ex. 3).

11. On Nov. 13 Mr. Howe buys from Mr. Long a 6% \$1000 bond at 101, with interest coupons payable on January 1 and July 1. Since Mr. Howe will receive the entire \$30 interest on the coupon for Jan. 1, he must pay Mr. Long the interest at 6% for the 135 da. from July 1 to Nov. 13, during which time Mr. Long held the bond. This is called **accrued interest**. How much is the accrued interest on this bond?

How much does Mr. Howe pay for the bond in all, including accrued interest and brokerage?

Compute interest by the 6% method or use the table on page 126.

12. Find the accrued interest on a 4% \$1000 bond bought on June 18 if the interest is payable Apr. 1 and Oct. 1. How much does the buyer pay in all for the bond if he buys it at $88\frac{1}{2}$?

FINDING THE RATE OF INCOME ON A BOND

1. Mr. Tate buys a 5% $1000 bond at 93. What rate of income does he get on his investment?

$$\begin{array}{r} .0537 = 5.4\% \\ \$930\overline{)\$50.0000} \\ 46\ 50 \\ \hline 3\ 500 \\ 2\ 790 \\ \hline 7100 \\ 6510 \\ \hline \end{array}$$

Since the interest is computed on the par value, Mr. Tate receives 5% of $1000, or $50, yearly interest. Since he has paid only $930 for the bond, he earns $\frac{50}{930}$, or 5.4%, every year on his investment. This 5.4% is called the **current yield** of the bond.

Brokerage is here disregarded because it is so small, compared with the price of the bond, as hardly to affect the rate of income.

2. Find the current yield on a 6% bond bought at 120.

The interest is 6% of $1000, or $60 a year. Since the price of the bond is $1200, the current yield on the price is $\frac{60}{1200}$, or 5%.

The price paid for a bond does not affect the yearly amount of interest. It does, however, affect the rate of income earned on the money invested.

Exercises

Disregard brokerage in the following examples:

1. Mr. Harris buys a 5% $1000 bond at 89. How much does he pay for it? How much interest does he get yearly? Find, to the nearest tenth of 1%, the current yield of the bond.

Find the current yield, to the nearest tenth of 1%, *if* $1000 *bonds are purchased as follows:*

2. A 4% bond at 87

3. A 5% bond at 96

4. A $3\frac{1}{2}$% bond at 79

5. A $4\frac{1}{2}$% bond at 92

6. A 6% bond at 103

7. A 7% bond at 103

8. A $6\frac{1}{2}$% bond at 111

9. A $5\frac{1}{2}$% bond at 120

10. Mr. Locke can buy a 4% $1000 bond at 85 or a $4\frac{1}{2}$% $1000 bond at par. On which bond would the current yield be higher? how much higher, to the nearest tenth of 1%?

FINDING THE YIELD TO MATURITY

In 1929 Mr. Lynn bought a 5% $1000 bond at 93. In 1936, when the bond is due, he will receive $1000 for it. To what yearly rate of income will his earnings then be equal?

The bond cost $930 and pays $50 interest yearly. In 7 yr. Mr. Lynn will receive $1000 for the bond, which is $70 more than he paid for it. This $70 is an average of $10 extra profit yearly. Hence his yearly income amounts to $50 interest + $10 extra profit, or $60. Therefore he has a yearly profit of $60 on an investment of $930. Since $60 is about 6.5% of $930, his income is equal to 6.5% yearly.

Since 6.5% is the average yearly yield to the bond's date of maturity, it is called the **yield to maturity.** How does this differ from the current yield as found on page 174?

Exercises

1. Mrs. Shaw buys an 8% bond, due in 5 yr., at 115. Show that the yield to maturity on her investment is 4.3%.

At the end of 5 yr. she will receive $1000 for a bond that cost her $1150. Hence she will lose $150 in 5 yr., or an average of $30 a year. Subtracting this $30 from her yearly interest of $80, you find that she really receives $50 a year on an investment of $1150. What per cent of $1150 is $50?

Find the yield to maturity on these $1000 *bonds:*

2. A 5% bond bought at 90 with 5 yr. still to run.

3. A 4% bond bought at 80 with 20 yr. still to run.

4. A $4\frac{1}{2}$% bond bought at 98 with 2 yr. still to run.

5. A 6% bond bought at 110 with 10 yr. still to run.

6. An 8% bond bought at 118 with 9 yr. still to run.

7. A 4% bond bought Jan. 1, 1928 at 90, due Jan. 1, 1938.

8. A 5% bond bought Jan. 1, 1928 at 95, due Jan. 1, 1933.

9. Is the yield to maturity more or less than the current yield on a bond bought below par? on a bond bought above par?

NOTE. The above method gives the approximate yield to maturity. Bond dealers and bankers use *bond tables* for more exact values.

REVIEW

Explain the meaning of each of the following terms:

1. bonds	**5.** par value	**9.** dividend
2. stocks	**6.** corporation	**10.** market value
3. broker	**7.** shareholder	**11.** current yield
4. coupon	**8.** speculation	**12.** stock quotation

13. A company has a capital stock of $860,000. Find the rate of dividend, if the amount distributed in dividends in a year is $34,400; if the dividend is $43,000; $51,600.

14. At each of the rates found in ex. 13, find the total dividend on 30 shares of stock, each of $100 par value.

15. What advantages has preferred over common stock?

Find the cost, including brokerage, of these stocks:

16. 75 shares at $65\frac{1}{2}$

17. 80 shares at $99\frac{1}{4}$

18. 150 shares at $105\frac{3}{8}$

19. 200 shares at $129\frac{1}{2}$

20. If the stocks in ex. 16 to 19 were sold at the prices quoted, what net amounts would the brokers send to the persons for whom the stocks were sold?

21. If the par value is $100, find the rate of income on stock purchased at 80 and paying a 4% dividend; on stock purchased at 150 and paying a 9% dividend.

22. What causes the market prices of bonds to fluctuate?

23. Find the current yield, to the nearest tenth of 1%, on a 4% $1000 bond bought at 90; on a 6% $1000 bond at 107.

24. Mr. Pearson buys a 5% bond, due in 5 yr., at 90. If he keeps the bond to the date of maturity, find, to the nearest tenth of 1%, the yield to maturity of the bond.

Find the cost, including brokerage, of these bonds and also the yearly interest on each bond:

25. One $1000 Redland Electric Co. 5s, 1954, at $94\frac{1}{2}$.

26. Fifteen $1000 Northern Railroad 6s, 1935, at $103\frac{3}{4}$.

IMPROVEMENT TEST No. 8

Test 8 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
9837	2369	6407	2603	7516
6895	3576	9869	7367	6982
9374	3347	4778	3457	5151
7063	4897	6987	8565	4771
6970	3464	8452	4527	6622
6499	8885	7633	4722	6488
9666	4547	9992	6994	4868
7785	5559	8983	1588	2748
8888	5157	5299	1328	8493

Test 8 B — Subtraction (Time 3 min.)

1.	2.	3.	4.
601074	754312	945228	333621
215316	578643	346667	136033
5.	**6.**	**7.**	**8.**
703659	600717	712802	952062
284665	355388	468235	156582
9.	**10.**	**11.**	**12.**
820837	961365	840411	953908
581379	847889	467732	567669

Test 8 C — Multiplication (Time 3 min. after copying)

1.	2.	3.
61759	52601	40289
67054	2189	1765

Test 8 D — Division (Time 4 min. after copying)

Find each result to the nearest hundredth:

1. $235\overline{)76895}$ 2. $798\overline{)719929}$ 3. $674\overline{)209899}$

GENERAL REVIEW

1. Give the formulas for finding the areas of these figures: a triangle, a rectangle, a trapezoid, a parallelogram, a circle.

2. Give the formula for finding the circumference of a circle if you know the radius; if you know the diameter.

3. The diameter of the earth is about 8000 mi. Find the circumference of the earth, letting $\pi = 3\frac{1}{7}$.

4. Give the formulas for finding the volumes of these solids: a rectangular solid, a cylinder, a pyramid, a sphere, a cone.

5. In the Taft Junior High School, the Athletic Association collected \$90 in all. Of this amount the 9th year class gave 3 times as much as the 7th year class, while the 8th year class gave twice as much as the 7th year class. How much did each class contribute? Solve this problem by using an equation.

6. The Riverdale baseball team has won 13 out of 15 games. The Harrison team has won 9 out of 11 games. Find the standing of each team and tell which one is ahead.

7. What is trade discount? What is bank discount?

8. The present enrollment in the Mead High School is 720 pupils, which is 60% greater than the enrollment 5 yr. ago. What was the enrollment 5 yr. ago?

Find the interest on these amounts at 6%, first by the 6% method, then by the table on page 126. Use exact days between dates:

9. \$800; Apr. 5 to May 11
10. \$450; Aug. 8 to Oct. 25
11. \$120; Mar. 4 to July 17
12. \$5000; June 10 to Aug. 15
13. \$2225; July 18 to Aug. 17
14. \$7500; Sept. 23 to Dec. 22

Solve the following equations and check the work:

15. $\frac{5x}{7} = 290$
16. $x + \frac{1}{2}x = 12$
17. $4x - x = 27$
18. $\frac{76x}{100} = \$114$
19. $\frac{27}{100}x = \$162$
20. $\frac{35}{100}x = \$245$
21. $\frac{4x}{5} + 3 = 451$
22. $x + \frac{5}{100}x = 84$
23. $x - \frac{35}{100}x = 13$

CHAPTER X

INSURANCE

FIRE INSURANCE

In a recent year fire destroyed the homes of about 1,150,000 people in the United States. Many of these people had all their money invested in their homes and could not rebuild them without outside help.

The wisest of these people had insured their houses against fire. To do this, a man pays to a *fire insurance company* a small sum of money called a **premium.** The company then gives him a written agreement, or **policy,** promising to pay for damage that may be done to his property by fire or smoke.

Mr. Austin has his house insured for \$8000; hence \$8000 is the **face** of his policy, and the company will give him \$8000 if the house is completely burned down. Mr. Austin's premium is at the rate of \$.24 per \$100. Since \$8000 is $80 \times \$100$, the premium is $80 \times \$.24$, or \$19.20. By paying \$19.20, Mr. Austin insures his house for 1 yr.

To find the premium, multiply the rate per \$100 *by the number of times* \$100 *is contained in the face of the policy.*

Exercises

1. Mr. Smith insures his house for \$10,000. At a yearly rate of \$.22 per \$100, what is his premium for 1 yr.?

Find the yearly premium on these insurance policies:

	Face	Rate per \$100		Face	Rate per \$100
2.	\$6000	\$.30	**6.**	\$15,000	\$.21
3.	\$4300	\$.28	**7.**	\$34,000	\$.80
4.	\$7500	\$.60	**8.**	\$12,800	\$1.05
5.	\$5400	\$2.75	**9.**	\$23,700	\$1.20

WHY FIRE INSURANCE RATES VARY

The insurance company can pay for the damage caused by fires because most buildings on which insurance is paid are never burned. By studying for many years the fire losses of different kinds of buildings, the company determines the rate for each type of building. The rate on a wooden house, a garage, or any building without fire protection and far from a fire house is higher than the rate on a well-protected and fireproof building.

This table shows the yearly rates for different buildings:

Kind of Building	Place Located	Rate per $100	Kind of Building	Place Located	Rate per $100
Brick dwelling	City	$.25	Frame church	Country	$1.50
Frame dwelling	City	.50	Frame hotel	Country	2.20
Brick garage	City	.35	Grist mill	Village	3.65
Frame garage	City	.80	Power mill	Village	4.22
Farmhouse	Near town	.58	Frame school	Country	1.40
Barn	Near town	.82	Soap factory	Village	6.50

Exercises

1. Which of the above kinds of buildings has the highest rate of insurance? Explain why. Give reasons why a frame hotel in the Adirondack Mountains pays a higher rate than a brick dwelling in Detroit.

2. Would a frame dwelling in a city pay a higher or a lower rate than one on a country road? Why?

Find the premiums on these policies at the above rates:

	Building	Face		Building	Face
3.	Brick garage	$1500	**7.**	Brick dwelling	$14,500
4.	Farmhouse	$7000	**8.**	Soap factory	$51,000
5.	Frame garage	$1200	**9.**	Frame school	$10,000
6.	Barn	$2200	**10.**	Power mill	$13,250

11. Name several uses to which a building might be put which would cause it to have a high rate of fire insurance.

INSURING A HOUSE FOR THREE OR FIVE YEARS

Mr. Law insures his house for \$14,000. The agent says that the rate for a 1-year policy is 30¢ per \$100; for a 3-year policy, it is $2\frac{1}{2}$ times as much; and for a 5-year policy, it is 4 times as much. Hence Mr. Law would pay $140 \times 30¢$, or \$42, for a 1-year policy. He would pay $2\frac{1}{2} \times \$42$, or \$105, for a 3-year policy. How much would a 5-year policy cost him?

Exercises

1. For 3 yr. Mr. Law buys 1-year policies. How much would he save by buying one 3-year policy?

2. How much cheaper would it be for Mr. Law to buy one 5-year policy than to buy five 1-year policies?

3. For 30 yr. Mr. Day renewed annually a \$31,600 insurance on his home, at 34¢ per \$100. How much would he have saved in this time by buying 3-year policies? 5-year policies?

4. For 30 yr. Mr. Day insured his house furnishings annually for \$5000, at 38¢ per \$100. How much would he have saved by buying 5-year policies?

Find the premiums at the given yearly rates per \$100:

	Face	Rate	Term		Face	Rate	Term
5.	\$2000	\$ .15	1 yr.	**10.**	\$15,500	\$ .30	3 yr.
6.	\$8700	\$ .42	3 yr.	**11.**	\$11,450	\$ .62	3 yr.
7.	\$3750	\$ .60	3 yr.	**12.**	\$24,500	\$3.75	5 yr.
8.	\$9460	\$1.00	5 yr.	**13.**	\$63,800	\$1.88	5 yr.
9.	\$2350	\$4.22	5 yr.	**14.**	\$17,850	\$6.75	5 yr.

15. A cotton merchant had a 3-year insurance policy for \$46,000 on his stock, at a yearly rate of \$4.50 per \$100. Find his premium. If \$2000 worth of his cotton was destroyed by fire and the company paid the damage, how much did the company lose or gain on this policy?

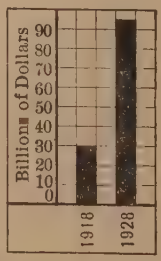
LIFE INSURANCE

Mr. Scott worked in an automobile factory. Besides his wife, there were four children in the family, all attending elementary school. When Mr. Scott was 35 yr. old, a life insurance agent called on him one day and tried in vain to persuade him to have his life insured for $10,000. Five years later Mr. Scott died suddenly.

After his death, Mrs. Scott found it very hard to get along. To help support her children, she got a position as clerk in a store; but as she did not earn very much, her son Jack, who was 14 yr. old, had to leave school and go to work. The Scotts had no relatives to help them and it was a hard struggle. A few years later, Mrs. Scott became ill from overwork and her son Ben, who was then 13 yr. old, also had to leave school in order to go to work.

All this distress could have been avoided if Mr. Scott had insured his life for $10,000. The life insurance company would then have paid Mrs. Scott $10,000 when Mr. Scott died, which would have kept all the children in school and prevented much hardship and worry.

People are rapidly realizing that life insurance is a very valuable form of protection, and much more life insurance is being sold each year than the year before. In 1918 the total amount of life insurance in force in this country was about $30,000,000,000, while in 1928 the total amount was about $95,000,000,000. This growth is shown by the graph at the right. The bar for 1928 is about how many times as long as the bar for 1918?



GROWTH OF LIFE INSURANCE IN THIS COUNTRY

At the present time, over half of all the men, women, and children in the United States carry some form of reliable life insurance.

BUYING AN ORDINARY LIFE POLICY

The kind of life insurance policy most frequently purchased is called an **ordinary life policy.** This type of policy is the most popular because it is the cheapest form of regular life insurance.

Suppose Mr. White, who is 30 yr. old, buys an ordinary life policy for $10,000. In this policy the life insurance company promises to pay $10,000 to Mr. White's wife if Mr. White dies. Mrs. White is the **beneficiary,** Mr. White is the **policyholder,** and $10,000 is the **face** of the policy.

Mr. White pays a yearly premium of $22.85 for each $1000 of insurance. Since the policy is for $10,000, his total yearly premium is $10 \times \$22.85$, or $228.50. This premium must be paid each year as long as Mr. White lives. For this reason an ordinary life policy is sometimes called a **continuous premium policy.**

The amount of the premium for an ordinary life policy depends on the age of the person *at the time he is insured,* as shown at the right. Thus, if a person is 35 yr. old when he takes out an ordinary life policy, he pays $26.35 per year for each $1000 insurance.

ORDINARY LIFE POLICY

Age	*Premium per* $1000
25	$20.14
30	22.85
35	26.35
40	30.94
45	37.08
50	45.45

Exercises

Using the rates above, find the yearly premiums for these ordinary life policies, purchased at the given ages:

1. $1000, age 25
2. $4000, age 25
3. $3000, age 40
4. $5000, age 50
5. $7000, age 30
6. $8000, age 45
7. $9000, age 35
8. $2000, age 50
9. $10,000, age 30
10. $12,000, age 45
11. $25,000, age 35
12. $42,000, age 40

13. Why must a man who takes out a policy at age 45 pay a higher premium than one who insures his life at age 30?

14. Why is it best to insure your life at an early age?

OTHER KINDS OF LIFE INSURANCE POLICIES

You have seen that the holder of an *ordinary life policy* must pay the premium each year as long as he lives.

On a **limited-payment policy** the premiums are required only for a definite number of years, like 20 yr. The insurance continues in force, however, until the policyholder dies, when the face of the policy is paid to the beneficiary.

Limited-payment policies cost much more than ordinary life policies. Such policies are preferred by those who wish to pay all their premiums before their earning power is decreased by old age.

On an **endowment policy,** the premiums are also paid for a definite number of years, like 20 yr. If the policyholder dies within the 20 yr., the face of the policy is paid to the beneficiary; but if he is living at the end of that period, the full amount is paid to him and the insurance ceases.

Endowment policies combine life insurance with a system of saving money to provide for old age. If a man lives to the end of the 20-year period, he usually gets back more money than he has paid in premiums, since the money has earned interest during that time. Although the premiums paid on an endowment policy do not yield as much, from the savings standpoint alone, as equal deposits in a savings bank for the same number of years, the endowment policy gives the additional benefit of life insurance for the period it is in force.

On a **term policy,** premiums are paid for a short term of years, like 5 yr., 10 yr., or 15 yr., and the company pays insurance only if the policyholder dies within that period. At the end of the term the insurance ceases.

Term insurance is purchased when a person wants extra protection for a short time. For example, a man may carry an ordinary life policy for $15,000 all his life; but during the 10 yr. that his children are in school, he may also take out a 10-year term policy for $10,000, to make certain that the education of his children will be completed if he dies during that period. The premium rates on term policies are lower than the rates on the other kinds of policies, since the company is taking the risk for a shorter period.

A person applying for life insurance has to pass a medical examination. Unless his health is good, he cannot be insured.

EVERYDAY PROBLEMS IN LIFE INSURANCE

The premium rates per $1000 vary according to the kind of policy and the age at which it is taken out, as shown below:

Age	Ordinary Life	20-Payment Life	20-Year Endowment	10-Year Term
25	$20.14	$30.12	$48.15	$11.40
30	22.85	32.87	48.83	12.10
35	26.35	36.22	49.85	13.19
40	30.94	40.38	51.48	15.02
45	37.08	45.73	54.22	18.40
50	45.45	52.87	58.81	24.50
55	56.93	62.68	66.36	34.80

1. From the table, find the premium per $1000 on a 20-payment life policy taken out at age 25; at age 30; at age 35.

2. Find the premium per $1000 on a 20-year endowment policy taken out at the age of 30 yr.; 40 yr.; 45 yr.; 50 yr.

3. At age 35, the 20-year endowment premium rate is about how many times as much as the ordinary life rate?

Using the rates above, find the yearly premium for these policies if purchased at the given ages:

4. A 10-year term policy for $15,000 issued at age 40.

5. A 20-payment life policy for $3000 issued at age 30.

6. An ordinary life policy for $25,000 issued at age 25.

7. A 20-year endowment policy for $5000 issued at age 35.

Reread pages 183 *and* 184 *before answering ex.* 8 *to* 10.

8. Why is the ordinary life policy the most popular?

9. Why are the premiums on a 20-payment life policy higher than those on an ordinary life policy? Why do some persons prefer the 20-payment policy?

10. How does a 20-year endowment policy differ from a 20-payment life policy? Why are the endowment rates higher?

EVERYDAY PROBLEMS IN LIFE INSURANCE

Use the premium rates given on page 185.

1. Mr. Stark is 30 yr. old. He wishes to protect his family with as much life insurance as possible. He can afford to pay about $200 a year in premiums. Approximately what amount of insurance can he buy for $200 yearly if he selects the ordinary life policy? the 20-payment life policy? the 20-year endowment? Which do you think it advisable for Mr. Stark to buy?

2. Give several reasons why it is unwise to wait until old age before taking out life insurance. Remember that a person who applies for life insurance must take a medical examination.

3. A motion-picture company took out an ordinary life policy for $1,000,000 on the life of its manager, upon whose business ability the success of the company depended. In case of the manager's death, the policy was payable to the company. What yearly premium did the company pay if the manager was 35 yr. old when the policy was issued? Why did the company carry such a large insurance on his life?

Life Insurance in the United States

1. Of the 120,000,000 inhabitants of the United States, about 65,000,000 carry life insurance. About what per cent of the inhabitants have their lives insured?

2. The total amount of life insurance in force in this country in 1928 was about $95,000,000,000. If 65,000,000 people were insured, find the average amount of insurance carried by each.

3. Find the per cent of increase in the amount of life insurance in force from 1918 to 1928 (see bottom of page 182).

4. One prominent man in the United States carries $7,500,000 life insurance; another carries $4,500,000 insurance. Find each man's yearly premium if the insurance is in ordinary life policies taken out at the age of 35 yr.

HOW THE COMPANY USES THE PREMIUMS

The premium on an ordinary life policy at age 35 is $26.35 per $1000. The company uses this premium as follows:

1. *Mortality cost.* About $9 of this $26.35 is set aside to pay the insurance of policyholders who die during the year.

From a careful study of past records of deaths, mortality tables have been made which show how many people may be expected to die each year.

2. *Expenses.* About $5 of the premium is used to pay overhead expenses, like rent, salaries, and commissions.

3. *Reserve.* The rest of the premium, about $12.35, forms a reserve fund, which is credited to the policyholder each year like a savings bank deposit.

$9.00	**Mortality Cost**
5.00	**Expenses**
12.35	**Reserve**
$26.35	**Premium**

This reserve, which usually earns 3% compound interest, is regarded as the property of the policyholder. If he gives up his policy, this reserve fund, which is then called the *cash value* of the policy, is returned to him. A policyholder can also borrow from the company an amount equal to his reserve fund, which is then known as the *loan value* of his policy.

Table of Cash or Loan Values

The table at the right gives the cash or loan values on an ordinary life policy issued at age 35. At the end of the fifth year the cash or loan value is $68.16. Hence the policyholder will receive $68.16 if he discontinues the policy at the end of 5 yr., or he may borrow $68.16 from the company at that time if he continues paying the premiums. This $68.16 equals the reserve fund of the policy.

Cash or Loan Value per $1000 Insurance	
End of 5th yr.	$ 68.16
End of 10th yr.	146.01
End of 15th yr.	233.28
End of 20th yr.	327.58
End of 25th yr.	425.49
End of 30th yr.	522.92

At the top of the page you found that $12.35 is put into the reserve fund each year. Notice that $68.16 is a little more than 5×$12.35, the extra amount being compound interest earned on this reserve fund.

BORROWING FROM AN INSURANCE COMPANY

Use the cash or loan values at the bottom of page 187.

1. At 35 yr. of age Mr. Baron purchased an ordinary life policy for $5000. At the end of 10 yr. he needed some money. How much could he borrow from the life insurance company?

The loan values on page 187 are for a $1000 policy. For a $5000 policy they are 5 times as much.

2. Mr. Baron borrowed $725 from the life insurance company and was charged interest at 6%. How much interest did he pay at the end of the first year?

3. If Mr. Baron had died before paying off his loan, the company would have paid the $5000 insurance less the amount of the loan and the interest due on it. If Mr. Baron had died 6 mo. after making the loan, how much insurance would the company have paid to his beneficiary?

4. Mr. Grady has a $15,000 policy issued at age 35. How much would the company lend him at the end of the 5th year?

Life Insurance Dividends

On page 187 you saw that life insurance premiums are used to pay the mortality cost and the company's expenses and to provide a reserve fund. It often happens that fewer people die during the year than the company expected and the *mortality cost* is therefore less than was estimated. The company may also save on *expenses*. On the *reserve fund* the interest may be over 4%, although only 3% was counted on.

The profit arising from these savings and from the extra interest above 3% is divided among the policyholders in the form of **dividends** at the end of each policy year. If a person whose premium is $26.35 receives a dividend of $5.31 at the end of the year, his insurance actually costs him only $26.35 − $5.31, or $21.04, for that year.

THE ACTUAL COST OF LIFE INSURANCE

1. Ten years ago, at age 35, Mr. Bell took out an ordinary life policy for $1000, with an annual premium of $26.35. His dividends have averaged $5.26 a year. How much has he paid each year, on the average, to carry this insurance? Find his total net payments for the 10 yr.

2. From the table on page 187, tell what cash value Mr. Bell (ex. 1) receives for his policy if he drops his insurance at the end of 10 yr. By subtracting this amount from the net amount paid in 10 yr. (found in ex. 1), find how much the 10 years' insurance has actually cost him. Also find the average cost per year of carrying this $1000 insurance.

3. Mr. Alden carried a 20-payment life policy for $1000 for 10 yr. His yearly premium was $36.22. During the 10 yr. his dividends totaled $61.69. Find his average yearly payment.

4. At the end of 10 yr. Mr. Alden (ex. 3) dropped his policy. Its cash value was then $255.78. How much per year, on an average, had the insurance actually cost him?

5. Mr. Rose took out a $10,000 ordinary life policy at age 30, the yearly premium being $22.85 per $1000. During the first 10 yr. the company paid him a total of $687.30 in dividends. Find the average net cost per year to Mr. Rose for each $1000 insurance that he carried.

6. During the second 10 yr. Mr. Rose (ex. 5) received $897.40 in dividends. Find the average net cost per year for each $1000 insurance for these 10 yr.; the average for the 20 yr.

NOTE. Instead of using the dividends each year to decrease the cost of their insurance, some policyholders leave them with the company, where they earn compound interest like savings deposits. If the policyholder dies, his beneficiary receives the face of the policy plus the accumulated dividends. There is also a plan by which each year's dividends may be used gradually to increase the face of the policy.

AUTOMOBILE INSURANCE

In the United States in a recent year about 25,000 people were killed and over 800,000 people were injured in automobile accidents. In the same year there were about 2,000,000 automobile accidents causing damage to cars or other property. Every year approximately 60,000 cars are totally destroyed by fire and 100,000 more cars are stolen.

An automobile accident often results in great financial loss to the owner of the car that is responsible for the accident. For example, recently a man was obliged to pay damages of $30,000 because his car seriously injured a child. This man carried no automobile insurance and in order to raise the $30,000 damages, he was forced to sell his business, which caused great hardship to himself and his family.

In an automobile collision, the owner of the car causing the accident may be sued for damages. The owner of a car also runs the risk of having his car stolen or destroyed by fire.

Mr. Hall, who lives in a medium-sized city, purchased a new, light, 6-cylinder car. To protect himself against damages and losses such as those described above, he bought the following kinds of automobile insurance:

(1) **Fire and Theft Insurance.** If Mr. Hall's car is stolen or is damaged or destroyed by fire, the insurance company will pay Mr. Hall up to 80% of the value of the car. For this protection Mr. Hall pays a premium of $6.40 a year.

Mr. Hall's premium is that charged for a new car. As a car gets older, the premium rate for fire insurance becomes higher, in proportion to the value of the car, since an old car may catch fire more easily than a new one because of defective parts.

For an expensive, 8-cylinder car the premium for fire insurance is proportionately less than for a light, 4-cylinder or 6-cylinder car, since an expensive car is better built and hence is less likely to take fire.

The premium for theft and fire insurance is much higher for people living in a city than for those living in the country. Why?

(2) **Public Liability Insurance.** If anybody is killed or injured by Mr. Hall's car, the insurance company will try to settle the claim peacefully. If the claim is unreasonable, the company will defend Mr. Hall in court. If damages are awarded, the insurance company will pay up to $10,000 in case one person has been injured, and up to $20,000 in case two or more persons have been injured. For this public liability insurance Mr. Hall pays a premium of $43.20 a year.

If Mr. Hall lived in a very large city, the premium would be very much higher because the streets of such cities are so crowded with cars that accidents are much more likely to happen. The premium is proportionately higher for an old car than for a new one, since accidents may occur more easily with an old car on account of defective parts.

For an additional premium of $5.76 the company would protect Mr. Hall up to $25,000 for injury to one person, and up to $50,000 for injuries to two or more persons, instead of limiting its responsibility to $10,000 and $20,000, respectively, as noted above. Even higher amounts may be provided for by paying extra premiums. This additional protection costs so little more that it is wise to carry it, especially since the courts so often award damages of large amounts, like $20,000 and $30,000.

(3) **Property Damage Insurance.** If a car or other property is accidentally damaged by Mr. Hall's car, the company will pay damages up to $1000. For this protection Mr. Hall pays a premium of $13.00 a year.

The premium would be higher for a heavy, 8-cylinder car because a heavier car can do more damage than a lighter one. It would also be higher for an old car or if Mr. Hall lived in a large city. Why?

For an extra premium of $1.95, the company would pay damages up to $2000. For an additional premium of $148.00, the company would also pay Mr. Hall for any damages *his own car* might suffer in a *collision.* **Collision insurance,** as this is called, is so expensive, however, that Mr. Hall decided not to buy it. The *property damage* insurance mentioned above applies only to the damage done by Mr. Hall's car to another car.

Find the total premium Mr. Hall pays for the year for the fire and theft insurance, the public liability insurance of $10,000 and $20,000, and the property damage insurance of $1000.

EVERYDAY PROBLEMS IN AUTOMOBILE INSURANCE

First read carefully pages 190 *and* 191.

1. If your father or a friend has a car, find out what kinds of automobile insurance he carries, what protection each policy provides, and what premium he pays for each policy.

2. Try to find out what the premiums are in your city for a new, light, 6-cylinder car of some particular make for policies providing *public liability* insurance with limits of $10,000 and $20,000, *property damage* insurance up to $1000, and also *fire and theft* insurance.

3. In ex. 2, find the premium for public liability insurance up to $25,000 and $50,000 and for property damage insurance up to $2000. Why is it wise to carry this extra protection?

4. Find the premium in your city for the same protection as in ex. 2 for a new, heavy, 8-cylinder car of some particular make; for a new, light, 4-cylinder car of some particular make. Explain why these rates differ from those in ex. 2.

5. Name some city near you where the premiums for automobile insurance are higher or lower than those charged in your city and tell why.

6. Give reasons why accidents are more likely to happen in driving an old car than in driving a new one.

7. Mr. Good's business and home are together worth $25,000 and he has no other property. He has two children in school and two in college. Mr. Fair is worth $250,000. Both men own automobiles. For which of these men is it more important to carry automobile insurance? Tell why.

8. Damages of $40,000 in all were awarded to three people injured at the same time by Mr. Bell's car. If Mr. Bell carried the policies in ex. 2, how much of these damages was paid by the insurance company? how much by Mr. Bell? Would the policy in ex. 3 have completely protected Mr. Bell?

IMPROVEMENT TEST No. 9

Test 9 A — Addition (Time 4 min.)

1.	2.	3.	4.	5.
$1\frac{5}{8}$	$2\frac{1}{3}$	$5\frac{1}{6}$	$2\frac{2}{5}$	$8\frac{5}{8}$
$4\frac{1}{8}$	$7\frac{1}{2}$	$3\frac{3}{4}$	$1\frac{4}{5}$	$7\frac{1}{3}$
$2\frac{3}{4}$	$9\frac{1}{4}$	$8\frac{5}{6}$	$1\frac{1}{2}$	$5\frac{1}{3}$
$9\frac{1}{16}$	$6\frac{1}{2}$	$3\frac{1}{6}$	$2\frac{7}{10}$	$6\frac{3}{8}$

Test 9 B — Subtraction (Time 4 min.)

1.	2.	3.	4.
$4\frac{1}{5}$	$6\frac{5}{12}$	$7\frac{1}{2}$	$8\frac{5}{6}$
$2\frac{9}{10}$	$3\frac{7}{12}$	$\frac{1}{3}$	$5\frac{1}{2}$
5.	**6.**	**7.**	**8.**
$3\frac{3}{8}$	8	$9\frac{4}{5}$	$7\frac{1}{2}$
$2\frac{5}{16}$	$7\frac{3}{8}$	$3\frac{1}{3}$	$2\frac{15}{16}$
9.	**10.**	**11.**	**12.**
$6\frac{1}{4}$	9	$8\frac{1}{6}$	$6\frac{2}{3}$
$2\frac{1}{5}$	$8\frac{5}{12}$	$1\frac{1}{4}$	$3\frac{5}{8}$

Test 9 C — Multiplication (Time 4 min. after copying)

1. \$1.53	**2.** \$2.76	**3.** \$1.90
$18\frac{1}{2}$	$11\frac{3}{4}$	$24\frac{2}{3}$
4. $\frac{3}{8}\times1\frac{5}{6}$	**5.** $18\times\frac{5}{6}$	**6.** $4\frac{1}{2}\times37$
7. $\frac{5}{12}\times\frac{9}{10}$	**8.** $98\times1\frac{1}{8}$	**9.** $4\frac{1}{6}\times6\frac{9}{10}$

Test 9 D — Division (Time 4 min. after copying)

1. $\frac{7}{8}\div2$	**2.** $5\frac{1}{4}\div7$	**3.** $29\div\frac{5}{6}$
4. $\frac{7}{10}\div\frac{7}{10}$	**5.** $2\frac{5}{8}\div\frac{14}{15}$	**6.** $3\frac{1}{2}\div4\frac{1}{4}$
7. $8\div3\frac{1}{5}$	**8.** $8\frac{3}{4}\div15$	**9.** $9\frac{3}{4}\div1\frac{1}{2}$
10. $6\div2\frac{5}{8}$	**11.** $3\frac{1}{3}\div16$	**12.** $12\frac{1}{4}\div1\frac{3}{4}$

ACCIDENT AND HEALTH INSURANCE

It is estimated that every person in the United States is disabled, on the average, 11 days a year on account of accidents and sickness; 7 of the 11 days are for sickness, and the remaining 4 for accidents. Every year over 11,000,000 people in this country are accidentally injured, 85,000 of these accidents resulting in death. If a man is disabled by accident or sickness, he may lose his wages, which may prove very serious to his family and himself. The total loss in wages in this country due to non-occupational accident and sickness amounts to the enormous sum of $1,500,000,000 a year.

To protect against the losses due to accidents and sickness, Mr. Griffin has purchased an accident and health insurance policy, for which he pays a premium of $55 a year. This policy gives him the following protection:

(1) If he is continuously disabled by accident or disease so that he cannot work, the insurance company pays him $25 a week as long as such disability continues. In the case of disability due to disease, he is not paid for the first 2 weeks of such illness nor for illness which continues beyond 52 weeks; but this restriction does not apply to disability due to accident.

(2) If he accidentally loses both hands or both feet or one hand and one foot or the sight of both eyes, he receives $7500.

(3) If he accidentally loses one hand or one foot, he receives $3750.

(4) If he dies as a result of accident, his family receives $7500.

(5) If the accident occurs while he is traveling on a train, a street car, or a vessel or while he is riding in a passenger elevator, the company pays double the amounts mentioned in (1) to (4) above.

In many states the workmen engaged in certain industries are protected by a kind of accident insurance which is provided by *Workmen's Compensation Laws.* These laws require that employers shall provide compensation for workmen in cases such as the following: If a workman is accidentally killed while at work, his family receives a certain amount; if a workman is totally disabled by accident while at work, he often receives $66\frac{2}{3}\%$ of his regular wages while disabled.

PROBLEMS IN ACCIDENT AND HEALTH INSURANCE

Before working ex. 1 *to* 6, *study carefully the conditions of Mr. Griffin's policy given on page* 194.

1. Mr. Griffin was ill with pneumonia for 17 weeks. How much did the company pay him in all during that time?

2. Mr. Griffin earns $40 a week when he is well. How much in wages did he lose due to his illness (ex. 1)? How much of the loss was not made up by his health insurance?

3. Instead of being disabled by illness, suppose Mr. Griffin had fallen on a defective sidewalk and had been continuously disabled by this accident for 17 weeks. How much in all would the company have paid him for his 17 weeks' disability?

4. If Mr. Griffin's disability in ex. 3 had been due to injuries received while he was riding in a train, how much would the company have paid him for 17 weeks' disability? What paragraph in Mr. Griffin's policy provides for this situation?

5. Mr. Brown carried an accident and health policy just like Mr. Griffin's. If he was accidentally killed by an automobile, how much did his family receive? If he had been killed while riding in an elevator, how much would his family have received?

6. For a premium of $110 a year, an accident and health insurance policy may be purchased, paying twice the weekly amounts and other insurance given in Mr. Griffin's policy. If Mr. Cook buys such a policy and a few months later is laid up for 39 weeks owing to an automobile accident, how much will the company pay him in all? If the premium paid by Mr. Cook is deducted, how much better off is he as a result of carrying accident and health insurance?

7. Each year in the United States about 85,000 deaths are due to accidents of all kinds; 25,000 of these deaths are due to automobile accidents. What per cent of all the accidental deaths are caused by automobiles?

REVIEW

1. Name a building in your neighborhood which you think would have a low rate of fire insurance. Also name a building which would have a high rate. Give reasons for each selection.

2. The fire insurance premium on Mr. Graham's factory is \$2.30 per \$100 for a 1-year policy and 4 times as much for a 5-year policy. Mr. Graham insures the factory for \$75,000. How much will he save in 15 yr. if he takes 5-year policies instead of 1-year policies?

3. Mr. Stewart is 30 yr. old and has two small children. How much less will his yearly premium be if he takes out an ordinary life policy for \$10,000 now than if he takes out the same policy 10 yr. from now? (See page 185.) What other reason, besides the lower premium, has he for insuring his life now?

4. In ex. 3, suppose that Mr. Stewart takes out a life insurance policy at age 30 and that he can afford to pay about \$250 a year in premiums. If he carries no other insurance, would it be better for him to buy an ordinary life policy, a 20-payment life policy, or a 20-year endowment policy? Why?

5. The workmen of a factory, *as a group*, sometimes have their lives insured for small amounts, such as \$1000 or \$2000 on each life. Such insurance is called **group life insurance.** The clerks of a large store and similar groups of 50 or more employees may also be protected by group insurance. The premiums for such insurance are paid either by the employer alone or by the employer and the employees jointly. Try to find out if any group of employees in your city is protected by group life insurance.

6. There are over 23,000,000 cars in use in the United States. Of these cars about 16% are covered by public liability insurance, 13% by property damage insurance, 2% by collision insurance, and from 35% to 50% by fire and theft insurance. Find the number of cars covered by each kind of insurance.

CHAPTER XI

TAXES

TOWN AND CITY TAXES

Mr. Livingston is the principal of an elementary school in a small city. He finds that it costs the city $102.83 a year to educate each pupil in his school.

Every city needs large sums of money to educate its girls and boys, to build and maintain roads, streets, and public buildings, and to pay its firemen, policemen, and governing officials. This money is raised by **taxation.** The amount which each inhabitant is *taxed* depends on the value of his possessions, because the man with much property is better able to pay a large tax, and also has more need of the town's protection.

Most towns have tax officers, called **assessors,** who fix a value each year for the property to be taxed, including *real estate,* like houses and lots, and *personal property,* like automobiles. This value is called the **assessed valuation.**

Suppose the total assessed valuation of the property in a town is $1,250,000 for a certain year. If the town needs $35,500 that year, the **tax rate** will be $35,500 ÷ $1,250,000, or .0284. Then every owner of property will have to pay a tax of .0284, or 2.84%, of the assessed value of his property.

To find the tax rate on property, find what per cent the amount to be raised is of the total assessed valuation.

A tax rate of 2.84% may also be expressed as a rate of 28.4 mills on $1, $2.84 on $100, or $28.40 on $1000.

City and town governments get most of their money by taxes on real or personal property. State and national governments usually raise money by other forms of taxation.

FINDING THE TAX RATE

1. A town whose total assessed valuation is \$2,388,000 needs \$70,446 for its expenses. Find the tax rate.

2. Express the tax rate in ex. 1 in three other ways.

3. A town with property having a total assessed valuation of \$685,000 needs to raise \$26,441 by taxation. Find the tax rate. Then express this rate in three other ways.

Find the tax rate per \$100 *for the following. Then express each rate in three other ways:*

	Assessed Valuation	Amount Needed		Assessed Valuation	Amount Needed
4.	\$38,000,000	\$836,000	**9.**	\$34,656,000	\$918,384
5.	\$20,850,000	\$781,875	**10.**	\$78,924,000	\$1,764,290
6.	\$51,464,800	\$1,981,364	**11.**	\$60,371,000	\$1,626,510
7.	\$85,000,000	\$3,110,000	**12.**	\$105,930,500	\$4,180,900
8.	\$93,700,000	\$4,046,000	**13.**	\$450,380,000	\$13,870,600

14. The actual value of the property in a town is \$4,562,500. The assessed valuation is \$3,650,000. If \$154,000 is to be raised, what is the tax rate? If the property were assessed at its actual value, what would the rate be?

15. What per cent of the actual value of the property is the assessed valuation in ex. 14?

16. The assessed value of property is often only from 50% to 80% of the real value. How does this affect the tax rate?

17. Find the tax rate in your town for last year and express it in four different ways. Also find what per cent the assessed valuation of some property was of the real value.

18. Hospitals, schools, public buildings, charitable institutions, and churches are not taxed. Can you suggest why? List some buildings in your town that are not taxed.

A TOWN BUDGET

To find the amount to be raised by taxation, many towns list their probable expenses for the coming year in a **budget.** Here are the 1929 and 1930 budgets of Ocean City:

	A 1930 Amount Needed	*B* Tax Rate per $100	*C* 1929 Amount Needed	*D* Tax Rate per $100
Municipal Requirements	$350,957.86	$1.177	$351,036.78	$1.349
Local School Tax	418,346.54	1.403	382,263.18	....
County Tax	153,860.88	.516	139,998.36	....
State School Tax	71,861.38	.241	69,478.74	...
State Road Tax	29,818.00	.100	26,022.00	....
State Institutional Tax	14,909.00	.050	13,011.00	....
Bridge and Tunnel Tax	10,734.48	.036	9,888.36	....
Soldiers' Bonus Tax	5,069.06	.017	4,944.18	....
Total Amount Needed	$1,055,557.20		$996,642.60	
Total Assessed Valuation	$29,818,000.00		$26,022,000.00	
Total Tax Rate	$3.54 per $100		.....	

Exercises

1. What was the total amount of money needed by this town for 1930? What was the total assessed valuation? Show how the total rate was found, using the rule on page 197.

2. Column *B* shows the rate of taxation for each item of expense. Thus, for the support of local schools in 1930, every inhabitant had to pay $1.403 for every $100 of the assessed valuation of his property. By dividing the amount needed for local schools by the assessed valuation, show how this rate was found.

3. Show how each tax rate in Column *B* was found.

4. Add all the rates in Column *B*. How does this sum compare with the total rate found in ex. 1?

5. Find each tax rate to complete Column *D*. Then find the total rate per $100 for 1929 in two different ways.

6. Name five purposes for which your town needs money

FINDING THE AMOUNT OF TAXES YOU MUST PAY

1. Suppose you lived in the town whose budget is given on page 199. If you owned property assessed in 1930 at $14,500, how much would you be taxed on it?

The rate for 1930 in this town was $3.54 per $100, which equals 3.54%. Hence your total tax would be 3.54% of $14,500, or .0354×$14,500.

2. If you owned property assessed at $8000 in the above town in 1930, how much would your local school tax be? your state school tax? your state road tax?

The state school tax helps schools in poorer districts.

3. Mr. Bird owns a lot 75 ft. wide, assessed at $35 a front foot. His house is assessed at $8500, his automobile at $950, and his other personal property at $1500. Find the total amount he is assessed. If the tax rate is $3.94 on $100, what is the total tax on his property?

Find the amount of taxes paid in a given year by persons whose property is assessed as follows:

	Assessed Value	Rate per $100		Assessed Value	Rate per $1000
4.	$20,000	$4.02	**8.**	$203,500	$34.50
5.	$97,540	$2.04	**9.**	$610,400	$28.75
6.	$51,230	$3.27	**10.**	$115,000	$41.60
7.	$74,000	$5.12	**11.**	$108,500	$49.50

12. In Hatton the assessed valuation of property is 45% of the real value and the tax rate is $5.83 on each $100 of assessed value. In Lakewood the property is assessed at 85% of its real value and the rate is $3.24 on each $100 of assessed value. Find the tax in each town on property actually worth $10,000; on a house actually worth $8400.

A tax rate means little unless the per cent which the assessed valuation is of the real value is also considered. A city with a high tax rate and a low valuation may be taxing its citizens less than one with a low rate and a high valuation.

USING A TAX TABLE

To save time in computing taxes, most cities use a tax table. Here is a portion of such a table, used by Ocean City:

Tax Table for Rate of $3.54 per $100

	$00	$1000	$2000	$3000	$4000	$5000	$6000	$7000
$ 00		35.40	70.80	106.20	141.60	177.00	212.40	247.80
50	1.77	37.17	72.57	107.97	143.37	178.77	214.17	249.57
100	3.54	38.94	74.34	109.74	145.14	180.54	215.94	251.34
150	5.31	40.71	76.11	111.51	146.91	182.31	217.71	
200	7.08	42.48	77.88	113.28	148.68	184.08	219.48	
250	8.85	44.25	79.65	115.05	150.45	185.85	221.25	
300	10.62	46.02	81.42	116.82	152.22	187.62	223.02	
350	12.39	47.79	83.19	118.59	153.99	189.39	224.79	
400	14.16	49.56	84.96	120.36	155.76	191.16		
450	15.93	51.33	86.73	122.13	157.53	192.93		
500	17.70	53.10	88.50	123.90	159.30	194.70		

To find the tax on property assessed at $6350, place a ruler horizontally under the number 350 in the column at the left. Then, in the column headed $6000, find the number just above the ruler, which is 224.79. Hence the tax on $6350 is $224.79.

Exercises

1. Using the table, find the tax on $6200; on $6150; on $6050; $6000; $5000; $5150; $4350; $3300; $2400; $7100.

2. Find the tax on $11,250 by adding the taxes on $6000 and $5250. Check by multiplying $11,250 by .0354.

3. Find the tax on $12,250 and on $9350, as in ex. 2.

4. How much greater is the tax on $6250 than on $6200? on $6300 than on $6250? on $6350 than on $6300? Explain how the table is made. Then fill in the amounts that have been omitted in the columns headed $6000 and $7000.

5. By the method used in ex. 4, continue the column headed $1000, giving the tax on $1550, $1600, etc., to $1950.

THE PENALTY FOR PAYING TAXES LATE

1. Mr. Luke's taxes for 1928 were \$522 and were due on Dec. 1, 1928, but he did not pay them until Oct. 14, 1929. As a penalty for paying the taxes late, he was charged 7% interest on the \$522 for the time it was overdue. Find the exact number of days the taxes were overdue. Then find, by the 6% method, the total amount he paid on Oct. 14, 1929.

2. Mr. Carter received a tax bill for \$274.50, due Dec. 1, 1927. He paid the bill on Jan. 3, 1930. If he was charged simple interest at 7% for the time the taxes were overdue, how much extra was he charged for paying late?

From Dec. 1, 1927 to Dec. 1, 1929 is 2 years. From Dec. 1, 1929 to Jan. 3, 1930 is 33 da. Hence the exact time is 2 full years and 33 da.

3. On May 2, Mr. Reed received a bill for \$570 for his 1927 taxes. In Mr. Reed's city taxes are payable in two installments. One half of the \$570 was due on June 1, 1927 and the other half, on Dec. 1, 1927. Mr. Reed was unable to pay either half until May 10, 1928. Then he paid both installments, with interest at 8%. What amount did he pay in all?

Find the amount of interest paid on these overdue taxes, counting the exact number of days between dates:

	Taxes	Date Due	Date Paid	Interest
4.	\$984.20	Dec. 1, 1928	Aug. 9, 1929	9%
5.	\$360.40	June 1, 1929	Sept. 5, 1929	8%
6.	\$670.16	Jan. 10, 1929	Apr. 17, 1929	8%
7.	\$1255.00	Dec. 1, 1926	Sept. 1, 1930	7%

8. In Georgetown the interest on unpaid taxes is 9%. What is the interest there on a tax of \$540, due June 1, 1929 and paid Feb. 14, 1930?

9. If the taxpayer in ex. 8 had borrowed \$540 at 6% in order to pay his taxes on time, how much would he have saved?

HELPING TO SUPPORT THE NATIONAL GOVERNMENT

When John and his mother were in Europe, they purchased many articles much cheaper than they could buy them in this country. But when they returned here, their baggage was examined and they had to pay taxes, known as **duties** (also called *customs* or *tariff*), on many of these purchases. The duty on an article may be an **ad valorem duty** (a.v.), which is a certain per cent of the price for which the article was bought, or a **specific duty** (s.), which is a fixed amount for each unit of measure. A duty of 50% of the cost is an ad valorem duty, while a duty of 25¢ a pound is a specific duty. Duties are collected by officials of the United States government and are used for Federal expenses.

John's mother was allowed to bring in $100 worth of taxable material free. Certain things, like foreign books and magazines, were not taxed. On the other purchases, she paid the duties given in the exercises below.

Exercises

Find what duty John's mother paid on the items in ex. 1 *to* 11:

	Cost	Duty		Cost	Duty
1. Bag,	$13.70	35% a.v.	**5.** Dresses,	$89.23	65% a.v.
2. Linens,	$80.00	55% a.v.	**6.** Jewelry,	$51.25	80% a.v.
3. Games,	$22.00	70% a.v.	**7.** Perfume,	$104.80	75% a.v.
4. Lace,	$65.50	90% a.v.	**8.** Fur coat,	$275.50	50% a.v.

9. 5 lace handkerchiefs, cost $2.90 each; a specific duty of 4¢ each and a 40% a.v. duty.

10. One Oriental rug 4 ft. 9 in. by 8 ft., costing $146.15. Duty 50¢ a square foot, but *not less* than 45% (a.v.).

11. 1 boys' suit and overcoat, weight 8 lb., cost $61.70. One duty of $.50 a pound (s.), and another of 50% (a.v.).

12. Find the total duties paid by John's mother.

PAYING TAXES TO THE GOVERNMENT

1. Mrs. Lawrence bought a French hat in Chicago for $25. The merchant who imported it paid $10 for it in France and paid a 50% ad valorem duty on it. In fixing the price he allowed $10 to cover selling expenses and profits. Who really paid the government duty, the merchant or Mrs. Lawrence?

A tax like this, which is paid by the final purchaser as part of the price of an article, is called an *indirect tax*.

2. There is a tax of 10% on all theater tickets that cost *more* than $3.00; there is no tax on tickets costing $3.00 or less. How much must you pay for a theater ticket regularly priced at $4.00? at $4.50? at $2.00? at $3.00?

3. If the tax was 10%, what was the regular price of a theater ticket for which Mr. Hughes paid $3.85 (including tax)? of a ticket for which Mr. Hughes paid $4.95?

4. On steamship tickets from the United States to Europe there is a tax of $5 on each ticket costing over $60. If Mr. Day bought 3 tickets to Europe that cost $200 each (without the tax), how much in all did he pay for the tickets?

5. John's grandfather died and left $586,000. The Federal *estate tax* on this amount was as follows: no tax on the first $100,000; 1% on the next $50,000; 2% on the next $50,000; 3% on the next $100,000; 4% on the next $200,000, and 5% on the remaining $86,000. How much did the Federal government get from this estate?

6. To settle the grandfather's estate (ex. 5), it was necessary to sell $375,800 worth of stock on which there was a *stamp tax* of 2¢ per $100. What was the total stamp tax paid?

Taxes on luxuries made within this country and on inheritances, incomes, etc. are called **internal revenue taxes.**

The **income tax,** which is a large source of revenue, is not included here because income tax laws change so often.

CHAPTER XII

METRIC SYSTEM

HOW OTHER COUNTRIES MEASURE LENGTH

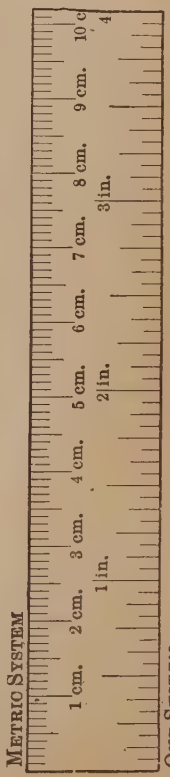
Instead of measuring length by the foot, the yard, etc., most countries of Europe, Asia, and South America use a system of units called the **metric system.** This system is simpler than ours because each larger unit is divided into tenths to form a smaller unit.

The basic unit in the metric system is the **meter,** which equals 39.37 in. To find the other units of length, join to the word *meter* the following prefixes:

milli = $\frac{1}{1000}$	deka = 10
centi = $\frac{1}{100}$	hekto = 100
deci = $\frac{1}{10}$	kilo = 1000

This gives the table of *metric length:*

1 millimeter (mm.)	= $\frac{1}{1000}$ meter (m.)
1 centimeter (cm.)	= $\frac{1}{100}$ meter
1 decimeter (dm.)	= $\frac{1}{10}$ meter
1 dekameter	= 10 meters
1 hektometer (hm.)	= 100 meters
1 kilometer (km.)	= 1000 meters



The illustration at the right shows the actual length of 1 decimeter. Each of the larger divisions is 1 centimeter. Each of the very small divisions is 1 millimeter

MEASURING AND COMPARING DISTANCES

1. A centimeter is what part of a meter? A cent is what part of a dollar? Does *cent* mean $\frac{1}{100}$ in both cases?

2. A millimeter is what part of a meter? A mill is what part of a dollar? What does *mill* mean in both cases?

3. In the decimeter shown on page 205, how many centimeters are there? how many millimeters? How many millimeters are there in 1 centimeter? in 1 meter?

4. Write the table of metric length another way, beginning: 10 millimeters = 1 centimeter, 10 centimeters = 1 decimeter.

5. Make a ruler 15 cm. long out of cardboard and mark it off into centimeters and millimeters, as on page 205.

6. With the ruler made in ex. 5, measure in centimeters and millimeters the width and the length of this book; the length of your middle finger; the length of the nail on your middle finger.

The middle finger of the average adult is about 1 dm. long; the nail of the middle finger is about 1 cm. long and .3 mm. thick.

7. At a French field meet, the best record for the pole vault was 3 m. 92 cm. Express this distance in meters.

Since 1 centimeter = $\frac{1}{100}$ meter, divide by 100 to change centimeters to meters. To change 92 cm. to meters, move the decimal point two places to the left, thus: 92 cm. = .92 m. Hence 3 m. 92 cm. = 3.92 m.

8. The winner of the hammer throw at a French field meet made a distance of 53 m. 29.5 cm. The second best distance was 50 m. 84 cm. Change each distance to meters and tell how much longer the first throw was than the second.

9. In a United States intercollegiate meet, the winning distance for the hammer throw was 156 ft. $\frac{1}{8}$ in.; the second best was 152 ft. $9\frac{7}{8}$ in. How much farther, in feet and inches, was the best throw than the second best?

10. Which is easier, comparing distances by metric units, as in ex. 8, or by feet and inches, as in ex. 9?

EQUIVALENTS TO METRIC UNITS OF LENGTH

You have seen that it is easier to compare distances by metric units than by our units (foot, inch, etc.). Since the metric system is not commonly used in this country, you should learn these approximate equivalents of the metric units of length:

1 meter (m.) = 39.37 in. or 1.1 yd.
1 kilometer (km.) = .6 mi., or $\frac{3}{5}$ mi.

Problems

Find answers to nearest tenth of a unit. To change meters to feet, use 1 *m.* = 39.37 *in.; to change to yards, use* 1 *m.* = 1.1 *yd.*

1. The waves sent out by a certain radio broadcasting station are 422 m. in length. How many yards long are they?

2. The world's walking champion covered 12.975 km. in 1 hr. How many miles an hour did he walk?

3. The Eiffel Tower of Paris, the tallest structure in the world, is 300 m. high. How many feet taller is it than the Washington Monument at Washington, which is $555\frac{1}{2}$ ft. high?

4. Lucy's mother bought 20 m. of silk in Paris. How many yards of silk did she buy?

5. Mr. and Mrs. Deane made a trip of 245 km. on a train in Italy. How many miles long was the trip?

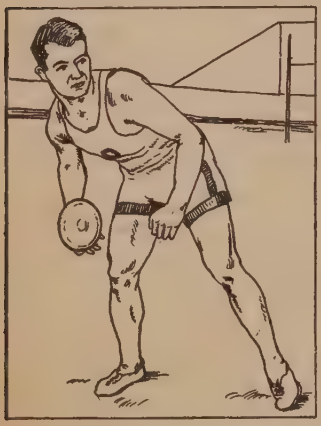
6. In a recent year an aviator from Argentina flew 15,000 km. in a seaplane in 114 hr. How many miles an hour did he fly?

7. A French aviator reached an altitude of 12,442 m. in an airplane. An American aviator later reached an altitude of 38,559 ft. Which altitude was higher? how many feet higher?

NOTE. The metric tables of square measure and of cubic measure, which may easily be constructed from the metric table of length, are omitted here, since the pupil does not have sufficient use for them at this time.

THE OLYMPIC GAMES

The Olympic Games, which take place every four years, were held at Amsterdam, Holland, in 1928. In these games athletes from all parts of the world take part. In the track, field, and other events all distances are measured by the metric system, because that system is used by most of the countries that compete in these contests.



THROWING THE DISCUS

1. The winner of the men's discus throw in the 1928 Olympic Games was an American who threw the discus 47.249 m. How many feet was that? In the 1924 Games the same American won, throwing the discus 46.153 m. How many feet was that?

2. The swimming events included the 100-meter back stroke, the 800-meter relay, the 200-meter breast stroke, and the 500-meter free style. Express each of these distances in yards.

3. The best records in some of the other events of the 1928 Olympic Games are given below. Express each distance in feet and inches, to the nearest whole inch. (Let 1 m. = 39.37 in.)

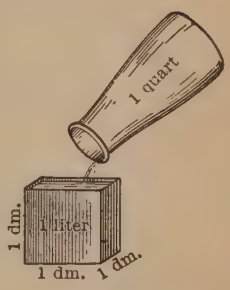
(*a*) High jump, 1.94 m.
(*b*) Pole vault, 4.204 m.
(*c*) Javelin throw, 66.462 m.
(*d*) 16 lb. shot put, 15.867 m.
(*e*) 16 lb. hammer throw, 51.39 m.
(*f*) Discus throw (women), 39.62 m.
(*g*) Running broad jump, 7.73 m.
(*h*) Running hop, step, and jump, 15.21 m.

4. Among the track events were the 10,000-meter run, the 3000-meter steeplechase, the 1600-meter relay, and the 400-meter relay. How many kilometers long was each of these races? how many miles long (to 10ths or 100ths of a mile)?

METRIC MEASURES OF CAPACITY

The metric unit of capacity, used chiefly in measuring liquids, is the **liter.** A cube measuring 1 dm. (or 10 cm.) on each side contains 1 liter. The liter is closely related to the meter, since a liter equals a cubic decimeter.

A complete table of metric capacity can be made by joining the prefixes *milli, centi, deci,* etc., whose meanings are given on page 205, to the word *liter.* Here are the most common measures in *metric capacity:*



1 Cubic Decimeter or 1 Liter

1 milliliter (ml.) = $\frac{1}{1000}$ liter (l.)
1 hektoliter (hl.) = 100 liters

A liter is equivalent to about 1 quart in our measure. Our quart, however, is not closely related to the yard or to the foot, as the liter is related to the meter.

Exercises

1. Write a table of capacity, thus: 1 milliliter = $\frac{1}{1000}$ liter, 1 centiliter = $\frac{1}{100}$ liter, etc.

2. Mr. Tilman is traveling in Europe in his car. He needs about 12 gal. of gasoline. How many liters shall he buy?

3. An aquarium is 5 dm. long, 4 dm. wide, and 3 dm. deep. How many cubic decimeters does it contain? How many liters of water will it hold?

4. An aquarium is 20 in. × 16 in. × 12 in. How many cubic inches does it contain? How many quarts of water will it hold?

5. Is the work shorter and easier with the metric system, as in ex. 3, or with our system, as in ex. 4?

METRIC MEASURES OF WEIGHT

The metric unit of weight is the **gram,** though the unit most commonly used is 1000 grams, or the **kilogram.** A kilogram is equal to the weight of 1 liter (1 cubic decimeter) of water. Hence a relationship exists between the liter and the kilogram.

Here are the most common measures in *metric weight:*

1 kilogram (kg.) = 1000 grams (g.)
1 metric ton (t.) = 1000 kilograms

One kilogram is approximately equal to 2.2 pounds.

Exercises

1. What part of a gram is 1 milligram? 1 centigram? 1 decigram? Write a complete table of metric weight, similar to the one given for metric length on page 205.

2. A United States dime (ten-cent piece) weighs 2 grams 5 decigrams. Write this in grams. If a half dollar weighs 5 times as much, find the weight, in grams, of a half dollar.

3. A United States nickel (five-cent piece) weighs 5 grams. One nickel weighs as much as how many dimes (see ex. 2)? How many nickels will it take to weigh 1 kg.?

4. A tank is 2 m. long, 2 m. wide, and 3 m. high. Express its dimensions in decimeters. Then find its volume in cubic decimeters. How many liters of water will this tank hold? How many kilograms will the water weigh?

5. A tank is 6 ft. long, 6 ft. wide, and 9 ft. high. How many cubic feet does it contain? If $7\frac{1}{2}$ gal. = 1 cu. ft., how many gallons of water will it hold? How many pounds will the water weigh if 1 gal. weighs 8.3 lb.?

6. Is it easier to find the weight of the water by the metric system, as in ex. 4, or by our system, as in ex. 5?

IMPROVEMENT TEST No. 10

Test 10 A — Addition (Time 3 min.)

1.	2.	3.	4.
34975	53926	80640	53598
69855	44285	69141	68364
7480	61638	1797	71959
90096	3869	51784	37876
77823	7805	78194	5896
98767	67535	86921	9570
3849	21363	99225	52721
47692	22357	3748	58168

Test 10 B — Addition (Time 3 min.)

1.	2.	3.	4.
244	601	736	852
132	627	405	144
913	851	833	489
238	103	946	372
155	346	697	410
614	480	241	362
326	602	268	274
505	573	502	100
430	565	980	821
174	974	899	523
273	541	125	165
921	339	278	988
242	180	678	953

Test 10 C — Subtraction (Time 3 min.)

1. 95742061 − 28228346	2. 81695640 − 15145860	3. 54451431 − 25064163
4. 74337627 − 36432775	5. 33173028 − 18875585	6. 60068426 − 11372750
7. 50621488 − 15227939	8. 68127215 − 45145569	9. 59705933 − 17814375

GENERAL REVIEW

1. Bertha deposits $25 in the bank every half year, beginning Jan. 1, 1920, at 4% compounded semiannually. Paul deposits $50 every year, beginning the same time, at 4% compounded annually. Using the table on page 141, find how much more Paul will have in the bank on Dec. 31, 1939 than Bertha.

2. To help Mr. Hale build a house, the Hamilton Building and Loan Association lends him $7200 at 7% interest, payable monthly. Mr. Hale must buy 36 $200 shares in the association at $1 a month per share. In 139 mo. these shares mature and he turns them over to the association to pay his debt. How much more than the amount he borrowed does Mr. Hale pay in all, including interest and payments on the shares?

3. Mrs. Morris wants to insure her house against fire for $12,500. The premium rate is $.32 per $100 for a 1-year policy and 4 times as much for a 5-year policy. How much can Mrs. Morris save in 15 yr. by buying 5-year policies?

4. At age 35 Mr. Cross bought an ordinary life policy for $12,000, at an annual premium of $26.35 per $1000. His total dividends averaged $62.16 a year. After 15 yr. he dropped the policy and received its cash value of $233.28 per $1000 insurance. How much had the insurance actually cost him per year?

5. Mr. Lewis imports from Europe $686 worth of cloth containing 612 yd. and weighing 511 lb. He pays duties of 30¢ a pound and 45% ad valorem. If he allows 32% of the selling price for overhead expenses, 12% for profits, and the rest for the cost and the duties, at what price per yard, to the nearest cent, shall he sell the goods?

6. If you wanted to buy about 1 lb. of butter in France, would you ask for 5 kg. or 1 kg. or $\frac{1}{2}$ kg.?

7. Albert Braun, who lives in Switzerland, writes that he weighs 46.8 kg. How many pounds does he weigh?

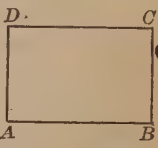
CHAPTER XIII

STUDYING GEOMETRIC FIGURES

PERPENDICULAR LINES

Last year you learned many facts about lines, angles, and triangles. You will now review some of these facts and also learn some interesting new things about geometric figures.

You have learned that the corners of a rectangle form **right angles.** Thus, in the rectangle shown at the right, angles A, B, C, and D are right angles. You also know that two lines that form a right angle are said to be **perpendicular** to each other. Thus, line AB is perpendicular to line BC at B. Likewise, line AB is perpendicular to line AD at A. What lines are perpendicular to each other at D? at C?



Drawing Perpendicular Lines

If you wish to draw a line perpendicular to line AB at point C, a quick way to do it is to use a draftsman's triangle, which is made of wood or celluloid.

First lay your ruler along line AB, as shown below in Fig. 1. Then slide the triangle along the ruler, as in Fig. 2, so that the

Fig. 1

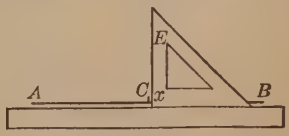
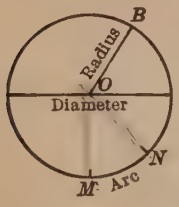


Fig. 2

horizontal edge of its right angle (x) is close to the ruler, while its vertical edge, E, just touches point C. Then draw a line from C along edge E of the triangle. This line will be perpendicular to line AB at point C.

USING COMPASSES TO DRAW PERPENDICULARS

1. Last year you used your compasses frequently to draw circles and arcs. An **arc** is a part of a circle, like the curved line MN. To draw a circle or an arc you must know the length of the radius, OB, and the position of the center, O, of the circle.



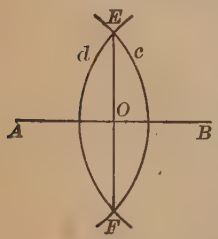
2. If you wish to draw one line perpendicular to another at a given point, you may do so by using your compasses instead of a draftsman's triangle. For example, to draw a line perpendicular to AB at point C you proceed as follows:

With C as center and with a radius of convenient length, such as 1 in., draw arcs cutting line AB at M and N. Then, with a radius a little longer than CM and with N as center, draw an arc s above AB. With the same radius and with M as center, draw another arc t, intersecting arc s at point O. With your ruler, join points O and C. Then CO is perpendicular to AB at C. To draw a perpendicular at the end of a line, first extend the line and then proceed as above.

3. Draw a square, making the right angles with compasses.

Using Compasses to Bisect a Line

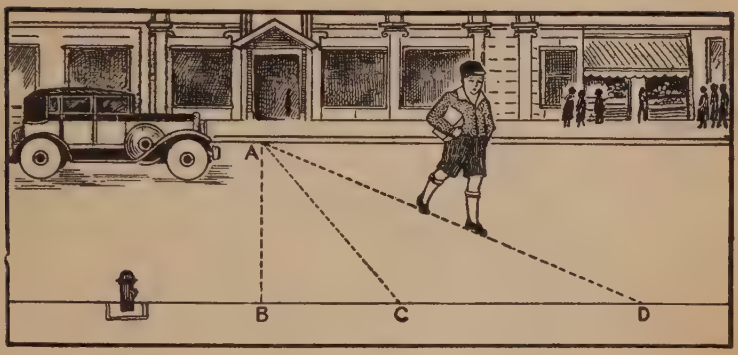
1. You may also use your compasses to find the middle point of, or to **bisect,** a line, such as line AB.



With A as center and with a radius somewhat longer than half of AB, draw arc c, as shown here. With the same radius and with B as center, draw arc d, cutting arc c at E and at F. Draw FE, which will cut AB at O, the midpoint of AB. Then $AO = OB$. AB is said to be bisected at O.

2. Bisect several different lines with your compasses. To test the work, measure the halves of each line with a ruler or compasses.

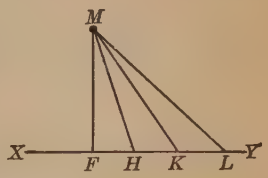
FINDING THE SHORTEST DISTANCE



1. Main Street is a very busy street with many automobiles passing to and fro. On this account Fred wanted to cross this street as quickly as possible. Hence, starting at point *A*, he crossed in the direction marked *AD* in the picture. Did he cross the street in the quickest way? What other route would have been shorter? What route would have been the shortest?

2. Which route from *A* across the street is perpendicular to the edge, *BD*, of the sidewalk? Which routes are not perpendicular to *BD*? What is the shortest distance from point *A* to the line *BD*?

3. Draw line *XY* and place point *M* above it, as shown here. Then from *M* draw line *MF* perpendicular to *XY*.



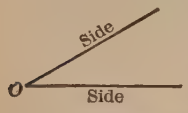
To draw *MF* lay a ruler along *XY*, as you did on page 213, and slide your triangle along the ruler until its vertical edge just touches point *M*. Then from *M* draw a line along the vertical edge of the triangle. This gives line *MF*, which is perpendicular to line *XY*.

From *M* also draw lines *MH*, *MK*, and *ML* and measure the length of each. Which line is shortest?

The perpendicular is the shortest distance from a point to a line.

ANGLES

1. Last year you learned that two straight lines meeting at a point form an angle. The lines are called the sides of the angle and the point O, the **vertex.**

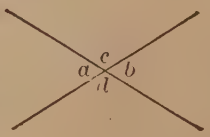


2. The commonest angles are: (1) the **right angle;** (2) the **acute angle,** which is less than a right angle; (3) the **obtuse angle,** which is greater than a right angle; and (4) the **straight angle,** which equals two right angles.

3. If you think of the two sides of an angle as hinged together at the vertex, you will see that the size of an angle depends on how far one side is turned away from the other side. In the right angle illustrated above, the sides are turned farther away from each other than they are in the acute angle.

4. The lengths of the sides of an angle have no effect on its size. Thus, the angles at the right are equal, since each represents the same amount of turning, though the sides of one angle are longer than those of the other.

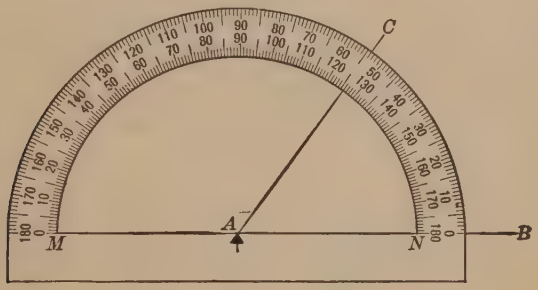
5. If two straight lines intersect, as shown here, angles a and b are called **vertical,** or **opposite, angles.** Similarly, c and d are vertical angles. You can readily see that angle a is the same size as angle b, since each would represent the same amount of turning if the two intersecting lines were hinged together at their point of intersection.



If two lines intersect, the vertical angles are equal.

MEASURING ANGLES WITH A PROTRACTOR

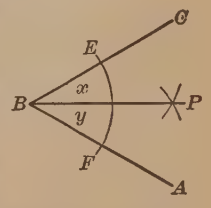
You have learned that angles are measured in degrees and that 90° make a right angle. The instrument for measuring angles, called a *protractor*, has its semicircular edge marked off into 180°, as shown below.



To measure angle BAC with a protractor, place the protractor over angle BAC so that the center of the protractor is at the vertex A, and the edge MN of the protractor lies along side AB of the angle, as shown above. Side AB of the angle should pass through the "zero" mark on the scale of the protractor. The side AC now points to 54°; hence angle BAC contains 54°.

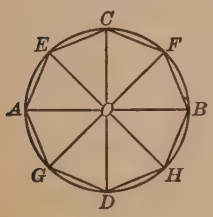
Bisecting an Angle with Compasses

To **bisect** an angle means to divide it into two equal angles. To bisect angle ABC, place one point of the compasses at B as center. With any convenient radius, draw an arc cutting the sides of the angle at E and F. Then, with EF as radius and E as center, draw an arc; with the same radius and with F as center, draw another arc cutting the first arc at P. Draw line BP, which bisects angle ABC, making angle x equal to angle y. BP is called the **bisector** of angle ABC.



DRAWING DESIGNS

1. Draw the figure shown here. First draw the circle; then draw AB. With your compasses draw DC perpendicular to AB at O. With the compasses bisect angles BOC and COA and extend the bisectors as diameters to cut the circle at F, G, E, and H. If you connect the eight points on the circle, you will have a **regular octagon.**



2. Divide a circle into eight equal parts, as in ex. 1, and draw these designs. Fig. 2 is based on Fig. 1; Fig. 4 is based on Fig. 3:

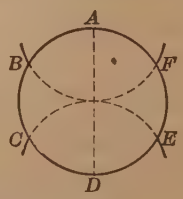
FIG. 1

FIG. 2

FIG. 3

FIG. 4

3. Draw a circle and a diameter AD. With A and D as centers and with the same radius as that of the circle, draw arcs cutting the circle at B and F and at C and E. Join points A, B, C, D, E, and F and erase the arcs and the diameter. You will then have a **regular hexagon.**



4. Divide a circle as in ex. 3, and draw these designs, basing Fig. 6 on Fig. 5, and Fig. 8 on Fig. 7.

FIG. 5

FIG. 6

FIG. 7

FIG. 8

STUDYING THE ANGLES OF A TRIANGLE

1. Draw a triangle, ABC, of any shape, with a base about 5 in. long. With a protractor carefully measure angle A and write down the number of degrees it contains. Measure angles B and C in the same way and record the size of each. Add the three results. How many degrees are there in all three angles together?

2. Draw three other triangles of different shapes, and find the sum of the three angles of each. Is each sum equal to or nearly equal to 180°, regardless of the shape of the triangle?

3. Draw a line AB 3 in. long. With a radius of 4 in. and with A as center, draw an arc above AB. With the same radius and with B as center, draw another arc intersecting the first arc at C. Draw AC and BC. You now have triangle ABC, which is called an **isosceles triangle** because two of its sides, AC and BC, are equal. Measure angles A and B, which are called **base angles.** How do these base angles compare in size?

4. Draw four other isosceles triangles of different sizes and compare the base angles in each. Are they equal?

5. Draw a triangle having each of its sides 3 in. long. This is called an **equilateral triangle** because all three of its sides are equal. Measure each of the three angles and compare them in size. How many degrees does each angle contain?

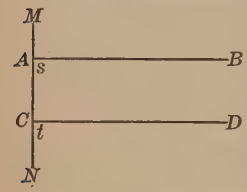
6. In triangle ABC suppose that angle $A = 40°$ and angle $B = 70°$. How many degrees does angle C contain?

7. The above exercises illustrate these facts:

The sum of the three angles of any triangle is always 180°.
The base angles of an isosceles triangle are equal.
The three angles of any equilateral triangle are equal.
Each angle of an equilateral triangle equals 60°.

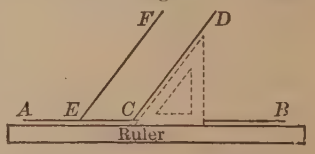
PARALLEL LINES

1. Draw a line, MN, about 6 in. long and mark points A and C on it. With your draftsman's triangle draw a line, AB, perpendicular to MN at A; also draw CD perpendicular to MN at C. Then angles s and t are right angles. Since lines AB and CD both make the same angle with line MN, they run in the *same direction.* Lines like AB and CD, which have the same direction, are called **parallel lines.**



2. You can easily see that parallel lines are everywhere the same distance apart. You also see that parallel lines never meet, no matter how far they are extended in either direction.

3. Draw line AB and place your ruler along it. Draw line CD along the slanting edge of your triangle, as shown at the right. This makes angle BCD equal to the larger acute angle of the triangle. Now slide the triangle along the ruler and draw EF in the same way. Then lines CD and EF are parallel, since they both make the same angle with line AB and hence have the same direction.



In the above figure, angles BCD and BEF are called **corresponding angles.** By making these angles equal to each other, you were able to draw CD parallel to EF.

To draw parallel lines it is easier to use one of the edges of the right angle of the triangle than to use the slanting edge.

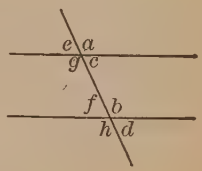
4. The above work illustrates the following facts:

If two lines are each perpendicular to a third line, the two lines are parallel.

If two lines make equal corresponding angles with a third line, the two lines are parallel.

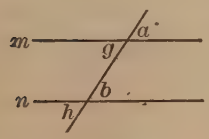
STUDYING PARALLEL LINES

1. When two lines are cut by a third line, angles a and b are called **corresponding angles.** Other pairs of corresponding angles are angles c and d, angles e and f, and angles g and h.



Angles g and b and angles c and f are called **alternate interior angles,** while angles a and h and angles e and d are called **alternate exterior angles.**

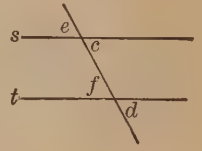
2. If lines m and n are cut by a third line so that alternate angles b and g are equal, show that lines m and n must be parallel.



You are told that angles b and g are equal. You also know that angles a and g are equal. Why? Hence angles b and a are equal because each is equal to angle g. Angles b and a are also corresponding angles. On page 220 you learned that if two lines make equal corresponding angles with a third line, the two lines must be parallel; hence lines m and n must be parallel.

3. In ex. 2, instead of beginning with angle b equal to angle g, suppose you begin with angle h equal to angle a. First show that angle h also equals angle b. Then tell why lines m and n must be parallel.

4. If angles e and d are equal, show that lines s and t are parallel. Also show that the lines are parallel if you start with angles c and f equal.



5. You have now learned these facts:

If two lines are cut by a third line, the lines are parallel

(1) *if any pair of corresponding angles are equal* or

(2) *if any pair of alternate interior angles are equal* or

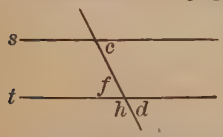
(3) *if any pair of alternate exterior angles are equal.*

MORE FACTS ABOUT PARALLEL LINES

1. On page 221 you started out knowing that certain angles were equal and you showed that the two lines would have to be parallel. Let us now start the other way, first knowing that the lines are parallel, and then see whether certain angles are equal.

2. If line s is parallel to line t, show that angles c and d are equal. Also show that angles c and f are equal.

Since lines s and t are parallel, they both run in the same direction. You learned on page 220 that if two lines have the same direction, each makes the same angle with a third line; hence angle c equals angle d. You may also verify this fact by drawing two parallel lines on paper and measuring angles c and d with a protractor.

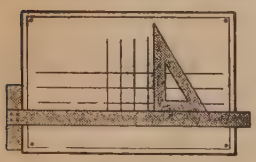


You have just shown that angle c equals angle d. Angle f also equals angle d. Why? Hence angle c equals angle f. Why?

3. In ex. 2, prove that another pair of alternate interior angles are equal. Also prove that two different pairs of alternate exterior angles are equal.

4. In ex. 2, if angle $c = 70°$, find the sizes of angles h, f, and d. Also find the sizes of the angles corresponding to h and f.

5. On this drawing, the horizontal parallel lines were drawn by sliding the T-square along the left-hand edge of the drawing board. The vertical parallel lines were drawn by holding the T-square firmly in one position and sliding the triangle along it. Explain why both sets of lines are parallel.



6. In ex. 2 to 4 above, you have learned the following:

If two parallel lines are cut by a third line, these facts are true:

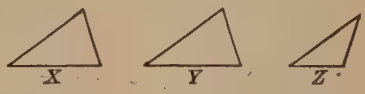
(1) *Any pair of corresponding angles are equal.*

(2) *Any pair of alternate interior angles are equal.*

(3) *Any pair of alternate exterior angles are equal.*

CONGRUENT TRIANGLES

1. Triangles of exactly the same size and shape are called **congruent triangles.** For example, triangles X and Y are congruent, but triangles Y and Z are not congruent.



2. If you have drawn two triangles, like X and Y, and wish to see if they are congruent, you can cut them out and place one over the other. If triangle X exactly fits over triangle Y, then they are congruent.

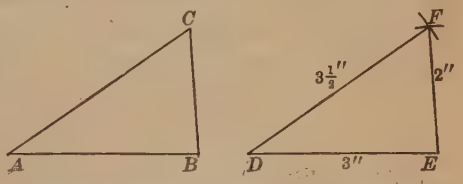
You will now learn some easier ways to tell whether triangles are congruent without cutting them out and matching them.

How to Copy a Triangle

If you have drawn a triangle, like ABC, and wish to copy it, you may do so, as shown below. This means that you will draw a new triangle that will be congruent to triangle ABC.

Measure the sides of triangle ABC. Suppose you find that $AB = 3$ in., $BC = 2$ in., and $CA = 3\frac{1}{2}$ in.

To copy triangle ABC, first draw line DE 3 in. long. With a radius of $3\frac{1}{2}$ in. and with D as center, draw an arc above DE, as shown in the figure. With a radius of 2 in. and with E as center, draw another arc cutting the first arc at F. Draw DF and EF. You now have triangle DEF, whose sides are equal to those of triangle ABC.

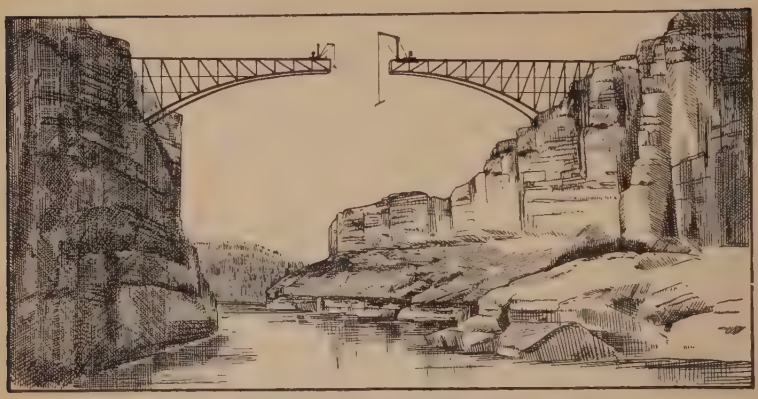


Cut out triangles ABC and DEF and place one over the other. Are the triangles congruent?

3. The work above illustrates the following fact:

If the three sides of one triangle are respectively equal to the three sides of another triangle, the triangles are congruent.

This is another way of saying that a triangle of only *one particular size and shape* can be constructed from three given sides.

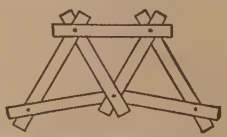


WHY DOESN'T THIS BRIDGE FALL?

1. Draw four triangles, each having side $AB = 6$ in., $BC = 5$ in., and $CA = 4$ in. Cut out the triangles and place them on top of one another. Have they all the same size and shape? Are the four triangles congruent?

2. Form a triangle by nailing three sticks together, using a single wire nail at each joint, as shown at the left. Can you change the shape of this triangle without removing the nails? Is this triangle a rigid figure?

3. Form three triangles by fastening seven sticks together, as shown at the right, using single wire nails at each joint. Is the entire figure rigid?



4. The picture at the top of the page shows the new bridge over the Grand Canyon in Arizona. Each half of the bridge was built out separately from the walls of the canyon and finally the two halves met in the middle. Each half of the bridge is constructed of many triangles fastened together. Explain why the two halves of the bridge did not fall before they met in the middle of the canyon.

TWO OTHER WAYS TO COPY A TRIANGLE

1. If you have already drawn triangle ABC, shown below, you may also copy it in either of the following ways:

First Method. Measure two sides and the included angle of triangle ABC (Fig. 1). Suppose you find that $AB = 5\frac{1}{2}$ in., $AC = 5$ in., and angle $A = 33°$. To copy triangle ABC, first draw DE $5\frac{1}{2}$ in. long, as in Fig. 2. At D draw angle EDX equal to 33°. With D as center and a radius of 5 in., draw an arc cutting DX at F. Then $DF = 5$ in. Draw line FE. Then triangle DEF is an exact copy of triangle ABC.

Cut out triangles ABC and DEF and place one over the other. Are both triangles of the same size and shape? Are the triangles congruent?

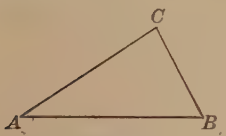


FIG. 1

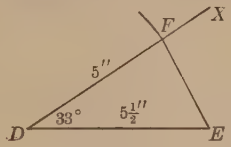


FIG. 2

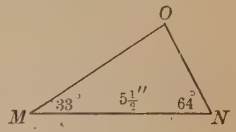


FIG. 3

Second Method. This time, in triangle ABC (Fig. 1) measure only side AB and angles A and B. Suppose you find that angle $A = 33°$, angle $B = 64°$, and side $AB = 5\frac{1}{2}$ in. To copy triangle ABC, first draw line MN $5\frac{1}{2}$ in. long, as in Fig. 3. At M draw an angle of 33°. At N draw an angle of 64°. Extend the upper sides of the two angles until they meet at O. Then triangle MNO is an exact copy of triangle ABC. Cut out the triangles and show that they are congruent.

Since triangles MNO and ABC are congruent, this means that each part of triangle MNO is equal to the corresponding part of triangle ABC. Hence you know *without measuring* that $NO = BC$ and that $MO = AC$; you also know, without measuring, that angle O = angle C.

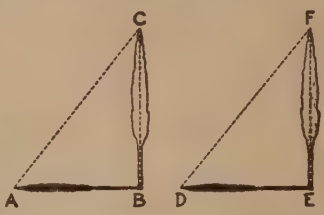
2. The above work illustrates the following facts:

If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another triangle, the triangles are congruent.

If two angles and the included side of one triangle are respectively equal to two angles and the included side of another triangle, the triangles are congruent.

USING CONGRUENT TRIANGLES

1. One morning at 9 o'clock Tom Brown observed that two trees, BC and EF, cast shadows of the same length. Tom said that the two trees must be of the same height. This is how he proved it:



Let AB and DE represent the shadows; then $AB = DE$. Why? Form triangles ABC and DEF by drawing lines AC and DF, which show the direction of the sun's rays. Then angle A = angle D, since the sun's rays are parallel and therefore strike the ground at equal angles. Similarly angle B = angle E. Why?

Tell why triangle ABC is congruent to triangle DEF. Then show why $EF = BC$, which proves that the trees have the same height.

2. George wanted to know the approximate distance, AC, across a pond. This is how he found it:

On the ground he paced off a distance, AB, 250 ft. long, and drove a stake at B. From B he continued pacing in the same direction for another 250 ft. and drove a stake at F. He then paced from C to B and found that $CB = 300$ ft. From B he continued in the same straight line for another 300 ft., driving a stake at E. He now paced the distance EF and found it to be 350 ft. long, which he said was the same as the distance, AC, across the pond.

You see that angles ABC and EBF are equal. Why? Show that triangles ABC and BEF are congruent by showing that two sides and the included angle of one triangle are equal to two sides and the included angle of the other. Then tell why side EF equals side AC (the distance across the pond).

Draw triangle ABC if three of its parts are as follows:

3. $AB = 5''$, $AC = 4''$, $A = 40°$

4. $AB = 6''$, $BC = 6''$, $AC = 7''$

5. $AB = 5''$, $A = 50°$, $B = 55°$

6. $AB = 6''$, $BC = 4''$, $B = 90°$

SYMMETRY

1. If you draw a line, *ST*, through the middle of the figure shown here, you see that the right side of the figure is just like the left side. Hence if you should cut out this figure and fold it on line *ST*, its right side would exactly fit over or match its left side.

2. This figure is a **symmetric figure** because, by one folding, you can make the right side of it exactly match the left side. The line of folding, or crease, is called **the axis of symmetry.**

3. Which figures below are symmetric? which are not?

4. Some figures have more than one axis of symmetry. For example, if the first figure below is folded on *any one* of the dotted lines, its two sides will match; hence this figure has four different axes of symmetry. In the other figures below, how many axes of symmetry are there and where are they located?

5. We see that symmetric figures are beautiful and pleasing because both sides of such figures balance or harmonize.

FACTS ABOUT SYMMETRIC FIGURES

1. Fold a piece of paper. Open the paper and in pencil draw half of a bird kite on the right side of the crease (ST), as in Fig. 1. Outline the drawing in ink and, while the ink is still wet, fold the paper over the same crease again, thus making an imprint of the drawing on the left side of ST. If you now open the paper, you will have a bird kite, as in Fig. 2. Is Fig. 2 symmetric? Why?

FIG. 1

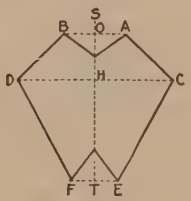
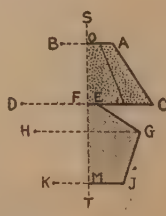


FIG. 2

2. In Fig. 2, points A and B, which are called **corresponding points,** are each the same distance from the axis of symmetry ST. Why? Hence $AO = BO$. You also see that AB is perpendicular to ST at O. Are the same things true about corresponding points C and D and corresponding points E and F?

In a symmetric figure, two corresponding points are each the same distance from the axis of symmetry; a line joining two corresponding points is perpendicular to the axis of symmetry.

3. If you have half of a symmetric figure, such as half of the table lamp shown at the right, you can draw the other half by using the facts given above.



First locate a point B to correspond to point A. To do this, draw from point A a line, AO, perpendicular to ST (see page 215). Why is AO drawn perpendicular to ST? Extend AO to the left and mark off $OB = OA$. Then A and B are corresponding points. Why? How do you locate point D, which corresponds to point C? How do you locate points F, H, and K? If you connect points O, B, D, F, H, K, and M, the lamp will be complete. Is the final figure symmetric? Why?

4. Copy the half of the lamp shown above, making it twice as large; then draw the other half.

ARE THESE FIGURES SYMMETRIC?

1. When it is not possible to fold a figure to see if it is symmetric, draw an axis of symmetry and see if corresponding points are the same distance from it (see page 228). Thus, in Fig. 1, show by measuring that points A and B are each the same distance from the dotted line. Test ten other pairs of corresponding points in Fig. 1 in the same way. Is Fig. 1 symmetric? Test Fig. 2 and Fig. 3. Are they symmetric?

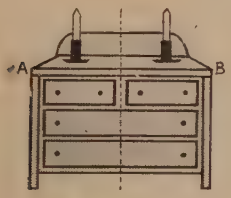


FIG. 1. DRESSER

FIG. 2. NAVAJO INDIAN RUG

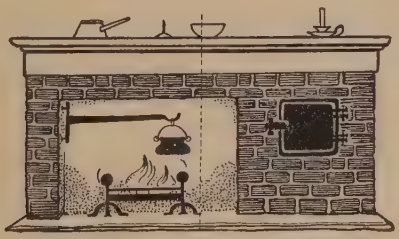
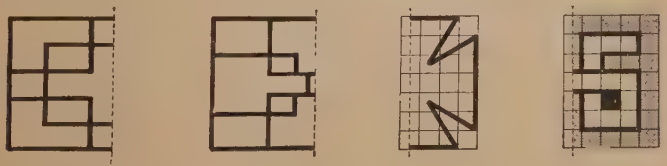


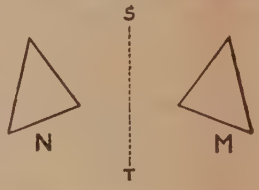
FIG. 3. COLONIAL FIREPLACE

2. These are halves of symmetric figures. Copy each on a larger scale and draw the other half. Use squared paper for the last two.



ARRANGING FIGURES SYMMETRICALLY

1. You have found that a *single* figure is symmetric if, on folding, its two halves exactly match. Two *different* figures, drawn on the same sheet of paper, are said to be **symmetric with respect to a given line** if they match when the paper is folded on that line. Thus, triangles *N* and *M* are symmetric with respect to line *ST* because they match when the paper is folded on *ST*. In this case line *ST* is also called the **axis of symmetry.**



2. Fold a sheet of paper and quickly draw a triangle in ink on the right of the crease. Fold the paper again, making an imprint of the triangle on the left of the crease. Are these two triangles symmetric with respect to the crease? Are these two triangles also congruent?

3. Is the pair of gloves in Fig. 1 arranged symmetrically with respect to the line? Are the gloves congruent figures? In Fig. 2, are the gloves symmetric with respect to the line? Are these gloves congruent? Are two symmetric figures always congruent? Are two congruent figures always symmetric?

FIG. 1

FIG. 2

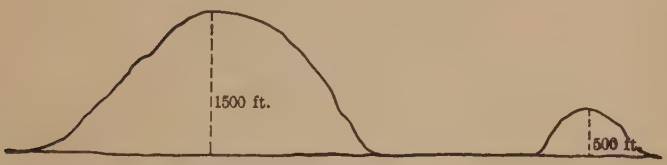
4. Cut out two congruent squares. Arrange them symmetrically with respect to a line. Also arrange them so that they are not symmetric with respect to a line.

5. Tell how to arrange some pictures symmetrically on a wall or some plates symmetrically on a table.

Figures symmetric with respect to a line are also congruent.

CHAPTER XIV

RATIO AND PROPORTION



COMPARING NUMBERS

John was telling Alice about his trip to Blue Mountain. "The mountain," he said, "is 1000 ft. higher than Bald Hill. I think it is three times as high." He was comparing the heights of the mountain and the hill in these two ways:

(1) He compared their heights by *subtraction* by showing the *difference* in the heights. In the picture above, the mountain is 1500 ft. − 500 ft., or 1000 ft., higher than the hill.

(2) He compared them by *division* by showing *how many times* one height is contained in the other. Thus, the mountain is $\frac{1500}{500}$ times, or 3 times, as high as the hill.

Similarly, if Ruth has \$5 and Sarah has \$2, Ruth has \$3 *more than* Sarah or $2\frac{1}{2}$ times *as much as* Sarah.

Exercises

1. 30 is how much greater than 10? 30 is how many times as large as 10? 10 is how much less than 30?

Compare the first number with the second in two ways:

2. 8, 2	**4.** 6, 1	**6.** 12, 4	**8.** 40, 5	**10.** 28, 14
3. 5, 2	**5.** 9, 6	**7.** 10, 3	**9.** 21, 6	**11.** 15, 15

COMPARING NUMBERS BY DIVISION

Will's fish pole is 12 ft. long, while Ted's is 6 ft. long. You can compare the length of Will's pole with the length of Ted's by dividing 12 by 6, to find *how many times* 12 contains 6. Will's pole is $\frac{12}{6}$, or 2, times as long as Ted's.

You can also compare the length of Ted's pole with the length of Will's by dividing 6 by 12 to find *what part* 6 is of 12. Ted's pole is $\frac{6}{12}$, or $\frac{1}{2}$, as long as Will's.

These comparisons are both made by division. The result is called the **ratio** of the one number to the other. Thus, 2 is the *ratio* of the length of Will's pole to Ted's, while $\frac{1}{2}$ is the *ratio* of the length of Ted's pole to Will's.

When two numbers are compared by division, the result is called their ratio.

Ratio is just a technical name for the quotient that results when numbers are compared by division. A ratio may be a fraction, like $\frac{7}{5}$ or $\frac{5}{7}$, or a whole number, like 2, or a mixed number, like $2\frac{1}{2}$.

Exercises

1. What part of 9 is 5? What is the ratio of 5 to 9?

2. How many times 3 is 9? What is the ratio of 9 to 3?

3. Ida has \$11 and Paul has \$6. What is the ratio of Ida's savings to Paul's? of Paul's savings to Ida's?

4. Alma weighs 96 lb. and Edna, 84 lb. What is the ratio of Alma's weight to Edna's? of Edna's weight to Alma's?

Reduce to lowest terms the fraction that expresses the ratio.

5. Margaret says that she weighs twice as much as her youngest sister. Say this in *ratio* language.

Find the ratio of the first number to the second:

6. 3, 5	**8.** 9, 9	**10.** 10, 3	**12.** 19, 20
7. 7, 9	**9.** 25, 5	**11.** 144, 8	**13.** 25, 10

HOW TO WRITE RATIOS

In comparing two quantities, such as 7 lb. and 5 lb., you may say that they have the ratio of 7 lb. to 5 lb. or of 7 to 5. This ratio may be written as $\frac{7}{5}$. It may also be written as $7 \div 5$ or as $7 : 5$. Hence $\frac{7}{5}$, $7 \div 5$, and $7 : 5$ all show the same ratio. 7 and 5 are the **terms** of the ratio.

The ratio of 2 to 6 may be written as $\frac{2}{6}$. But $\frac{2}{6} = \frac{1}{3}$; hence the ratio of 2 to 6 is the same as the ratio of 1 to 3. This means that 2 compares with 6 in the same way that 1 compares with 3.

Similarly, by multiplying both terms of $\frac{2}{6}$ by 4, you find that the ratio $\frac{2}{6} = \frac{8}{24}$. This means that 2 compares with 6 in the same way that 8 compares with 24.

You can multiply or divide both terms of a ratio by the same number without changing its value.

Exercises

1. Write in three ways the ratio of 3 to 11; of 18 to 13.

Write each of the following ratios in two other ways:

2. $5 \div 2$ **3.** $7 \div 8$ **4.** $6 \div 7$ **5.** $9 \div 9$ **6.** $1 : 8$ **7.** $7 : 1$ **8.** $\frac{5}{12}$ **9.** $\frac{100}{19}$

Write each ratio as a fraction in its simplest form:

10. $6 \div 12$ **11.** $22 \div 4$ **12.** $36 \div 8$ **13.** 4 in. $\div$ 8 in. **14.** 6 ft. $\div$ 9 ft. **15.** 9 lb. $\div$ 6 lb. **16.** $\frac{27}{45}$ **17.** $\frac{14}{21}$ **18.** $\frac{80}{30}$ **19.** 18 yd. : 12 yd. **20.** 36 in. : 84 in. **21.** 48 oz. : 16 oz.

22. Which of these ratios equal $\frac{3}{4}$: $\frac{9}{12}$, $\frac{12}{20}$, $\frac{18}{24}$, $\frac{24}{32}$, $\frac{6}{9}$?

23. Write as fractions three ratios each equal to $\frac{14}{16}$.

24. Last week Ann earned \$5 and spent \$4. Dora earned \$15 and spent \$12. Write the ratio of each girl's expenses to her earnings. How do these ratios compare?

FINDING RATIOS

Rose wanted to know the ratio of the length of her desk to its width. The desk was 2 ft. long and 18 in. wide. Since she could not compare feet with inches, Rose first changed 2 ft. to 24 in. Then the ratio of the length to the width was $\frac{24}{18}$, or $\frac{4}{3}$.

If the ratio of two quantities is to be found, the quantities must first be expressed in the same unit of measure.

Ratios in Your Classroom

1. Measure your desk and find the ratio of its width to its length. What is the ratio of its length to its width?

2. Have a member of the class measure the teacher's desk. Then find the ratio of its length to its width.

3. Measure the blackboard in your room and compare its length with its width by finding their ratio.

4. Using the dimensions found in ex. 1 and 2, find the ratio between the area of the teacher's desk and of yours.

5. Measure the floor of your classroom. What is the ratio of its length to its width? Tell why you cannot compare its area with its length.

6. How many girls are there in your class? how many boys? Find the ratio of the number of boys to the number of girls; of the number of girls to the total number of pupils.

Express these ratios as fractions in their simplest forms:

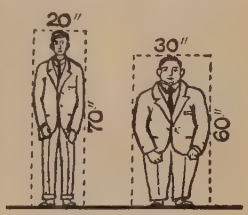
7. 2 in. : 2 ft.
8. 7 yd. : 2 ft.
9. 3 qt. : 2 bu.
10. 8 oz. : 5 lb.
11. 9 in. : $1\frac{1}{2}$ yd.
12. 5 lb. : 10 oz.
13. 3 gal. : 3 qt.
14. 15 qt. : 1 pt.
15. 14 ft. : 6 in.
16. $1\frac{1}{2}$ hr. : 1 da.
17. 15 ft. : 9 in.
18. 1320 ft. : 2 mi.

19. What is the ratio of 1 in. to 1 ft.? of 1 in. to 1 yd.?

20. What is the ratio of a nickel to \$1? of a dime to \$1?

PROPORTION

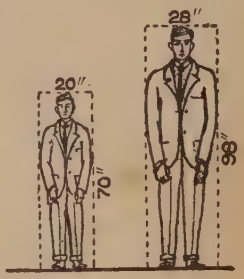
Joe is standing beside the fat man at the circus. Joe's shape is that of a normal young man, his height being 70 in. and his breadth, 20 in. The ratio of Joe's height to his breadth is $\frac{70}{20}$, or $3\frac{1}{2}$. If the fat man's height, 60 in., is compared with his breadth, 30 in., the ratio is $\frac{60}{30}$, or 2. The fat man is *out of proportion* because the ratio of his height to his breadth is only 2, while the ratio of a normal man is $3\frac{1}{2}$



Here Joe is beside the circus giant, who is 98 in. tall and 28 in. broad. The ratio of the giant's height to his breadth is $\frac{98}{28}$, or $3\frac{1}{2}$, which is the same as Joe's. The giant is *well proportioned*, or *in proportion*, because his ratio is normal.

The equality of Joe's ratio and the giant's is shown thus:

$$\frac{70}{20} = \frac{98}{28}$$

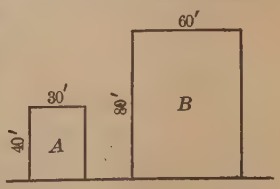


An expression like the above, which shows that two ratios are equal, is called a **proportion.**

Exercises

1. What is the ratio of the width of A to its height? of the width of B to its height? Are the ratios equal? If so, write the proportion showing it.

Do not reduce the ratios to lowest terms.



2. One picture is 2 ft. wide and 3 ft. long. Another picture is 4 ft. wide and 6 ft. long. Write the proportion showing that the ratios of their widths to their lengths are equal.

PROPORTIONS AS EQUATIONS

Last year the Athletic Association of the Taft School collected \$75 in dues and gave \$30 of this amount to help support the baseball team. This year they have collected \$90 in dues. If they give the same proportion of the dues to the team this year as they did last year, what amount will the team receive?

Let x = the number of dollars the team gets this year. Then this year the ratio of the team's allowance to the total dues is $\frac{x}{90}$. Last year the ratio was $\frac{30}{75}$. Since this year's allowance is the same proportion of the dues as it was last year, these two ratios are equal. Hence

$$\frac{x}{90} = \frac{30}{75}$$

The above expression is a **proportion** because it shows that the two ratios are equal. This expression is also an equation like those which you studied on page 40. Every proportion is also an equation. By solving this equation for x, you can find this year's allowance for the team thus:

Start with $\frac{x}{90} = \frac{30}{75}$

To have only x on one side, multiply both sides by 90.

This gives $x = \frac{90 \times 30}{75}$, or 36

Hence the baseball team will receive \$36 this year.

Check. $\frac{36}{90} = \frac{30}{75}$, which reduces to $\frac{2}{5} = \frac{2}{5}$.

When one term of a proportion is unknown, you can easily find it by using the principles of equations.

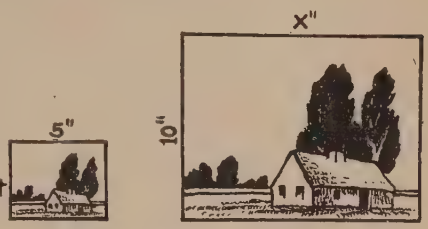
Exercises

Find the unknown number in these proportions, and check:

1. $\frac{x}{16} = \frac{5}{8}$
2. $\frac{x}{15} = \frac{7}{5}$
3. $\frac{12}{24} = \frac{x}{6}$
4. $\frac{12}{18} = \frac{x}{3}$
5. $\frac{x}{6} = \frac{3}{9}$
6. $\frac{x}{3} = \frac{6}{8}$
7. $\frac{x}{14} = \frac{9}{21}$
8. $\frac{x}{72} = \frac{16}{9}$

ENLARGING PICTURES

1. Raymond's camera makes pictures 5 in. long and 4 in. wide. He wishes to enlarge one of his pictures so that it will be 10 in. wide. How long will the enlarged picture be?



When pictures are enlarged, the length and the width of the enlarged picture have the same ratio as the length and the width of the small picture. This can be expressed as a proportion as follows:

Let x = length of large picture, in inches

Form the proportion: $\frac{x}{10} = \frac{5}{4}$

Multiply by 10: $x = \frac{5 \times 10}{4}$, or $12\frac{1}{2}$

Hence the enlarged picture will be $12\frac{1}{2}$ in. long. Check the result.

2. If a picture 5 in. long and 4 in. wide is enlarged so as to be 12 in. wide, how long will the enlarged picture be?

3. The pictures made by Elsie's camera are 3 in. long and 2 in. wide. Find how long these pictures will be if they are enlarged so as to be 6 in. wide; 8 in. wide; 10 in. wide; 5 in. wide.

4. A map is 8 in. long and 6 in. wide. How wide will an enlarged map of the same proportions be if it is 40 in. long?

Let x = the width in inches. The proportion may be written as $\frac{40}{x} = \frac{8}{6}$ or as $\frac{x}{40} = \frac{6}{8}$. The latter form is preferable because it is easier to solve the equation when x is the numerator of the first fraction.

5. A picture is now 5 in. long and 4 in. wide. If it is enlarged so as to be $7\frac{1}{2}$ in. long, how wide will it be?

6. Make up three problems about enlarging photographs.

HOW GRANDFATHER USED TO WRITE PROPORTIONS

You know that the ratio of 7 to 5 may be written as $\frac{7}{5}$ or as 7 : 5. A proportion may also be written in several ways.

$\frac{7}{5} = \frac{14}{10}$ may be written as $7 : 5 = 14 : 10$.

This proportion is read "7 is to 5 as 14 is to 10."

When your grandparents went to school, proportions were written in the horizontal form $7 : 5 : : 14 : 10$, which is another way of writing $7 : 5 = 14 : 10$. The sign : : means *equals*. These horizontal forms are still found in many science books.

Books often print proportions in the horizontal form because it is easier to set type in that form than in the fractional form.

When a proportion containing an unknown term is given in the form $x : 36 = 2 : 9$, you may rewrite it in the fractional form and then solve for x, as shown below.

Given the proportion $x : 36 = 2 : 9$

Write in fractional form: $\frac{x}{36} = \frac{2}{9}$

Multiply by 36: $x = \frac{36 \times 2}{9}$, or 8

Check. $\frac{8}{36} = \frac{2}{9}$

Exercises

Write in fractional form and solve for x. Check the work:

1. $x : 6 = 2 : 4$ **4.** $8 : 12 = x : 54$ **7.** $x : 12 = 16 : 24$

2. $x : 9 = 2 : 6$ **5.** $10 : 7 = x : 28$ **8.** $x : 48 = 56 : 64$

3. $x : 8 = 1 : 4$ **6.** $10 : 15 = x : 27$ **9.** $x : 30 = 19 : 20$

10. Ada earns \$12 a week. For each \$4 that Ada earns, Rob earns \$5; hence Ada's earnings are to Rob's as 4 is to 5. Express this as a proportion and find how much Rob earns.

11. A team played 27 games and won 2 out of every 3. Express this as a proportion and find how many games it won.

IMPROVEMENT TEST No. 11

Test 11 A — Addition (Time 3 min.)

1.	2.	3.	4.
89.406	8.1244	5318.9	338.94
23.721	.9755	8599.4	744.22
4.585	2.3727	140.3	793.58
3.746	4.6202	1596.4	37.63
91.087	.7395	6885.6	681.35
40.536	3.8656	432.3	969.69
2.728	.1908	9817.1	57.76
61.973	8.5629	3142.6	324.38

Test 11 B — Subtraction (Time 3 min.)

1.	2.	3.	4.
40.6051	73871.4	5266.57	9.
12.3405	8426.6	3527.82	5.12346
5.	**6.**	**7.**	**8.**
562.371	89551.8	84.3786	5.
206.649	22856.5	6.4967	2.27069
9.	**10.**	**11.**	**12.**
6248.93	464.173	95719.1	997.142
1286.07	44.339	37544.2	227.824

Test 11 C — Reduction (Time 4 min. after copying)

Change to decimals. If there is a remainder, find the result to the nearest thousandth:

1. $\frac{5}{8}$	**2.** $\frac{3}{20}$	**3.** $\frac{29}{40}$	**4.** $\frac{37}{45}$
5. $\frac{13}{14}$	**6.** $\frac{17}{25}$	**7.** $\frac{15}{16}$	**8.** $\frac{61}{125}$

Test 11 D — Multiplication (Time 3 min. after copying)

1.	2.	3.	4.	5.
9.46	972	.368	.517	.083
16.8	9.18	.023	.417	590

A QUICKER WAY TO SOLVE $x : 36 = 2 : 9$

In any proportion, the first and last terms are called the **extremes** and the two middle terms, the **means.** In the proportion $7 : 5 = 14 : 10$, 7 and 10 are the *extremes* and 5 and 14 are the *means.* In this proportion, what is the product of the extremes? of the means? Compare these products.

In any proportion, the product of the extremes equals the product of the means.

This rule enables you to solve for x in a proportion like $x : 36 = 2 : 9$ without rewriting it in fractional form.

Given $x : 36 = 2 : 9$

The product of the extremes is $9x$. The product of the means is 72
Since these products are equal, you get the equation

$$9x = 72$$

Divide by 9: $x = 8$

Check. $8 \times 9 = 36 \times 2$

Exercises

Solve in the above way, and check your results:

1. $x : 21 = 2 : 3$	**9.** $8 : x = 16 : 6$	**17.** $21 : 75 = x : 25$
2. $x : 30 = 1 : 5$	**10.** $5 : x = 10 : 2$	**18.** $56 : 91 = x : 13$
3. $x : 12 = 4 : 16$	**11.** $8 : x = 12 : 6$	**19.** $21 : 30 = x : 20$
4. $x : 40 = 5 : 80$	**12.** $4 : x = 16 : 4$	**20.** $10 : 50 = x : 90$
5. $x : 15 = 30 : 45$	**13.** $4 : x = 16 : 32$	**21.** $12 : 18 = 14 : a$
6. $x : 57 = 13 : 19$	**14.** $8 : x = 40 : 15$	**22.** $18 : 30 = 15 : x$
7. $x : 17 = 44 : 68$	**15.** $6 : x = 26 : 39$	**23.** $72 : 12 = 12 : x$
8. $x : 50 = 14 : 35$	**16.** $7 : x = 28 : 14$	**24.** $24 : 14 = 24 : x$

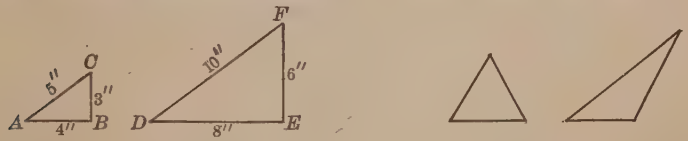
25. In this country the number of people injured yearly is to the total population as 1 is to 9. If there are 118,629,000 people in this country, how many are injured in a year?

CHAPTER XV

SIMILAR TRIANGLES

STUDYING SIMILAR TRIANGLES

Similar figures are figures with the same shape. The first two triangles below are similar; the last two are not similar.



The bases AB and DE of the two similar triangles are called **corresponding sides** because they have like positions in their triangles. BC and EF are also corresponding sides. What other pair of corresponding sides have these triangles?

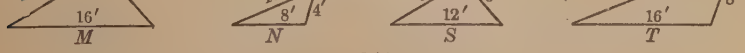
In the similar triangles, side AB is 4″ and side DE is 8″. The ratio of these two corresponding sides is $\frac{4}{8}$, or $\frac{1}{2}$. Side $BC = 3''$ and side $EF = 6''$. What is the ratio of BC to EF? What is the ratio of side AC to side DF? How do the ratios of the three pairs of corresponding sides compare?

In similar triangles, the ratio of any pair of corresponding sides equals the ratio of any other pair.

Exercises

1. In the similar triangles above, show that each of these proportions is true: $\frac{AB}{DE} = \frac{BC}{EF}$, $\frac{AB}{DE} = \frac{AC}{DF}$, $\frac{BC}{EF} = \frac{AC}{DF}$.

2. Which triangle below is similar to M? to N? In each pair of triangles, do the corresponding sides have equal ratios?

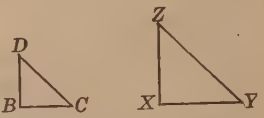


FINDING AN UNKNOWN SIDE OF A TRIANGLE

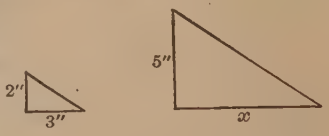
Copy and complete these proportions for the similar triangles at the right:

1. $\frac{BC}{XY} = \frac{BD}{\quad}$ 3. $\frac{CD}{YZ} = \frac{\quad}{XZ}$ or $\frac{\quad}{XY}$

2. $\frac{XY}{BC} = \frac{YZ}{\quad}$ 4. $\frac{XZ}{BD} = \frac{XY}{\quad}$ or $\frac{\quad}{CD}$

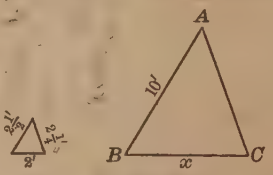


5. In a book Bertha sees an embroidery design in the form of a right triangle. The sides forming the right angle are 2 in. and 3 in. She wants to enlarge the design so that the shortest side will be 5 in. How long shall she make side x?

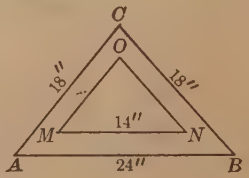


Since these triangles will be similar, the ratio of their bases will be $\frac{x}{3}$ and the ratio of their heights will be $\frac{5}{2}$. Since the ratios are equal, you can form the proportion $\frac{x}{3} = \frac{5}{2}$; then solve for x.

6. Roger is making a triangular sail for his canoe similar to a pattern for a sail the sides of which measure 2 ft., $2\frac{1}{4}$ ft., and $2\frac{1}{2}$ ft. Roger wants the longest side, AB, of his sail to be 10 ft. Let x stand for the length of the shortest side, BC, of Roger's sail. Find the length of side BC and of side AC.



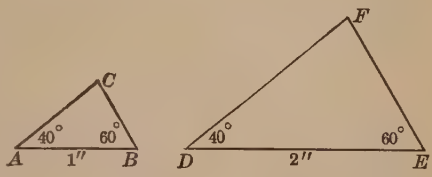
7. Mrs. Bruce wants to make a centerpiece, MNO, similar in shape to a triangular table top, ABC. The longest side of the table is 24 in. and the other sides are both 18 in. If she wishes the longest side of the centerpiece to be 14 in., how long should each of the other sides be?



THE ANGLES OF SIMILAR TRIANGLES

Here is an easy way to tell whether two triangles are similar:

1. Draw line AB 1 in. long and line DE 2 in. long. With your protractor draw angles A and D, each equal to 40°. Also draw angles B and E, each equal to 60°. Extend the sides of these angles to form triangles ABC and DEF. You then have two angles of triangle ABC equal to two angles of triangle DEF.



2. Triangles ABC and DEF appear to be similar. To make sure that they are similar, see if each pair of corresponding sides has the same ratio. You can do this as follows:

Since $DE = 2$ in. and $AB = 1$ in., the ratio of DE to AB is 2, which means that DE is twice as long as AB. To see if DF is twice as long as AC, open your compasses to a length equal to line AC. Then, without changing the opening of the compasses, apply them to line DF and see if DF contains the length of AC twice. In the same way, see if FE is twice as long as CB. If you do this carefully, you will find that each side of the larger triangle is twice the length of the corresponding side of the smaller triangle; hence each pair of corresponding sides has the same ratio, which is 2. This shows that triangles ABC and DEF are similar.

If two angles of one triangle are equal to two angles of another triangle, the triangles are similar.

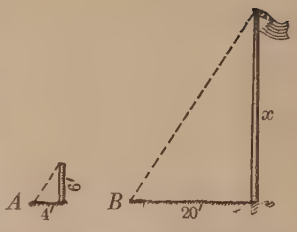
Exercises

1. Without measuring, show that angle C equals angle F.

2. Draw line MN 2″ long and line RS 6″ long. With MN as base, draw a triangle having angle $M = 70°$ and angle $N = 35°$. With RS as base, draw another triangle having angle $R = 70°$ and angle $S = 35°$. Find the ratio of RS to MN. Then test the other pairs of corresponding sides to see if each pair has the same ratio as RS to MN. Are the triangles similar?

FINDING HEIGHTS BY SHADOWS

The shadow of the flagpole in front of a school is 20 ft. long when the shadow of a 6-foot post is 4 ft. long. How can you find the height of the flagpole without measuring it?



If you connect the top of the pole and the end of its shadow by a dotted line, which shows the direction of the sun's rays, you have a triangle. By drawing a line from the top of the post to the end of its shadow, you form another triangle. We shall now show that these triangles are similar.

In these triangles the right angles are equal. The angles at A and B are also equal, since the sun's rays are parallel and therefore strike the earth at equal angles. Hence the triangles are similar because two angles of one triangle are equal to two angles of the other.

Let x = height of the flagpole, in feet

Since the triangles are similar, you can write this proportion:

$$\frac{x}{6} = \frac{20}{4}$$

Solve: $x = 30$

Hence the flagpole is 30 ft. high. Check the result.

Exercises

1. A tree casts a shadow of 44 ft. when a post 3 ft. high casts a shadow of 2 ft. Find the height of the tree.

2. When Ben, who is 5 ft. tall, casts a shadow of 6 ft., a telegraph pole casts a shadow of 42 ft. How high is the pole?

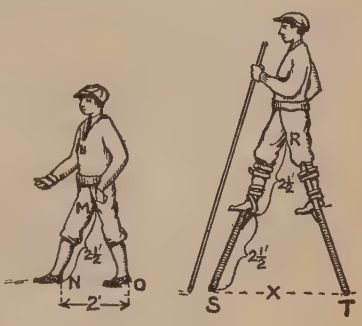
3. A monument casts a shadow of 45 ft. when a post 8 ft. high casts a shadow of 2 ft. How high is the monument?

When a 5-foot post casts a shadow of 4 ft., tell how high trees are whose shadows have these lengths:

4. 20 ft. **5.** 12 ft. **6.** 30 ft. **7.** 23 ft. **8.** 36 ft.

HOW LONG A STEP CAN CARL TAKE ON STILTS?

Carl's legs are $2\frac{1}{2}$ ft. long and his normal step on the ground is 2 ft. He wants to know how long a step he can make on stilts that measure $2\frac{1}{2}$ ft. from his feet to the ground.



When Carl walks on the ground, his legs form triangle MNO and his normal step forms its base. When he walks on stilts, triangle RST is formed. Its base ST (or x) is Carl's enlarged step. Since the stilts are $2\frac{1}{2}$ ft. high and Carl's legs are $2\frac{1}{2}$ ft., RS and RT are each 5 ft. long. Triangles MNO and RST are similar; hence $x : 2 = RS : MN$.

Let x = length, in feet, of enlarged step

Form the proportion: $\frac{x}{2} = \frac{5}{2\frac{1}{2}}$

Solve: $x = 4$

Hence Carl can make a 4-foot step on his stilts. Check the result.

Exercises

The measurements given for the stilts below are their lengths from the person's feet to the ground.

1. Tell why triangles MNO and RST, shown above, are similar.

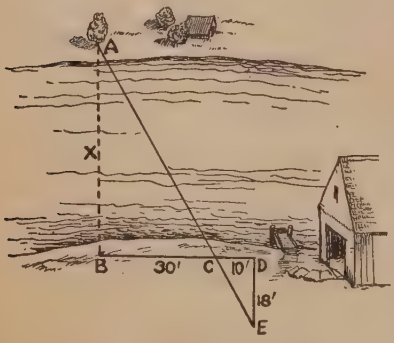
2. How long a step can a boy make on 5-foot stilts if his legs are $2\frac{1}{2}$ ft. long and his normal step is 2 ft.?

3. Helen's legs are 2 ft. long and her normal step is $1\frac{1}{2}$ ft. How long a step will she take on stilts $1\frac{2}{3}$ ft. long?

4. The tall man in the circus was a man on stilts. His legs were 3 ft. long and his normal step was $2\frac{1}{2}$ ft. The stilts were 9 ft. long. How long a step did he make on stilts? How high would his stilts have to be to take steps $12\frac{1}{2}$ ft. long?

FINDING THE WIDTH OF A RIVER

A scoutmaster shows his boy scouts how to measure the width of a river without crossing it.



From point B, directly across the river from tree A, he measures off $BC = 30$ ft., making BC perpendicular to BA, and drives a stake at C. He measures 10 ft. further in the same direction and sets another stake at D. Then he walks from D, at a right angle to BD, until he reaches a point E, where tree A can be seen in line with stake C. He drives a stake at E. He then measures DE and finds it to be 18 ft. He is now ready to compute the width of the river.

In triangles ABC and CDE, the right angles B and D are equal; the vertical angles BCA and DCE are also equal; hence triangles ABC and CDE are similar. Their shortest sides, BC and CD, are corresponding sides; AB and DE are also corresponding sides. Hence the scoutmaster forms the proportion $AB : DE = BC : CD$. Letting $x = AB$, since AB is the unknown side, and substituting the values of the other lines, he gets $x : 18 = 30 : 10$. Solve for x and tell how wide the river is.

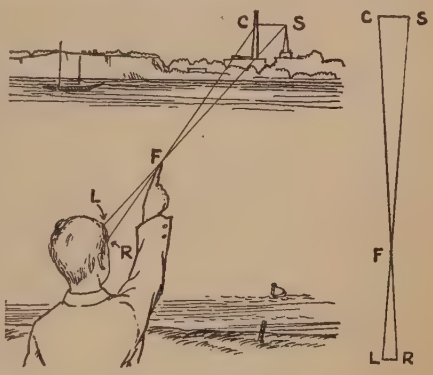
Exercises

Find the width of the river if the lines above measured thus:

	BC	*CD*	*DE*		*BC*	*CD*	*DE*
1.	90′	5′	35′	**7.**	200′	15′	45′
2.	30′	4′	10′	**8.**	450′	20′	75′
3.	72′	6′	15′	**9.**	366′	12′	80′
4.	55′	3′	36′	**10.**	220′	16′	50′
5.	80′	20′	40′	**11.**	125′	$6\frac{1}{3}$′	38′
6.	40′	10′	20′	**12.**	100′	$2\frac{1}{2}$′	10′

AN EASY WAY TO ESTIMATE DISTANCES

Tom wants to know how far it is to the tall chimney, *C*, across the river. He estimates this distance as follows:



Stretching out his right arm at full length and shutting his left eye, he points at the chimney, *C*, so that the line of sight from his right eye, *R*, to the chimney passes over the tip of his finger, *F*. Then, without moving his arm or finger, he shuts his right eye and opens his left eye, sighting with the left eye, *L*, across the tip of his finger, as before. He now finds that his finger is pointing at a steeple, *S*, instead of at the chimney. Tom estimates that the distance, *CS*, from the chimney to the steeple is about 60 ft. He then multiplies 60 ft. by 10 and gets 600 ft., which he says is the approximate distance from him to the chimney.

This method makes use of two triangles, *FLR* and *FSC*, which are shown above at the right of the picture. Since these triangles are similar, it can be shown by geometry that the ratio of *RF* to *RL* in triangle *FLR* equals the ratio of *FC* to *CS* in triangle *FSC*. It has been found that the distance, *RF*, from one's right eye to the tip of one's outstretched right arm is about 10 times the distance, *RL*, between the pupils of the eyes; therefore *RF* is 10 times as long as *RL*. Hence *FC* is also 10 times as long as *CS*. Since Tom estimated *CS* to be about 60 ft. long, it follows that *FC* is about 600 ft. long.

The above method uses the fact that it is much easier to estimate a *short* distance, like *CS*, which is nearly perpendicular to the line of sight than a *long* distance, like *FC*, which is straight ahead.

To estimate a distance straight ahead of you, estimate the distance CS, as described above, and multiply the result by 10.

ESTIMATING DISTANCES

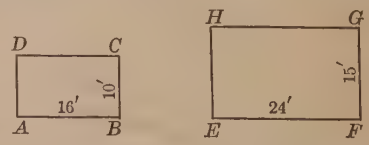
1. Irma's house is on a hill. From her window she sights with her right eye over the forefinger of her outstretched arm the tower of Town Hall, as described on page 247. Then, closing her right eye and opening her left eye, she notices that her finger now points at Mr. Day's store, which she knows to be two blocks from Town Hall. If each block in her city is 300 ft. long, about how many feet from Town Hall does Irma live?

2. A sailor at sea sights a large steamship on the horizon by the method on page 247. As he closes his right eye and opens his left eye, the finger with which he points at the steamship marks off on the horizon a distance (corresponding to *CS* on page 247) equal to about 3 times the length of the steamship. He knows that the steamship is about 700 ft. long. About how many miles away is the steamship?

SIMILAR RECTANGLES

1. On page 243 you learned that two triangles are similar if two angles of one triangle equal two angles of the other. Two rectangles, however, are not necessarily similar, as is shown in these figures, though the four angles of the one equal the four angles of the other.

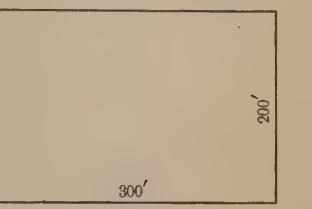
2. Two rectangles are similar, however, if, in addition to having equal angles, *their corresponding sides have the same ratio*. Thus, in the rectangles *ABCD* and *EFGH*, the ratio of *EF* to *AB* is $\frac{24}{16}$, or $\frac{3}{2}$; likewise the ratio of *FG* to *BC* is $\frac{15}{10}$, or $\frac{3}{2}$. The other corresponding sides also have a ratio of $\frac{3}{2}$, hence the rectangles are similar.



3. Draw two rectangles, one 4″ by 2″ and the other 6″ by 4″. Are these rectangles similar? Give a reason for your answer.

SCALE DRAWINGS AS SIMILAR FIGURES

1. Mr. Ward has a rectangular garden 300 ft. long and 200 ft. wide. John makes a scale drawing of it, as shown at the right. If he uses a scale of 1 in. = 200 ft., how many inches will represent 300 ft.? Check the drawing and see if each side is of the correct length.



SCALE: 1 in. = 200 ft.

2. How many feet wide is the garden? By changing feet to inches, express the width of the actual garden in inches. How many inches wide is John's drawing? Show that the ratio of the width of the drawing to the width of the garden is $\frac{1}{2400}$.

3. How many feet long is the garden? Express this length in inches. How many inches long is the scale drawing? Show that the ratio of the length of the drawing to the length of the garden is $\frac{1\frac{1}{2}}{3600}$. Since this ratio is a fraction, you may multiply its numerator and its denominator by 2 without changing its value. Show that this gives $\frac{3}{7200}$, or $\frac{1}{2400}$.

4. Do all the corresponding sides of the drawing and the garden have the same ratio? Do the four angles of the drawing equal the corresponding angles of the garden? Tell why the drawing and the garden are similar figures.

5. Make a scale drawing of a garden 500 ft. long and 400 ft. wide, letting 1 in. = 200 ft. Show that the drawing and the actual garden are similar figures, by comparing their angles and by finding the ratio of each pair of corresponding sides.

6. Jane's room is 15 ft. long and 12 ft. wide. She draws a floor plan of it 5 in. long and 3 in. wide. Show that her drawing is not similar to the floor of the room and hence is wrong.

Every scale drawing is similar to the figure it represents.

REVIEW OF SIMILAR AND CONGRUENT FIGURES

1. On page 223 you learned that congruent triangles are exactly alike both in size and in shape. Are similar triangles alike in size? Are similar triangles alike in shape?

2. Are congruent triangles also similar triangles? Are similar triangles always congruent triangles?

3. How do the corresponding sides of two triangles compare if the triangles are congruent? if the triangles are similar?

4. How do the corresponding angles of congruent triangles compare? the corresponding angles of similar triangles?

5. How do the corresponding sides of two rectangles compare if the rectangles are congruent? How do they compare if the rectangles are similar?

6. Draw a 2-inch square. Then draw a 3-inch square. Are they similar figures? Are all squares similar figures? Are all rectangles similar figures?

7. Draw triangle ABC, with sides measuring 4 in., 3 in., and 2 in. Draw triangle DEF, whose sides are equal to those of triangle ABC. Then draw triangle MNO, whose sides are half as long as those of triangle ABC. Which triangle is similar to ABC? Which is congruent to ABC?

8. If two triangles are congruent, what is the ratio of each pair of corresponding sides? If necessary, draw two congruent triangles to help you answer this question.

9. If two angles of a triangle are equal to two angles of another triangle, how do the two triangles compare?

10. Draw a rectangle 5 in. long and 4 in. wide. Then draw a second rectangle 6 in. wide, which is similar to the first rectangle. How many inches long is the second rectangle?

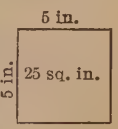
11. When a man 6 ft. tall casts a shadow of 4 ft., the Ulm Cathedral spire, which is the highest spire in Europe, casts a shadow of 352 ft. How many feet tall is the spire?

CHAPTER XVI

SQUARE ROOT

SQUARES AND SQUARE ROOTS

If a square measures 5 in. on a side, its area is 5×5 sq. in., or 25 sq. in. 25 is called the **square** of 5.



A short way of writing 5×5 is 5^2, which is read "5 square." You see that $5^2 = 25$.

If you know that the area of a square is 25 sq. in. and wish to find the length of its side, you must find the **square root** of 25. This means that you must find a number which, when multiplied by itself, gives 25. Since $5 \times 5 = 25$, the *square root* of 25 is 5. Hence 5 in. is the length of the side of the square. The sign for square root is $\sqrt{\ }$. Thus, $\sqrt{25}$ is read the *square root of* 25 and $\sqrt{25} = 5$.

Exercises

1. Since $7 \times 7 = 49$, the square of 7 is 49. What is the square root of 49? What does $\sqrt{49}$ equal?

2. How many are 6^2? What does $\sqrt{36}$ equal?

3. Give the squares of all the numbers from 1 to 12.

Find the following, remembering that 14^2 *means* 14×14:

4. 14^2	**6.** 15^2	**8.** 25^2	**10.** 30^2	**12.** 100^2
5. 20^2	**7.** 18^2	**9.** 16^2	**11.** 90^2	**13.** 200^2

14. Give the square roots of the following numbers: 1; 4; 9; 64; 16; 81; 49; 100; 144; 121; 169; 196; 900.

Read, and tell what numbers should be put in place of the dots:

15. $\sqrt{81} = \ldots$ **16.** $\sqrt{400} = \ldots$ **17.** $\sqrt{225} = \ldots$

USING A TABLE OF SQUARES

This table gives the squares of numbers from 1 to 100:

Number	Square	Number	Square	Number	Square	Number	Square
1	1	26	676	51	2601	76	5776
2	4	27	729	52	2704	77	5929
3	9	28	784	53	2809	78	6084
4	16	29	841	54	2916	79	6241
5	25	30	900	55	3025	80	6400
6	36	31	961	56	3136	81	6561
7	49	32	1024	57	3249	82	6724
8	64	33	1089	58	3364	83	6889
9	81	34	1156	59	3481	84	7056
10	100	35	1225	60	3600	85	7225
11	121	36	1296	61	3721	86	7396
12	144	37	1369	62	3844	87	7569
13	169	38	1444	63	3969	88	7744
14	196	39	1521	64	4096	89	7921
15	225	40	1600	65	4225	90	8100
16	256	41	1681	66	4356	91	8281
17	289	42	1764	67	4489	92	8464
18	324	43	1849	68	4624	93	8649
19	361	44	1936	69	4761	94	8836
20	400	45	2025	70	4900	95	9025
21	441	46	2116	71	5041	96	9216
22	484	47	2209	72	5184	97	9409
23	529	48	2304	73	5329	98	9604
24	576	49	2401	74	5476	99	9801
25	625	50	2500	75	5625	100	10000

1. Find the square of 19; of 23; 37; 51; 65; 89.

To find the square of 19 in the table, find 19 in the left-hand column headed "Number." Then, in the column headed "Square," just to the right of 19, you will find 361. 361 is the square of 19.

2. Find the square root of 2401.

Find 2401 in one of the columns headed "Square." Then find the number just to the left of 2401, which is 49. 49 is the square root of 2401.

3. Find the square root of 81; of 256; 625; 841; 1444; 1681; 2209; 3025; 4096; 5184; 7569; 8464; 9216; 10,000.

A number like 81, 256, or 10,000, whose square root is a whole number, is called a **perfect square.**

FINDING THE APPROXIMATE SQUARE ROOT

The table on page 252 also helps you to find the square root of a number that is not a perfect square.

1. Find, to the nearest tenth, the square root of 149.

Although 149 is not in the table of squares, you find that it lies between 144 (the square of 12) and 169 (the square of 13). Hence the square root of 149 is between 12 and 13.

Since 149 is much closer to 144 than to 169, the square root of 149 is a little over 12. Find 12.1×12.1, 12.2×12.2, and 12.3×12.3, as shown at the right. Which of these squares is closest to 149? Then 12.2 is, to the nearest tenth, the square root of 149.

$12.1^2 = 146.41$
$12.2^2 = 148.84$
$12.3^2 = 151.29$

2. Find, to the nearest tenth, the square root of 145.

Since 145 is a little more than 144 (the square of 12), you may try 12.1 as the square root. $12.1^2 = 146.41$. But 144 is closer to 145 than 146.41 is; hence the approximate square root of 145 is 12.

If you cannot tell by inspection whether 144 or 146.41 is closer to 145, find the difference between 145 and each square by subtracting, as shown at the right. This shows that 144 is only 1 less than 145, while 146.41 is 1.41 greater than 145.

145	146.41
144	145
1	1.41

Exercises

1. Anna is finding the square root of 968. She says that 968 lies between 961 (the square of 31) and 1024 (the square of 32). Is she right? To which of these squares is 968 closer?

2. Since 968 is a little more than the square of 31, Anna tries both 31.1 and 31.2 as the square root of 968. Find 31.1^2 and 31.2^2 and see which result is closer to 968. Show that 31.1 is the approximate square root of 968.

3. Show that the square root of 401 is a little over 20. Is 20.0 or 20.1 the approximate square root of 401?

Find, to the nearest tenth, the square root of:

4. 27 **5.** 52 **6.** 87 **7.** 789 **8.** 914 **9.** 2331

FINDING APPROXIMATE SQUARE ROOTS

1. Find, to the nearest tenth, the square root of 164.

In the table of squares on page 252, you find that 164 lies between 144 (the square of 12) and 169 (the square of 13); but it is much nearer to 169 than to 144. Hence the square root of 164 is closer to 13 than to 12. Square 12.7, 12.8, and 12.9 to find which square is closest to 164. Show that, to the nearest tenth, 12.8 is the square root of 164.

$12.7^2 = 161.29$
$12.8^2 = 163.84$
$12.9^2 = 166.41$

2. Find, to the nearest tenth, the square root of 156.

Since 156 lies about halfway between 144 and 169, its square root is about halfway between 12 and 13. Try 12.4, 12.5, and 12.6, squaring each as shown at the right. You find that 12.5 is the approximate square root of 156.

$12.4^2 = 153.76$
$12.5^2 = 156.25$
$12.6^2 = 158.76$

Exercises

In the following exercises, use the table on page 252.

1. Mary Dodd wishes to make a square garden that will contain 130 sq. ft. of land. Find, to the nearest tenth of a foot, how long each side of the garden should be.

2. Find, to the nearest tenth of a foot, the length of the side of a square garden whose area is 750 sq. ft.; of a square garden whose area is 650 sq. ft.; 600 sq. ft.

Find, to the nearest tenth, the square root of:

3. 46	**7.** 50	**11.** 174	**15.** 449	**19.** 1450
4. 42	**8.** 56	**12.** 185	**16.** 458	**20.** 2024
5. 90	**9.** 60	**13.** 190	**17.** 471	**21.** 3230
6. 96	**10.** 24	**14.** 194	**18.** 480	**22.** 8875

23. A factory is going to make 2-quart cans which will be $5\frac{1}{2}$ in. high and square on the bottom. Find, to the nearest tenth of an inch, the length of one side of the bottom of the can.

HINT. 1 gal. = 231 cu. in.; hence 2 qt. = $115\frac{1}{2}$ cu. in. $115\frac{1}{2} \div 5\frac{1}{2} = 21$, the area of the bottom in square inches.

FINDING SQUARE ROOTS TO THE NEAREST HUNDREDTH

The method on pages 253 and 254 will also enable you to find the square root of a number correct to any number of decimal places. For example, to find the square root of 164 to the nearest hundredth, you proceed as follows:

First find, as on page 254, the square root of 164 to the nearest tenth, which gives 12.8. Since $12.8^2 = 163.84$, and since 164 is a little more than 163.8, the square root of 164 is a little over 12.8 (which equals 12.80). Trying 12.81, you find that $12.81^2 = 164.0961$.

Since 164.0961 (the square of 12.81) is a little more than 164 and since 163.84 (the square of 12.80) is a little less than 164, you must decide which is closer to 164. Subtracting, you find that $164.0961 - 164 = .0961$ and that $164 - 163.84 = .16$, or .1600. Since .0961 is less than .1600, 164.0961 is closer to 164. Therefore 12.81 is the square root of 164, to the nearest hundredth.

Exercises

1. Dorothy found that, to the nearest tenth, the square root of 85 is between 9.2 and 9.3, but nearer to 9.2. Hence to get the root to the nearest hundredth she tried 9.21, 9.22, and 9.23. Square each of these numbers and show that 9.22 is the square root of 85.

2. Show that the square root of 48 is closer to 6.9 than it is to 7.0. Then find the square root of 48, to the nearest hundredth, by squaring 6.91, 6.92, and 6.93 to see which of these results is closest to 48.

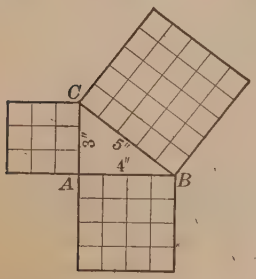
Find these square roots correct to the nearest hundredth:

3. $\sqrt{17}$	**8.** $\sqrt{200}$	**13.** $\sqrt{247}$	**18.** $\sqrt{1085}$
4. $\sqrt{44}$	**9.** $\sqrt{428}$	**14.** $\sqrt{342}$	**19.** $\sqrt{2018}$
5. $\sqrt{61}$	**10.** $\sqrt{128}$	**15.** $\sqrt{530}$	**20.** $\sqrt{3230}$
6. $\sqrt{74}$	**11.** $\sqrt{139}$	**16.** $\sqrt{593}$	**21.** $\sqrt{4635}$
7. $\sqrt{65}$	**12.** $\sqrt{140}$	**17.** $\sqrt{945}$	**22.** $\sqrt{6815}$

USING SQUARE ROOT IN RIGHT TRIANGLES

Square root is useful in finding the length of one side of a right triangle when the other sides are known.

In any right triangle, there is one right angle. The side opposite the right angle is called the **hypotenuse;** the other sides are called the **legs.**



Draw a right triangle, ABC, whose legs, CA and AB, measure 3 in. and 4 in. Measuring the hypotenuse, BC, you find that it is exactly 5 in. long.

Now construct three squares having CA, AB, and BC as sides. Divide these squares into square inches.

The square on leg CA contains 9 sq. in. The square on leg AB contains 16 sq. in. The square on the hypotenuse BC contains 25 sq. in., which is the sum of 9 sq. in. and 16 sq. in. Hence the square on BC equals the sum of the squares on the legs.

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

This rule is called the *Pythagorean Theorem* and was discovered by Pythagoras, a Greek philosopher, about 540 B.C.

Exercises

1. Draw a right triangle whose legs measure 6 in. and 8 in. Measure the length of its hypotenuse. Find the area of a 6-inch square; of an 8-inch square; of a 10-inch square. Then find 6^2, 8^2, and 10^2. Does the square of the hypotenuse here equal the sum of the squares of the legs?

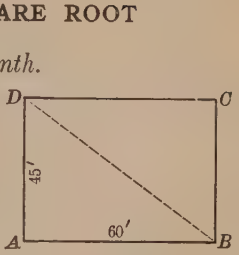
2. The legs of a right triangle are 9 in. and 12 in. Find the length of the hypotenuse, without drawing a triangle.

The sum of the squares of 9 and 12 is $81 + 144$, or 225. Hence the length of the hypotenuse is the square root of 225 sq. in.

FINDING DISTANCES BY SQUARE ROOT

In ex. 1 *to* 4, *find answers to the nearest tenth.*

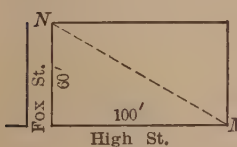
1. Dan can swim from one corner, B, of the swimming pool to the opposite corner, D. The pool measures 60 ft. by 45 ft. How far can Dan swim?



BD is the hypotenuse of triangle DAB.

2. A rectangular playground is 45 ft. wide and 85 ft. long. Draw a diagram and find how far Agnes runs when she runs from one corner of the playground to the opposite corner.

3. Wilson Green is 70 ft. square. A path is to be made across it from one corner to the opposite corner. How long will the path be?



4. An empty corner lot 100 ft. long and 60 ft. wide has a diagonal path, MN, leading across it. In walking from M to N, how many feet shorter is it to take the path than to follow the sidewalks on High Street and Fox Street?

The number 13,600 does not appear in the table on page 252. Show that it lies between 116^2 and 117^2; then find the square root of 13,600 to the nearest tenth.

5. At a May party, the Maypole is to be 24 ft. high and the children holding the streamers are to stand 7 ft. from the pole. How long must the streamers be?

Do not count the height of the children.

Find, to the nearest hundredth, the hypotenuses of right triangles whose legs are as follows:

6. 6 ft., 9 ft.

7. 12 ft., 18 ft.

8. 31 ft., 43 ft.

9. 9 mi., 10 mi.

10. 60 ft., 35 ft.

11. 35 ft., 15 ft.

THE SQUARE ROOT OF A MIXED DECIMAL

The legs of a right triangle measure 2.1 in. and 5.3 in. Find, to the nearest tenth of an inch, the length of the hypotenuse.

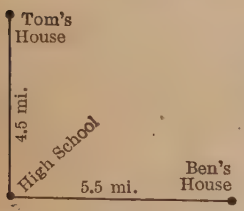
$2.1^2 = 4.41$, and $5.3^2 = 28.09$. Hence the square of the hypotenuse, which is the sum of the squares of the legs, is $4.41 + 28.09$, or 32.50. You must find the square root of 32.50.

Since 32.50 lies between the perfect squares 25 and 36, the square root of 32.50 is between 5 and 6. 32.50 is nearer to 36 than to 25; hence try 5.6, 5.7, and 5.8, as shown at the right. Since 32.49 (5.7^2) is the closest to 32.50, the square root of 32.50 is 5.7. The hypotenuse is 5.7 in. long.

$5.6^2 = 31.36$
$5.7^2 = 32.49$
$5.8^2 = 33.64$

Problems

In ex. 1 *and* 2, *find answers to the nearest tenth of a mile.*



1. Tom Allen lives 4.5 mi. north of the new high school. Ben Lewis lives 5.5 mi. east of it. How many miles, in a straight line, does Ben live from Tom?

2. Oldstown is 15.1 mi. south of Smokum. Mine City is 12.2 mi. west of Smokum. Draw a diagram showing the route of an airplane flying in a straight line between Oldstown and Mine City. How long is this route?

3. In a skating race the boys were allowed to use sails containing not more than 18.5 sq. ft. Albert wanted to make a square sail. Find, to the nearest hundredth of a foot, the greatest length which a side of the square sail could have.

Find, to the nearest tenth of a mile, the hypotenuses of right triangles whose legs are:

4. 2.1 mi., 3.7 mi.
5. 3.4 mi., 6.2 mi.
6. 2.8 mi., 3.5 mi.
7. 8.6 mi., 8.6 mi.
8. 5.3 mi., 7.5 mi.
9. 1.5 mi., 8.5 mi.
10. 9 mi., 9.9 mi.
11. 3 mi., 4.8 mi.
12. 5 mi., 6.2 mi.

IMPROVEMENT TEST No. 12

Test 12 A — Comparing Numbers (Time 4 min. after copying)

Find results to the nearest tenth of 1%:

1. 18,200,000 is what per cent of 76,500,000?
2. 147,260,000 is what per cent of 125,000,000?

Test 12 B — Comparing Numbers (Time 4 min. after copying)

Find results to the nearest tenth of 1%:

1. What per cent of $125,000,000 is $45,650,000?
2. What per cent of $369,500,000 is $486,500,000?

Test 12 C — Finding the Base (Time 4 min. after copying)

What numbers should be put in place of the dots?

1. 36 is 8% of . . .
2. 68% of . . . is 3366
3. 96 is 15% of . . .
4. $66\frac{2}{3}$% of . . . is 274
5. 235.05 is 15% of . . .

Test 12 D — Trade Discount (Time 4 min. after copying)

Find, to the nearest cent, the net price when the list price and the discount are as follows:

1. $8; $33\frac{1}{3}$%
2. $485; $16\frac{2}{3}$%
3. $750; 20%, 15%, 2%
4. $8.97; 10%
5. $752; $12\frac{1}{2}$%
6. $2780; 25%, 20%, 3%

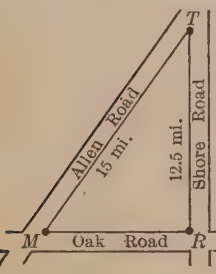
Test 12 E — Bank Discount (Time 3 min. after copying)

Find, to the nearest cent, the bank discount on the following, using the 6% *method:*

1. $495 for 96 da. at 6%
2. $870 for 150 da. at 5%
3. $298 for 54 da. at 7%

PROBLEMS THAT USE SQUARE ROOT

1. It is 15 mi. along Allen Road from the Town Hall (T) to the mill (M). It is 12.5 mi. along Shore Road from the Town Hall (T) to Reed Farm (R). How far is it along Oak Road from Reed Farm to the mill?

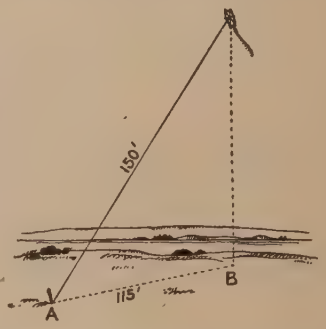


You know the length of the hypotenuse and of one leg of the triangle and you must find the length of the other leg. Let x stand for MR. Then $x^2 + 12.5^2 = 15^2$, or $x^2 + 156.25 = 225$. Show that $x^2 = 68.75$. Then show that $x = \sqrt{68.75}$.

2. An acre contains 43,560 sq. ft. If a square field contains an acre, find, to the nearest hundredth of a foot, the length of one side of the field.

The table on page 252 does not contain squares as large as 43,560. First show by trial that 43,560 lies between 208^2 and 209^2. Then find the result to the nearest hundredth.

3. George is flying his kite. When he has let out 150 ft. of string, he fastens the end to a stick, A, in the ground and walks until he is directly under the kite at B. He then measures AB and finds that it is 115 ft. long. How high, to the nearest tenth of a foot is George's kite?



4. The formula for the area, A, of a circle is $A = \pi r^2$. From this formula show how to make the following formula for the radius: $r = \sqrt{\frac{A}{\pi}}$.

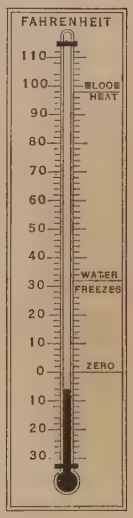
5. Ruth has a garden 11 ft. by 8 ft. Dan wants to make a circular garden with the same area as Ruth's. Find the radius of Dan's garden, to the nearest tenth of a foot. (See ex. 4.)

CHAPTER XVII

POSITIVE AND NEGATIVE NUMBERS

A NEW KIND OF NUMBER

One cold morning an outdoor thermometer read 6° below zero. In the evening this temperature was reported in the newspaper as $-6°$.



A temperature of 6° *below* zero is often written $-6°$ and is read "minus 6 degrees," while a temperature of 6° *above* zero is written $+6°$ and is read "plus 6 degrees."

Thus the signs + and − are placed before numbers to show that they are *opposite in meaning* or *direction*. A number preceded by the sign +, like $+6$, is called a **positive number**; a number preceded by the sign −, like -6, is called a **negative number**. Positive and negative numbers are also called **signed numbers.**

You see that the signs + and − may be used to express a kind of number as well as to indicate addition and subtraction.

Exercise

This weather report gives the highest and lowest temperatures at these cities on a certain day. Read the temperatures.

Station	High	Low
Atlanta	48	42
Chicago	20	− 12
St. Paul	16	− 20
Omaha	28	0

When the temperature is above zero, the sign + is omitted in this report. Whenever the sign of a number is omitted, the number is positive.

EVERYDAY USES OF SIGNED NUMBERS

In ex. 1 *to* 7, *use signed numbers to answer the questions.*

1. If + 9 represents \$9 *earned*, what represents \$5 *spent?*

2. If + 8 mi. indicates 8 mi. *east*, what indicates 6 mi. *west?*

3. If − \$5 represents *a debt* of \$5, what represents \$8 *saved?*

4. If − \$4 represents \$4 *lost*, what represents \$3 *gained?*

5. If + 3 ft. indicates 3 ft. *up*, what indicates 7 ft. *down?*

6. If + 10° represents a latitude of 10° *north* of the equator, what indicates a latitude of 30° *south* of the equator? In measuring latitude, where is 0° located?

7. Represent a *gain* of \$10 and a *loss* of \$7; a trip of 10 mi. *east* and one of 14 mi. *west;* a *debt* of \$100 and *savings* of \$90.

8. The first Olympic games were held in the year 1453 B.C. Since this year came before the birth of Christ, it is often written − 1453. What does 1492 A.D. mean? Write this date as a signed number.

9. Write as signed numbers: 1620 A.D.; 51 B.C.; 1776 A.D.

10. In the larger newspapers the daily prices of stocks are printed. The *net change* shows by signed numbers whether the closing price of to-day is *higher* or *lower* than the closing price of yesterday. For example, the last sales of Atlas Telephone stock to-day were at $146\frac{1}{2}$, which was $2\frac{1}{2}$ points *higher* than the last price yesterday. Hence the change was + $2\frac{1}{2}$. This means that 144 was the last price yesterday. For which stocks was the price to-day *higher* than the price yesterday? For which was it lower?

Name of Stock	Closing Price	Net Change
Atlas Telephone	$146\frac{1}{2}$	$+2\frac{1}{2}$
Brazil Rubber	$84\frac{1}{2}$	$-1\frac{1}{2}$
U. S. Electric	$112\frac{3}{4}$	$+1\frac{1}{4}$
Northern R. R.	$98\frac{1}{2}$	$-3\frac{1}{2}$

11. Find the closing price yesterday for each stock in ex. 10.

Positive and negative numbers are opposite numbers.

ADDING NUMBERS WITH LIKE SIGNS

1. Margaret has \$4 and earns \$5 more. She is therefore worth + \$4 and + \$5, or + \$9. Hence the sum of + 4 and + 5 is + 9.

$$\begin{array}{r} +4 \\ +5 \\ \hline +9 \end{array}$$

2. Robert owes a friend \$4 and borrows \$5 more. He then owes \$9 in all. Since owing money is the opposite of having it, you may say that Robert is worth − \$4 plus − \$5, or − \$9. Hence the sum of − 4 and − 5 is − 9.

$$\begin{array}{r} -4 \\ -5 \\ \hline -9 \end{array}$$

3. The **arithmetic value** of a signed number is obtained by disregarding its sign. Thus, the arithmetic value of − 4 is 4. Likewise the arithmetic value of + 4 is 4.

The arithmetic value is also called the *absolute value*.

To add two numbers with like signs, find the sum of their arithmetic values and prefix to it their common sign.

Exercises

In ex. 1 *and* 2, *add, giving the sums orally:*

1.

+ 9	− 7	+ 5	+ 1	+ 4	− 2	+ 8
+ 4	− 4	+ 5	+ 2	+ 3	− 5	+ 2
+ 13	− 11	+ 10	+ 3	+ 7	− 7	+ 10

2.

− 3	− 4	+ 3	− 7	+ 1	− 3	+ 6
− 7	− 1	+ 6	− 7	+ 7	− 5	+ 3
− 10	− 5	+ 9	− 14	+ 8	− 8	+ 9

Work ex. 3 *to* 5 *by using signed numbers.*

3. If you owe \$.50 to Howard and \$1.50 to your sister, show that you are worth − \$2.

4. The price of the stocks which Mr. Hayward owns dropped 3 points on Monday and $4\frac{1}{4}$ points more on Tuesday. What was the total change in price for these two days? $-7\frac{1}{4}$

5. Mr. Taylor deposited \$25.00 in the bank yesterday and \$28.00 to-day. Express his total deposits as a signed number. $+53$

ADDING NUMBERS WITH UNLIKE SIGNS

1. A grocer gained \$3 on one barrel of apples and lost \$2 on another barrel. Find his net gain on the two barrels.

$$\begin{array}{r} +3 \\ -2 \\ \hline +1 \end{array}$$

Let + \$3 represent his gain and − \$2 his loss. You know that his *net gain* is \$1. Hence the sum of + 3 and − 2 is + 1.

You can get the same result if you subtract 2 from 3 and prefix the sign +, which is the sign of the greater arithmetic number, 3.

2. Mr. Foster sells secondhand automobiles. Yesterday he gained \$45 on one car and lost \$90 on another car. What was his net gain or loss for the day?

$$\begin{array}{r} +45 \\ -90 \\ \hline -45 \end{array}$$

You know that his *net loss* was \$45, which may be represented by − \$45. Hence the sum of − \$90 and + \$45 is − \$45.

You can get the same result if you subtract 45 from 90 and prefix to the result the sign −, which is the sign of the greater arithmetic number, 90.

To add two numbers with unlike signs, find the difference between their arithmetic values and prefix to it the sign of the number having the greater arithmetic value.

Exercises

1. Alice won 2 tennis games this morning and lost 2 games this afternoon. What was her score at the end of the day?

2. By studying ex. 1, show that the sum of + 2 and − 2 is 0.

3. Mr. Allen builds houses. He lost \$500 on one house and gained \$900 on another. Find his net gain or loss.

Add the following orally, using the rule given above:

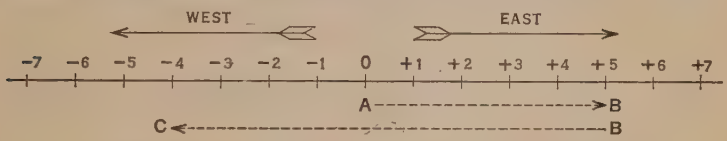
4.	+ 8	− 9	+ 3	+ 4	− 1	+ 2	− 4
	− 3	+ 1	− 6	− 4	+ 3	− 1	+ 2
5.	+ 7	+ 3	− 6	− 9	+ 7	− 9	− 3
	− 7	− 9	+ 7	+ 9	− 2	+ 3	+ 3
6.	− 1	+ 9	− 4	+ 2	+ 6	+ 5	− 4
	+ 1	− 8	+ 8	− 6	− 6	− 2	+ 9

ADDING SIGNED NUMBERS GRAPHICALLY

1. Mr. Day drove his car east 5 mi. and then west 9 mi. In what direction and how far was he then from his starting point?

If + 5 mi. represents 5 mi. *east*, what represents 9 mi. *west?* The total is the sum of + 5 mi. and − 9 mi. What does the result, − 4 mi., mean?

2. The numbers in ex. 1 may be added graphically by representing distances east and west on a horizontal scale, thus:



Mr. Day started at zero, or *A*, on the scale. After driving 5 mi. east, he was at *B*. He then drove 9 mi. west. By counting off 9 spaces to the left of *B*, you reach − 4, or *C*. Hence Mr. Day was then at − 4. Was this east or west of his starting point? how many miles east or west? How does this result agree with that obtained in ex. 1?

3. By doing the work in two ways, as in ex. 1 and 2, find in what direction and how far from the starting point Mr. Cox is

(*a*) if he drives 5 mi. east and then 4 mi. further east.
(*b*) if he drives 12 mi. east and then 7 mi. west.
(*c*) if he drives 13 mi. west and then 6 mi. east.
(*d*) if he drives 7 mi. west and then 14 mi. east.
(*e*) if he drives 6 mi. west and then 8 mi. further west.
(*f*) if he drives 10 mi. west and then 10 mi. east.

4. Find the year of death of a man if he was born in 90 B.C. and lived 80 yr.; if he was born in the year − 29 and lived 60 yr.

Add, using the rules on pages 263 *and* 264:

5.	+ 9	+ 4	+ 3	− 8	+ 7	− 5	+ 5
	+ 6	− 5	+ 3	0	− 3	+ 7	+ 9
6.	+ 8	− 9	− 8	− 1	− 2	0	− 2
	− 8	+ 5	− 5	+ 1	− 9	+ 3	− 2

ADDING THREE OR MORE SIGNED NUMBERS

1. In five successive "downs," a football team gained 4 yd., was penalized 5 yd., gained 3 yd., gained 9 yd., and lost 7 yd. How much did the team gain in the five "downs"?

Represent each gain by a positive number and each loss or penalty by a negative number. First add the positive numbers, $+4$, $+3$, and $+9$, which gives $+16$. Next add the negative numbers, -5 and -7, which gives -12. Then add the two results, $+16$ and -12, which gives $+4$. Hence the team gained 4 yd. in the five "downs."

$$\begin{array}{r} +4 \\ -5 \\ +3 \\ +9 \\ \underline{-7} \end{array}$$

To find the sum of several positive and negative numbers, first add the positive numbers, then add the negative numbers, and then add the two results.

Exercises

Solve the following problems by using signed numbers:

1. Mr. Powers sells goods at auction. Yesterday he gained \$60 on the first sale, lost \$40 on the second, lost \$10 on the third, and gained \$20 on the fourth. Find his net gain or loss.

2. Mr. Monroe deposited the following amounts in the bank: \$248.32, \$75.83, \$26.42. He also drew money out of the bank by writing checks for the following amounts: \$25.83, \$45.16, \$15.75. Which of these numbers will you consider positive and which negative? Find Mr. Monroe's balance.

Add, and check your work by going over it:

3. $+8$	**5.** $+12$	**7.** $+22$	**9.** -210	**11.** $+\$6.50$
-3	-10	$+10$	$+116$	$-\ 1.20$
-2	-18	$-\ 8$	-221	$+\ 8.37$
$+3$	-16	$+24$	-315	$+\$13.67$
4. -4	**6.** $+16$	**8.** -19	**10.** $+710$	**12.** $+\$2.93$
-2	$+14$	$+17$	-615	$+\ 2.00$
$+5$	$+17$	$+14$	$-\ 95$	$-\ 3.50$
-3	-15	-22	$+300$	$+\ 1.65$
-4	$+32$	-10	$+306$	$+\$3.08$

SUBTRACTING POSITIVE AND NEGATIVE NUMBERS

1. A thermometer reads + 2° at 8 A.M. and + 10° at noon. Find the difference between these two temperatures.

To find the difference, you subtract. In Fig. 1 you see that the difference between + 2° and + 10° is 8°, because the temperature *went up* 8°; hence if you subtract + 2° from + 10°, you get + 8°.

+ 10°	Minuend
+ 2°	Subtrahend
+ 8°	Difference

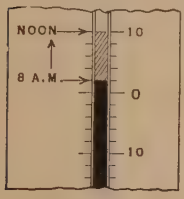


FIG. 1

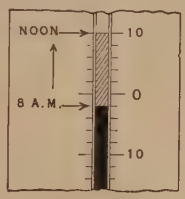


FIG. 2

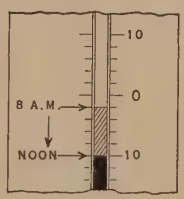


FIG. 3

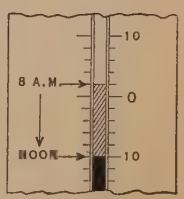


FIG. 4

2. A thermometer reads − 2° at 8 A.M. and + 10° at noon. Find the difference between these temperatures.

+ 10°
− 2°
+ 12°

Explain how Fig. 2 shows that the difference is + 12°; hence if you subtract − 2° from + 10°, you get + 12°.

In the work at the right, change the sign of the subtrahend to +, thus making the subtrahend + 2°. Then add + 10° and + 2°. This is an easy way to get the same result, + 12°, as above.

3. A thermometer reads − 2° at 8 A.M. and − 10° at noon. Find the difference between − 2° and − 10°.

− 10°
− 2°
− 8°

Fig. 3 shows that the temperature *went down* 8°, making a difference of − 8°. Hence − 2° subtracted from − 10° gives − 8°. An easy way to get the same result is to change − 2° to + 2° and then add + 2° to − 10°.

4. A thermometer reads + 2° at 8 A.M. and − 10° at noon. Find the difference in these temperatures.

− 10°
+ 2°
− 12°

Explain how Fig. 4 shows that the difference is − 12°. What is an easy way to get the same result?

To subtract signed numbers, change the sign of the subtrahend mentally and add.

USING SUBTRACTION

In ex. 1 *to* 6, *use signed numbers to find the results.*

1. Find the difference between these temperatures:

(*a*) if the thermometer reads + 3° at 7 A.M. and + 9° at noon.
(*b*) if the thermometer reads − 3° at 7 A.M. and + 9° at noon.
(*c*) if the thermometer reads − 3° at 7 A.M. and − 9° at noon.
(*d*) if the thermometer reads + 3° at 7 A.M. and − 9° at noon.

2. A man was born in the year − 140 and died in the year − 106. How old was he at his death?

Since − 106 is a later year than − 140, you must subtract − 140 from − 106 to find the man's age.

3. If a man was born in 20 B.C., how old was he in 18 A.D.?

4. A ship sailed from latitude 15° south of the equator to latitude 25° north of the equator. How many degrees of latitude did it sail north? How many degrees of latitude had it sailed north when it reached latitude 7° south of the equator?

5. Ellenburg and Meadville are in a straight line with Newtown. How far is it from Ellenburg to Meadville if Ellenburg is 5 mi. west of Newtown and Meadville is 7 mi. east of Newtown?

6. At the end of a game Thomas had gained 28 points and Charles had lost 17 points. How many points was Thomas ahead of Charles?

Subtract orally, using the rule on page 267:

7.	+ 7	+ 4	− 4	− 7	+ 5	− 8	0
	+ 4	− 1	0	+ 8	+ 9	+ 5	+ 7
8.	+ 8	− 4	+ 1	+ 6	+ 3	− 8	− 7
	− 8	− 3	− 8	0	+ 6	− 9	− 7
9.	− 3	0	+ 9	+ 6	− 3	− 3	+ 5
	+ 3	− 4	+ 3	− 7	− 5	0	+ 5

EXERCISES

Tell what words should be put in place of the dotted lines:

1. If numbers with like signs are added, the result has the same sign as the . . .

2. If numbers with unlike signs are added, the result has the same sign as the . . .

3. In subtracting signed numbers, you change the sign of the . . . and then you . . .

Add, and check the work by going over it:

4. -7	**5.** -8	**6.** $+43$	**7.** $+91$	**8.** $-\$2.63$
$+9$	-6	$+62$	-91	$+\ 1.74$
9. -6	**10.** -9	**11.** $+19$	**12.** $+78$	**13.** $-\$1.20$
-5	$+5$	-35	-23	$-\ 3.75$
-1	$+1$	-11	$+50$	$+\ 5.10$
14. $+8$	**15.** $+3$	**16.** -83	**17.** -27	**18.** $+\$6.51$
$+2$	-2	$+72$	$+58$	$-\ 5.00$
-1	-7	-11	$+64$	$+\ 2.07$
-4	$+6$	-42	-39	$+\ 4.26$

Subtract, and check the work by going over it:

19. $+6$	**20.** -7	**21.** $+36$	**22.** -77	**23.** $+\$9.62$
-5	$+7$	$+54$	$+25$	$+\ 9.75$
24. -9	**25.** $+1$	**26.** -48	**27.** $+49$	**28.** $+\$7.35$
$+3$	$+3$	-29	$+49$	$-\ 2.69$
29. -7	**30.** $+6$	**31.** -61	**32.** -68	**33.** $-\$1.10$
-4	-8	$+53$	$+37$	$-\ .57$
34. $+1$	**35.** -2	**36.** $+12$	**37.** -19	**38.** $+\$6.39$
$+9$	-2	-99	-52	$-\ 8.98$

PROBLEMS FOR REVIEW

1. The net daily changes in the prices of Hamilton Motor stock last week were $+1\frac{1}{2}$, $+3$, $+2\frac{1}{2}$, -4, -8, $-7\frac{1}{4}$. Find the total rise or fall during the week.

2. When Tom was ill, he had a temperature of 101°. During the night his temperature showed a change of $-2°$. What was his temperature then?

3. Mr. Stephen wrote a check for \$76.45. If he had only \$59.43 in the bank at that time, by how much would this "overdraw" his account?

4. On Tuesday two stocks were selling at exactly the same price. On Wednesday the price of one went up $2\frac{1}{2}$ points and the price of the other dropped $5\frac{1}{4}$ points. What was then the number of points of difference in the prices of these stocks?

5. On a winter's day when the temperature was $-35°$ in North Dakota, it was $+65°$ in Southern Florida. What was the difference in temperature between these places?

6. Name all the uses of signed numbers that you can.

Add the following, and check. When no sign appears before a number, the number is understood to be positive. 0 *has no sign.*

7.	3	-8	4	-1	5	-2	-4
	-1	2	-7	6	-5	0	4
8.	8	-8	1	5	0	-4	-3
	-7	5	-7	0	-3	8	3

Subtract the following, and check the work:

9.	7	-7	4	4	-9	-2	0
	-1	2	-6	-4	9	8	-3
10.	9	-9	2	8	-1	-3	-4
	-7	3	-4	-8	1	9	0

CHAPTER XVIII

REVIEW

CAN YOU ANSWER THESE QUESTIONS?

1. How is a commercial bank paid for the trouble and work of letting its depositors have checking accounts?

2. Mr. Hall lends \$500 to a friend at 4% simple interest, payable quarterly. What should he do with each payment of interest on this loan to make his money earn the equivalent of compound interest?

3. Why is a large factory or a railroad usually owned by a corporation rather than by an individual?

4. Mrs. Reed has \$2240 in the savings bank at $4\frac{1}{2}\%$ compound interest. Mr. Fox advises her to use the money to buy stock with a par value of \$100, which is selling at $38\frac{3}{4}$, with few prospects ahead. What advice would you give her? Why?

5. If you buy a 4% bond at 94 and keep it until it matures, will your investment earn more or less than 4% a year? Why?

6. Mr. Gregg, a married man, 36 yr. old, earns \$40 a week. What kind of life insurance policy should he purchase?

7. The fire insurance rate on the Red Brook Mountain House, a frame structure 4 mi. from the village of Red Brook, is \$2.29 per \$100. The rate on Mr. Gray's brick dwelling in St. Louis is \$.28 per \$100. Explain this difference in rates.

8. The actual values of the property in East Milton and in West Milton are about the same; but the assessed valuation of East Milton is only 75% of that of West Milton. If both towns must raise about the same amount for their expenses, which will have the higher tax rate?

FORMULAS AND EQUATIONS

1. Give the formulas for finding the areas of these figures: triangle, parallelogram, trapezoid, circle.

2. Give the formula for the circumference of a circle.

3. Using the formula in ex. 2, make a new formula for finding the diameter of a circle if you know the circumference. Use this new formula to find the distance across a circular pond whose circumference is 660 ft. Let $\pi = 3\frac{1}{7}$.

4. Tom was asked how old he was. He said, "Multiply my age by 5, add 5, and you get 75." Find his age.

5. A flagpole casts a shadow 64 ft. long when a boy $5\frac{1}{2}$ ft. tall casts a shadow 4 ft. long. How many feet high is the pole?

6. The population of a city is 55,430, which is 15% greater than it was 10 yr. ago. What was the population 10 yr. ago?

7. The formula $M = \frac{D}{G}$ shows how many miles (M) a car averages on each gallon of gasoline. In this formula D stands for the total number of miles traveled on a trip and G stands for the number of gallons used on the trip. Using this formula, find M when $D = 396$ mi. and $G = 22$ gal.; when $D = 540$ mi. and $G = 24$ gal.; when $D = 12$ mi. and $G = \frac{1}{2}$ gal.

8. Starting with the formula given in ex. 7, make a formula for D if you know M and G.

9. Make a formula showing the cost (C) of any number of gallons (n) of gasoline at 24¢ a gallon. Make a graph of this formula, using values for n from 1 to 10.

Solve these equations and check your results:

10. $2x + 3x = 5$

11. $4x - 18 = 24$

12. $\frac{1}{2}x - 19 = 37$

13. $\frac{1}{3}x = 12$

14. $\frac{3}{4}x = 18$

15. $\frac{2}{3}x = 11$

16. $\frac{33}{100}x = \$478.50$

17. $x + \frac{4}{100}x = \$5.72$

18. $x - \frac{9}{100}x = \$3.64$

MEASUREMENTS

1. A swimming pool 80 ft. long and 50 ft. wide is 4 ft. deep at one end and 8 ft. deep at the other end. How much does it cost to fill this pool, if the water costs 30¢ per 100 cu. ft.?

2. Give the formulas for the volumes of a cylinder, a cone, a pyramid, and a sphere.

3. A marble monument consists of a ball 4 ft. in diameter on top of a cylindrical column $3\frac{1}{2}$ ft. in diameter and 21 ft. high. The column stands on a base that is a rectangular solid 12 ft. square and 3 ft. deep. Draw to scale a side view of the monument. If 1 cu. ft. of marble weighs 168 lb., what is the weight of the monument in pounds? in tons?

4. A rectangular lot is 180 ft. long and 120 ft. wide. What is the length, to the nearest tenth of a foot, of a diagonal path from one corner of the lot to the opposite corner?

Find the curved surfaces and the volumes of these cylinders:

5. $h = 14$ in., $r = 6$ in.

6. $h = 12$ in., $r = 7$ in.

7. $h = 5\frac{1}{4}$ ft., $r = 1\frac{1}{2}$ ft.

8. $h = 5.6$ ft., $r = 1.4$ ft.

Find the volumes of the following pyramids:

	Area of Base	Height		Area of Base	Height
9.	9 sq. in.	5 in.	**11.**	$23\frac{1}{4}$ sq. ft.	8 ft.
10.	13 sq. in.	7 in.	**12.**	16.5 sq. ft.	3.5 ft.

Find the volumes of the following cones:

	Radius of Base	Height		Radius of Base	Height
13.	4 ft.	14 ft.	**15.**	$4\frac{3}{8}$ in.	16 in.
14.	0.7 ft.	2.9 ft.	**16.**	9.1 yd.	18.5 yd.

Find the areas and the volumes of spheres having these radii:

17. 21 ft. **18.** $3\frac{1}{2}$ in. **19.** $10\frac{1}{2}$ yd. **20.** 7.7 ft. **21.** 3.5 ft.

22. In what ways is the metric system easier to compute with than our system of measures?

MEASURING THE ENERGY YOU GET FROM FOODS

Food is needed to build up your body, to give you energy, and to keep you warm. The energy and heat that foods give are measured by a unit called a **calorie**. A calorie is the amount of heat needed to raise the temperature of 1 kg. of water 1° centigrade or of 1 lb. of water 4° Fahrenheit. If you eat a large apple, it will give you 100 calories of heat and energy.

The energy that you get from food comes from three substances called *carbohydrate*, *protein*, and *fat*. Carbohydrate is contained in sweet and starchy foods like sugar, bread, cereals, and potatoes. Much protein is found in meat, eggs, cheese, and milk, while fat is furnished by butter, olive oil, and the fat of meat. Most foods give more than one of these substances, as is shown in the table below.

The United States government publishes tables showing the per cent of carbohydrate, protein, and fat in the more common foods. In the table below, the first line shows that 9% of the *weight* of a piece of white bread is protein, 53% is carbohydrate, and 1% is fat; the rest is water or refuse, which explains why the sum of these three per cents is not 100%.

	Protein	*Carbohydrate*	*Fat*
White Bread	9%	53%	1%
Beefsteak	19%	—	18%
Chicken	21%	—	3%
Whole Egg	13%	—	10%
Whole Milk	3%	5%	4%
Butter	1%	—	85%
Cheese	28%	—	36%
Doughnut	7%	52%	21%
Cabbage	1%	5%	—
Carrots	1%	9%	—
Potato	2%	18%	—
Spinach	2%	3%	—
Banana	1%	22%	0.6%

FINDING THE NUMBER OF CALORIES IN FOODS

Mary has a loaf of white bread weighing 1 lb. She wishes to know how much protein, carbohydrate, and fat it contains, and how many calories of energy it will furnish.

The table on page 274 shows that 9% of white bread is protein. Since 9% of 16 oz. is 1.44 oz., the pound of bread contains 1.44 oz. of protein. Similarly, it contains 53% of 16 oz., or 8.48 oz., of carbohydrate, and 1% of 16 oz., or 0.16 oz., of fat.

To find the number of calories, use this table:

1 oz. carbohydrate	= 115 calories
1 oz. protein	= 115 calories
1 oz. fat	= 255 calories

You have found above that 1 lb. bread contains 1.44 oz. of protein, 8.48 oz. of carbohydrate, and 0.16 oz. of fat. Hence it furnishes:

$1.44 \times 115 = 166$, number of calories from protein
$8.48 \times 115 = 975$, number of calories from carbohydrate
$0.16 \times 255 = 41$, number of calories from fat
1182, number of calories in all

Hence 1 lb. bread gives 1182 calories, of which 166 come from protein, 975 from carbohydrate, and 41 from fat.

Exercises

From the table on page 274, find how many ounces of protein, of carbohydrate, and of fat each of the following contains. Then find the total calories in each:

1. 1 lb. spinach
2. 1 lb. cabbage
3. 1 lb. carrots
4. 1 lb. bananas
5. 3 lb. milk
6. $\frac{1}{2}$ lb. eggs
7. $\frac{1}{4}$ lb. cheese
8. 1 lb. butter
9. $\frac{1}{2}$ lb. chicken
10. 2 lb. potatoes
11. 1 lb. beefsteak
12. $\frac{1}{4}$ lb. doughnuts
13. A quart of whole milk, which weighs 34 oz.

A DAY'S MENU FOR A BOY

A sensible meal contains the right number of calories and also the right proportion of carbohydrate, protein, and fat.

Here is a menu for a day, suitable for a boy 11 yr. old:

	Total Calories	Kinds of Calories		
Breakfast		*Protein*	*Fat*	*Carbohydrate*
Banana	100	5	6	89
Shredded Wheat Biscuit	100	13	5	82
Glass of Milk	160	31	83	46
Bread	100	14	6	80
Butter	100	1	99	—
Luncheon				
Cup of Bean Soup	200	30	90	80
French Rolls	200	24	16	160
Butter	100	1	99	—
Cup Custard	300	51	117	132
Dinner				
Beef Loaf	200	80	120	—
Baked White Potato	100	11	1	88
Canned Peas	134	35	4	95
Lettuce, French Dressing	100	1	95	4
Bread	100	14	6	80
Butter	100	1	99	—
Baked Apple, Whipped Cream	200	2	62	136
Gingerbread	100	12	1	87

See the footnote about 100-calorie portions on page 277.

Exercises

1. Find the number of calories in each meal and the total calories for the day on the above menu.

2. How many calories in each meal are protein calories? How many are fat? How many are carbohydrate?

3. Find the number of each kind of calorie in the total day's menu and what per cent each is of the total calories for the day.

EATING THE RIGHT AMOUNT OF PROTEIN

In a well-balanced menu, from 10% to 15% of the total calories for the day are protein. Hence a day's menu of 2200 calories should contain from 220 to 330 protein calories.

1. In the menu on page 276, do the total calories for the day include the right per cent of protein? (See ex. 3, page 276.)

2. Suppose these were added to the menu on page 276:

		Total Calories	*Protein Calories*
BREAKFAST:	2 Eggs	150	54
LUNCHEON:	2 Lamb Chops	200	80
DINNER:	3 Crackers	50	5
	Cheese	100	25

What would then be the total number of calories consumed during the day? How many of these would be protein? Is the per cent of protein more or less than it should be? by how much?

Selecting School Lunches

Below is a list of foods served in a school lunchroom. Select five different luncheons, each having at least one article from each group and a total of from 750 to 900 calories. Let each luncheon have the right per cent of protein.

GROUP 1	CALORIES *Total*	*Protein*	GROUP 2	CALORIES *Total*	*Protein*
Cream of Celery Soup	200	22	Cocoa . . .	250	40
Split Pea Soup . .	167	44	Milk . . .	160	31
Macaroni and Cheese	200	34	GROUP 3		
Baked Beans . . .	300	63	Orange . . .	100	7
Lentil Meat Loaf . .	200	56	Rice pudding .	200	32
Scalloped Potatoes .	160	14	Brown Betty .	200	6
Egg Salad	250	35	Chocolate Bar	250	18
Bread and Butter . .	200	15	Ice Cream .	200	12

Note. In making menus, tables are often used giving 100-calorie portions of each food, thus making it easy to select portions of 100, 200, or 300 calories each, as in the items above and on page 276.

HOW MANY CALORIES DO YOU NEED?

The amount of food you need each day depends on your weight and age and on how you spend your time. An active child 12 yr. of age needs 32 calories a day for every pound of his weight; hence a child 12 yr. old weighing 78 lb. needs 32×78 calories, or 2496 calories, a day.

The table at the right gives the average number of calories which children of different ages need per day for every pound of their weight. To find how many calories a day a child needs, multiply his weight by the average number of calories for a child of his age.

Age in Years	Calories per Pound of Weight
1 to 2	43
3 to 5	38
6 to 9	34
10 to 13	32
14 to 17	27

Exercises

1. What is your weight and age? Then how many calories a day do you need?

2. Billie is 6 yr. old. How many calories a day should he have if he weighs 40 lb.? if he weighs 44 lb.? 48 lb.? 50 lb.?

Tell how many calories a day these children need:

3. Doris, who is 13 yr. old and weighs 89 lb.

4. Joseph, who is 7 yr. old and weighs 54 lb.

5. George, who is 14 yr. old and weighs 94 lb.

6. Evelyn, who is 16 yr. old and weighs 104 lb.

7. Ann is 12 yr. old and weighs 84 lb. She has 450 calories for breakfast, 700 calories for lunch, and 900 calories for dinner. How many calories less than she needs has she?

8. Has the menu on page 276 enough calories for you?

9. How many calories (see pages 274 and 275) are there in a pound of spinach? in a pound of bread? in a pound of butter? Why is it impossible for a person to live on spinach alone? Why do people eat so much bread and butter?

TABLES FOR REFERENCE

Linear Measure

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$16\frac{1}{2}$ feet or $5\frac{1}{2}$ yards = 1 rod (rd.)
5280 feet or 320 rods = 1 mile (mi.)

Square Measure

144 square (sq.) inches = 1 square foot
9 square feet = 1 square yard
$30\frac{1}{4}$ square yards = 1 square rod
43,560 square feet or 160 square rods = 1 acre (A.)
640 acres = 1 square mile

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) or 365 days = 1 year (yr.)
366 days = 1 leap year
100 years = 1 century

Calendar

Thirty days have September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone,
To which we twenty-eight assign
Till leap year gives it twenty-nine.

Weight

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (T.)

Angles and Arcs

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circumference.

Counting Articles

12 units = 1 dozen (doz.)
12 doz. or 144 units = 1 gross
20 units = 1 score

Counting Paper

24 folded sheets or 25 unfolded sheets = 1 quire
20 quires = 1 ream

Useful Equivalents

1 bushel = about 2150 cu. in. or about 1¼ cu. ft.
1 cubic foot = about 0.8 bu.
1 gallon = 231 cu. in.
1 cubic foot = about 7½ gal.
1 barrel = 31½ gal.

1 ton hard coal = about 35 cu. ft.
1 ton soft coal = about 42 cu. ft.
1 long ton = 2240 lb.
1 cord of wood = 128 cu. ft.
1 cubic foot water = about 62½ lb.
1 cubic foot ice = about 57½ lb.

Weight per Bushel

The legal weights per bushel are not the same throughout the country. The following weights per bushel, however, are accepted in many states.

Apples	48 lb.	Clover seed	60 lb.	Peas	60 lb.
Barley	48 lb.	Corn (shelled)	56 lb.	Potatoes (Irish)	60 lb.
Beans	60 lb.	Corn (on cob)	70 lb.	Rye	56 lb.
Buckwheat	48 lb.	Oats	32 lb.	Wheat	60 lb.

Metric Length

10 millimeters (mm.) = 1 centimeter (cm.)
10 centimeters = 1 decimeter (dm.)
10 decimeters = 1 meter (m.)
10 meters = 1 dekameter
10 dekameters = 1 hektometer (hm.)
10 hektometers = 1 kilometer (km.)

Metric Weight

1000 grams (g.) = 1 kilogram (kg.)
1000 kilograms = 1 metric ton (t.)

Metric Capacity

1000 milliliters (ml.) = 1 liter (l.)
100 liters = 1 hektoliter (hl.)

Metric Equivalents

1 meter = 39.37 in. or 1.1 yd.
1 kilometer = 0.6 mi.
1 liter = 1.06 liquid qt. or 0.91 dry qt.
1 kilogram = 2.2 lb.

1 yard = 0.914 m.
1 mile = 1.609 km.
1 dry quart = 1.10 l.
1 liquid quart = 0.95 l.
1 pound = 0.454 kg.

INDEX

Acceptance, trade, 130, 132.
Accident insurance, 194, 195.
Account, bank, 113, 114.
  savings, 135.
Addition, improvement tests in, 23, 81, 133, 177, 211.
  of signed numbers, 263–266.
Altitude (*see* Height).
Amount, in interest, 135.
Angles, bisecting, 217.
  kinds of, 213, 216, 220, 221.
  measuring, 217.
  of similar triangles, 243.
  right, 213, 216.
Arc, 214.
Area, lateral, 67.
  of circle, 6, 260.
  of cylinder, 63.
  of parallelogram, 2.
  of pyramid, 67.
  of rectangle, 1, 2, 75, 76.
  of sphere, 69.
  of trapezoid, 2, 4.
  of triangle, 2, 4, 77, 78.
Automobile insurance, 190–192.

Bank account, 113, 114, 135.
Bank discount, 128, 131, 132.
Bank draft, 120–122.
Banks, commercial, 113–134.
  savings, 135.
Bar graphs, 97.
Base, 1, 2.
Bisection, of a line, 214.
  of an angle, 217.
Bonds, 170–175.
Brokerage, 165–167, 172.
Building and loan associations, 157–159.

Calorie, 274.
Check, bank, 113–117.
  certified, 119.
Checking computations in percentage, 83.
Circle, 5, 6, 80, 214, 260.
Circle graphs, 103.
Circumference, 5, 80.
Clearing house, 118.
Collateral, 129.
Commercial forms, 113–123, 128–132, 134.
Commission, 98.
Comparing numbers, by division, 231–234.
  by per cents, 88–93.
  by subtraction, 231.
Compound interest, 135–146.
Cone, 68.
Congruent triangles, 223–226.
Corporation, 161, 162.
Corresponding sides, 241.
Corresponding values, 8.
Coupons, 170.
Cube, 51, 57.
Customs, 203.
Cylinder, 59–63, 80.

Decimals, improvement tests in, 49, 239.
  in per cents, 87, 88, 94.
Degree of accuracy,
  of measurements, 53–56.
  of results, 88–92, 253–255, 258.
Demand note, 129.
Deposit slip, 114.
Diameter, 80.
Discount, bank, 128, 131, 132.
  trade, 85, 104–106.
Dividends, 161–163, 188.
Division, improvement tests in, 81, 133, 177.
  in formulas, 13.
Draft, bank, 120–122.
  sight, 121.
Duties, 203, 204.

Electricity formula, 14.
Equations, 31–50.
  in percentage, 31, 107–112.
  solving, by addition, 37.
  solving, by division, 39.
  solving, by multiplication, 38.
  solving, by subtraction, 35, 36.
  solving problems by, 42–48.
Estimating answers, 86, 253, 254.
Estimating distances, 247, 248.
Evaluating formulas, 1.

Face of policy, 179, 183.
Fire insurance, 179–181.
Formulas, 1–20, 51, 57, 59, 63, 66–70, 272.
  as equations, 74–80, 260.
Fractions, improvement tests in, 65, 111, 193.

Graphs, bar, 97.
  circle, 103.
  line, 9, 25, 26, 29, 30.
  of a formula, 10–12, 17.

Health insurance, 194, 195.
Height, 1, 2, 57, 59, 63, 66–68.
Hemisphere, 69.
Hexagon, 218.
Hypotenuse, 256.

Improvement tests, in decimals, 49, 239.
  in fractions, 65, 111, 193.
  in percentage, 145, 259.
  in whole numbers, 23, 81, 133, 177, 211.
  keeping records in, 25–29.
  scoring, 24–29.
  taking, 21, 22, 64.
Indorsement, 115, 117.
Installment buying, 147–159.
Insurance, 179–195.
Interest, accrued, 173.
  by 6% method, 124.
  compound, 135–146.
  in installment buying, 148–156.
  simple, 124–132, 144, 147–156.
  tables, 126, 136, 141, 146.

Lateral area (*see* Area).
Life insurance, 182–189.
Line graphs, 9, 25, 26, 29, 30.
Lines, bisecting, 214.
  parallel, 220–222.
  perpendicular, 213–215.
List price, 104.

Loans, 124–132, 147–159, 187, 188.

Margin, 99.
Market value, 165.
Maturity, day of, 131.
  yield to, 175.
Measures (*see* Tables).
Median score, 21, 27–30.
Metric system, 205–210.
Money order, 123.
Mortgage, 154, 156.
Multiplication, improvement tests in, 81, 133, 177.

Negative numbers, 261–270.
Net price, 104.
Net proceeds, 98.
Notes, 128, 129 131.

Octagon, 218.
Overhead expenses, 99.

Parallel lines, 220–222.
Parallelogram, 2, 3.
Par value, 161, 162, 165, 172.
Per cents, finding, 83–87, 94, 95.
  fractions equivalent to, 86.
  large, 95.
  smaller than 1%, 94.
Percentage, applications of, 98-106,124–132,147–156.
  comparing numbers in, 83, 84, 88–93, 96, 97.
  equation in, 31, 107–112.
  finding the whole in, 83.
  improvement tests in, 145, 259.
Perpendicular lines, 213–215.
Pi (π), 5.
Policy, insurance, 179, 183, 184, 187.
Positive numbers, 261–270.
Premium, 179, 187.
Prism, 51, 57, 58.
Problems about
  Automobiles, 16–18.
  Baseball, 89–91.
  Boats, 72, 73.
  Buying homes, 153–156, 158, 159.
  Camping, 82.
  Electricity, 14, 15.
  Farms, 61, 62.
  Foods, 274–278.
  Gardens, 75–77.
Problems about (*Continued*)
  Government receipts and expenses, 92, 199.
  Installment buying, 147–159.
  Investments, 144, 159, 168, 169, 173, 174.
  Olympic games, 208.
  Savings, 138–146.
  Shopping, 85.
  Silos, 61, 62.
  Sports, 84, 89–91, 206, 208, 245.
  Stores, 134.
  Thrift, 85, 138–140, 142–146, 151, 152, 159.
Proceeds, net, 98.
  of note, 128.
Profit and loss, 87, 99–103.
Promissory note, 128, 131.
Proportion, 235–246.
Protractor, 217.
Pyramid, 66, 67.
Pythagorean theorem, 256.

Radius, 5, 6, 69, 260.
Rate, of commission, 98.
  of interest, 147, 149.
  of taxation, 197.
Ratio, 231–234, 241, 248.
Rectangles, 1–3.
  similar, 248–250.
Rectangular solids, 51, 79.
Reviews, 19, 20, 50, 112, 134, 160, 176, 178, 196, 212, 250, 269–278.
Round numbers, 92, 93.
Rules as formulas, 7.

Sales dollar, 103.
Savings banks, 135.
Scale drawing, 249.
Signed numbers, 261–270.
Similar figures, 241–250.
Simple interest, 124–132, 144, 147–156.
Slant height, 67.
Sphere, 69–71.
Square root, 251–260.
Squared paper, use of, 10, 11, 17, 97, 229.
Squares of numbers, 6, 251, 252.
Stock exchange, 164, 165.
Stocks and bonds, 161–175.
Subtraction, improvement tests in, 23, 81, 133, 177, 211.
Subtraction, of signed numbers, 267–269.
Symmetry, 227–230.

Tables
  angles and arcs, 279.
  calories, 274, 275, 278.
  compound interest, 136, 141, 146.
  cubic measure, 52, 279.
  dry measure, 55, 279.
  equivalents, 53, 55, 207, 280.
  life insurance, 183, 185, 187.
  linear measure, 279.
  liquid measure, 53, 279.
  made from formulas, 8, 10, 13, 16.
  metric measures, 205, 207, 209, 210, 280.
  simple interest, 126.
  square measure, 279.
  squares of numbers, 252.
  tax, 201.
  time, 279.
  weight, 279, 280.
Tariff, 203.
Taxes, 197–204.
Time between dates, 125.
Trade acceptances, 130, 132.
Trade discount, 85, 104–106.
Trapezoid, 2, 4.
Triangles, area of, 2, 4, 77, 78.
  congruent, 223–226.
  drawing, 219, 223, 225.
  kinds of, 219, 223, 230, 241, 256.
  right, 256–260.
  similar, 241–250.
  sum of angles of, 219.
  uses of, 4, 224, 226, 246, 247, 257, 258, 260.

Unknown number, 74, 75.

Vertex, 66, 68, 216.
Volt, 14.
Volume, of cone, 68.
  of cylinder, 59–62, 80.
  of prism, 57, 58.
  of pyramid, 66.
  of rectangular solid, 51, 79.
  of sphere, 70, 71.

Watt, 14.

ANSWERS TO STRAYER–UPTON JUNIOR MATHEMATICS

BOOK TWO

Page 3. — **1.** \$126.72. **2.** $3\frac{1}{2}$ gal. **3.** \$52.50. **4.** .29 A. **5.** \$365.50; \$677.25; \$674.03. **6.** \$92.40. **7.** 48 sq. ft. **8.** 5 sq. yd. **9.** $22\frac{3}{4}$ sq. ft. **10.** $12\frac{11}{16}$ sq. yd. **11.** 13.95 sq. in. **12.** 51.84 sq. in. **13.** 42 sq. in. **14.** 20 sq. in. **15.** $16\frac{1}{2}$ sq. ft. **16.** $1\frac{7}{8}$ sq. yd. **17.** 131.4 sq. ft. **18.** 62.56 sq. ft.

Page 4. — **1.** *M*, 10,000 sq. ft.; *N*, 35,000 sq. ft.; *R*, 60,000 sq. ft. **2.** 137 sq. ft. **3.** $7\frac{1}{2}$ sq. ft. **4.** 14 sq. ft. **5.** 15 sq. ft. **6.** 13 sq. in. **7.** $6\frac{3}{8}$ sq. ft. **8.** $5\frac{3}{4}$ sq. yd. **9.** 3.875 sq. ft. **10.** 9.24 sq. ft. **11.** 7.25 sq. ft. **12.** 18 sq. yd. **13.** $41\frac{1}{6}$ sq. yd. **14.** 340 sq. ft. **15.** $1662\frac{1}{2}$ sq. ft. **16.** 3720 sq. ft. **17.** 91 sq. ft. **18.** $89\frac{1}{4}$ sq. ft. **19.** 150 sq. ft.

Page 5. — **1.** 88 in. **2.** $7\frac{1}{3}$ ft.; 2160 times. **3.** $81\frac{5}{7}$ in.; $6\frac{1}{21}$ ft. **4.** $55\frac{5}{13}$ revolutions. **5.** 132 ft.; 40 times. **6.** 44 in. **7.** 88 ft. **8.** 176 yd. **9.** $157\frac{1}{7}$ ft. **10.** $6\frac{2}{7}$ yd. **11.** 308 in. **12.** $7\frac{1}{3}$ ft. **13.** $29\frac{1}{3}$ ft. **14.** 110 in. **15.** $31\frac{3}{7}$ mi. **16.** 242 ft. **17.** $56\frac{4}{7}$ rd. **18.** 132 ft. **19.** $314\frac{2}{7}$ ft. **20.** $27\frac{1}{2}$ rd. **21.** $17\frac{3}{8}$ mi.

Page 6. — **1.** 2464 sq. ft.; $1169\frac{1}{7}$ sq. ft. **2.** 3850 sq. mi. **3.** \$171. **4.** 154 sq. in. **5.** $12\frac{4}{7}$ sq. ft. **6.** $28\frac{2}{7}$ sq. in. **7.** $3\frac{1}{7}$ sq. ft. **8.** 616 sq. ft. **9.** 12,474 sq. ft. **10.** 1386 sq. in. **11.** 2464 sq. in. **12.** $314\frac{2}{7}$ sq. ft. **13.** 9856 sq. in. **14.** 7546 sq. in. **15.** 5544 sq. in.

Page 7. — **1.** \$52.80. **2.** \$36. **3.** \$88. **4.** \$74.10. **5.** \$57.60. **6.** $D = 30\,n$. **7.** $D = RT$. **8.** 150 mi. **9.** 175 mi. **10.** 60 mi. **11.** 162 mi. **12.** 54 mi. **13.** 273 mi.

Page 8. — **2.** 4 tickets, \$1.40; 5 t., \$1.75; 6 t., \$2.10; 7 t., \$2.45; 8 t., \$2.80; 9 t., \$3.15; 10 t., \$3.50; 11 t., \$3.85; 12 t., \$4.20; 13 t., \$4.55; 14 t., \$4.90; 15 t., \$5.25; 16 t., \$5.60; 17 t., \$5.95; 18 t., \$6.30; 19 t., \$6.65; 20 t., \$7.00; 21 t., \$7.35; 22 t., \$7.70; 23 t., \$8.05; 24 t., \$8.40; 25 t., \$8.75. **3.** \$.54; \$.90; \$4.68.

Page 8 (*continued*). — **4.** 15 mi., \$.54; 20 mi., \$.72; 25 mi., \$.90; 30 mi., \$1.08; 35 mi., \$1.26; 40 mi., \$1.44; 45 mi., \$1.62; 50 mi., \$1.80; 55 mi., \$1.98; 60 mi., \$2.16; 65 mi., \$2.34; 70 mi., \$2.52; 75 mi., \$2.70; 80 mi., \$2.88; 85 mi., \$3.06; 90 mi., \$3.24; 95 mi., \$3.42; 100 mi., \$3.60; 105 mi., \$3.78; 110 mi., \$3.96; 115 mi., \$4.14; 120 mi., \$4.32; 125 mi., \$4.50. **5.** $C = .30\,n$; 4 doz., \$1.20; 5 doz., \$1.50; 6 doz., \$1.80; 7 doz., \$2.10; 8 doz., \$2.40; 9 doz., \$2.70; 10 doz., \$3.00; 11 doz., \$3.30; 12 doz., \$3.60.

Page 12. — **5.** 1 da., \$8; 2 da., \$16; 3 da., \$24; 4 da., \$32; 5 da., \$40; 6 da., \$48; 7 da., \$56; 8 da., \$64; 9 da., \$72; 10 da., \$80.

Page 13. — **1.** \$.05; \$.04$\frac{1}{2}$; \$.03$\frac{1}{3}$. **2.** \$.50; \$.45; \$.40; \$.33$\frac{1}{3}$; \$.25; \$.20$\frac{5}{6}$; \$.16$\frac{2}{3}$; \$.15; \$.12$\frac{1}{2}$; \$.08$\frac{1}{3}$. **3.** 32 mi. an hour; 39 mi. an hour; 25 mi. an hour. **4.** 25 mi. **5.** 30 mi. **6.** 28 mi. **7.** 36 mi. **8.** 32.9 mi. **9.** 40 mi. **10.** $M = \frac{D}{G}$. **11.** 25 mi.; 20 mi.; 18 mi.

Page 14. — **2.** 660 watts; 660 watts. **3.** 60 watts. **4.** 3840 watts. **5.** 1800 watts. **6.** 90 watts. **7.** 1100 watts. **8.** 2200 watts.

Page 15. — **9.** 300 watts; yes. **10.** 1100 watts; no; 5 lamps. **2.** 1650 watts. **3.** No. **4.** 13 lamps.

Page 16. — **1.** 20 mi., 36 ft.; 25 mi., 56$\frac{1}{4}$ ft., or 56.25 ft.; 30 mi., 81 ft.; 35 mi., 110$\frac{1}{4}$ ft., or 110.25 ft.; 40 mi., 144 ft. **2.** 4 times as great. **4.** Yes.

Page 17. — **1.** 40 ft.; 75 ft.; 60 ft.; 26 ft.; 98 ft.; 7 ft. **2.** 45 mi., 182$\frac{1}{4}$ ft., or 182.25 ft.; 50 mi., 225 ft.; 55 mi., 272$\frac{1}{4}$ ft., or 272.25 ft.; 60 mi., 324 ft.

Page 18. — **1.** 24 ft.; 36 ft.; 12 ft. less. **2.** 5 mi., 1$\frac{1}{2}$ ft., or 1.5 ft.; 10 mi., 6 ft.; 15 mi., 13$\frac{1}{2}$ ft., or 13.5 ft.; 20 mi., 24 ft.; 25 mi., 37$\frac{1}{2}$ ft., or 37.5 ft.; 30 mi., 54 ft.; 35 mi., 73$\frac{1}{2}$ ft., or 73.5 ft.; 40 mi., 96 ft.; 45 mi., 121$\frac{1}{2}$ ft., or 121.5 ft.; 50 mi., 150 ft.; 55 mi., 181$\frac{1}{2}$ ft., or 181.5 ft.; 60 mi., 216 ft. **4.** 3 ft. greater; 12 ft.; 27 ft.; 48 ft.; 75 ft.; 108 ft.

Page 19. — **1.** (*d*). **2.** (*g*). **3.** (*i*). **4.** (*a*). **5.** (*h*). **6.** (*b*). **7.** (*e*). **8.** (*f*). **9.** (*c*).

Page 20. — **1.** $P = .22\,n$. **7.** 15; 4$\frac{1}{2}$. **8.** 270; 150. **9.** 28. **10.** 36. **11.** 3850; 1257$\frac{1}{7}$. **12.** 92.16, or 92$\frac{4}{25}$; 380$\frac{1}{4}$, or 380.25; 441. **13.** 52$\frac{1}{2}$ watts. **14.** 18 watts, 34$\frac{1}{2}$ watts less; 33 watts, 19$\frac{1}{2}$ watts less; 27 watts, 25$\frac{1}{2}$ watts less; 22$\frac{1}{2}$ watts, 30 watts less; 30 watts, 22$\frac{1}{2}$ watts less; 36 watts, 16$\frac{1}{2}$ watts less.

Page 23. — *See page* xx.

Page 36. — **8.** 268. **9.** 276. **10.** 168. **11.** 123. **12.** 201 **13.** 440. **14.** .96. **15.** 3.44. **16.** 1.90. **17.** 1.49. **18.** 48.1. **19.** 2.04. **20.** 25. **21.** 29. **22.** 317. **23.** 588.

Page 37. — **1.** 12. **2.** 13. **3.** 22. **4.** 19. **5.** 45. **6.** 403. **7.** 601. **8.** 1150. **9.** 1101. **10.** 1080. **11.** 5. **12.** 4.5. **13.** $22\frac{3}{4}$. **14.** 1.40. **15.** 3.30.

Page 38. — **1.** 36. **2.** 24. **3.** 12. **4.** 120. **5.** 28. **6.** 20. **7.** 56. **8.** 425. **9.** 13. **10.** 1.35. **11.** 105. **12.** 151.

Page 39. — **1.** 7. **2.** 6. **3.** 3. **4.** 3. **5.** 1. **6.** $\frac{1}{2}$. **7.** $\frac{1}{5}$. **8.** $1\frac{3}{4}$. **9.** $\frac{1}{2}$. **10.** 3. **11.** 3. **12.** 3.03. **13.** .06.

Page 40. — **17.** 9. **18.** 14. **19.** 35. **20.** 9. **21.** 33. **22.** 27. **23.** 46. **24.** 4. **25.** 1. **26.** 32. **27.** 34. **28.** 10. **29.** $2\frac{3}{4}$. **30.** 625. **31.** 2. **32.** 800. **33.** 24. **34.** 25. **35.** 4. **36.** $36\frac{3}{4}$. **37.** 1285. **38.** 4.5. **39.** $3\frac{3}{4}$. **40.** 522.3. **41.** 60.8. **42.** $40\frac{1}{4}$. **43.** $17\frac{1}{2}$.

Page 41. — **2.** $(m - 6)$ beads. **3.** $2y$ dollars; $\frac{1}{2}y$ dollars. **4.** $(x - 3)$ dollars; $(x + 5)$ dollars; $2x$ dollars; $\frac{1}{2}x$ dollars. **5.** $\frac{1}{3}d$ dollars; $2d$ dollars. **6.** $(n + 5)$ years; $(n + 1)$ years; $(n + 10)$ years. **7.** $(x - 2)$ years; $(x - 5)$ years; $(x - 7)$ years; $(x - x)$ years, or 0 years. **8.** $(x - 2.50)$ dollars. **9.** $\frac{1}{2}y$ dollars. **10.** 17; 5; 30. **11.** $(x - 5)$ dollars.

Page 43. — **1.** 2485 stamps. **2.** $27. **3.** $25. **4.** 9000. **5.** 252 shells. **6.** $8.50. **7.** $8x$ dollars; 12 wk. **8.** $24x$ miles; $5\frac{1}{3}$ hr. **9.** $10. **10.** $2\frac{1}{2}$ ft. **11.** 15 ft.

Page 44. — **1.** $5x$. **2.** $3x$. **3.** $2\frac{1}{2}r$. **4.** $3x$. **5.** $3b$. **6.** c. **7.** $2x$. **8.** 0. **9.** $2m$. **10.** $2\frac{1}{2}x$. **11.** $\frac{1}{3}x$. **12.** x. **13.** 4. **14.** 6. **15.** 2. **16.** 8. **17.** $2\frac{1}{2}$. **18.** 3. **19.** 4. **20.** 4. **21.** 15. **22.** 1. **23.** $3\frac{1}{2}$. **24.** 17. **25.** 4. **26.** 1. **27.** 3. **28.** $2\frac{1}{2}$. **29.** 20. **30.** 48. **31.** 33. **32.** 4. **33.** 9. **34.** 5. **35.** 150. **36.** 2. **37.** 2. **38.** 5. **39.** 13 yr.

Page 45. — **11.** 120. **12.** 5. **13.** 8. **14.** 4. **15.** 8. **16.** 4. **17.** 11. **18.** 27. **19.** 20. **20.** 18. **21.** 36. **22.** 42. **23.** 500. **24.** 10. **25.** $21. **26.** $40. **27.** $70. **28.** 480. **29.** 140. **30.** 200. **31.** $\frac{1}{2}$.

Page 46. — **1.** 13 yr. **2.** 13 yr. **3.** 2. **4.** 2. **5.** $1100. **9.** 20¢.

Page 47. — **2.** Sweater, $8; racket, $4. **3.** Book, $1.30; pencil, $.10. **4.** House, $33,000; lot, $15,000. **5.** 6 cups milk, 3 cups water. **6.** 1 yd.; 4 yd. **7.** Jane, $42; Mary, $130.

Page 48. — **3.** Frank, 4 yr.; Betty, 12 yr. **4.** 15 yr. **5.** Susan, 6 yr.; Henry, 24 yr. **6.** Mother, 32 yr.; Edna, 8 yr. **7.** Joe, 8 yr.; Harry, 21 yr. **8.** Lena, 7 yr.; Hugh, 14 yr. **9.** 11 yr.

Page 49. — *See page* xx.

Page 50. — **1.** \$2100. **2.** 290 pupils. **3.** \$3.20; \$6.40. **4.** \$9. **5.** Roy, \$20; Tom, \$10; Joe, \$15. **6.** \$1. **7.** \$.42. **8.** 8. **9.** 32. **10.** $22\frac{1}{2}$. **11.** 160. **12.** 4. **13.** 16. **14.** $33\frac{1}{3}$. **15.** \$36. **16.** 5. **17.** 2. **18.** $8\frac{1}{2}$. **19.** 736. **20.** 126. **21.** 130. **22.** 10. **23.** \$50. **24.** 840. **25.** 32. **26.** 195. **27.** 60. **28.** 50. **29.** 80. **30.** 75.

Page 51. — **1.** 105 cu. in. **2.** 512 cu. in. **3.** 432 cu. in., or $\frac{1}{4}$ cu. ft. **4.** 14 cu. ft.

Page 52. — **3.** 12,600 cu. ft.; 63 pupils. **4.** 167 cu. yd.; \$108.55. **5.** 21,547$\frac{1}{2}$ lb. **6.** 6120 cu. ft.; \$16.52. **7.** 1512 cu. in.; 54$\frac{11}{16}$ lb. **8.** 72 cans.

Page 53. — **1.** Third can. **2.** More; $9\frac{5}{8}$ cu. in. **3.** 24 gal. **4.** 141,750 gal. **5.** 105 cu. ft.; 787$\frac{1}{2}$ gal.; 25 bbl. **6.** 300 gal. **7.** 315 gal. **8.** 525 gal. **9.** 1 gal. **10.** $2\frac{1}{2}$ gal. **11.** 74,250 gal.

Page 54. — **2.** $2\frac{1}{16}$ cu. in.

Page 55. — **2.** 160 bu.; 272 bu. **3.** $9\frac{1}{2}$ T.; $11\frac{2}{3}$ T. **4.** 5 ft. **5.** 408 bu. **6.** 396 bu. **7.** 186 bu. **8.** 336 bu. **9.** 342 bu. **10.** $94\frac{1}{2}$ bu. **11.** 157$\frac{1}{2}$ bu. **12.** 364 bu. **13.** 117 bu. **14.** 16.8 bu. **15.** 81.12 bu. **16.** 271.32 bu.

Page 56. — **1.** 2160 cu. in.; 10 cu. in.; about 1 in 200. **2.** 2150 cu. in.; 50 cu. in.; about 1 in 40. **3.** 1 in 86; greater. **4.** More. **5.** Box (*f*). **6.** Former.

Page 58. — **1.** 196 cu. in. **2.** 27 cu. ft. **3.** $10\frac{1}{2}$ cu. in. **4.** 28 cu. in. **5.** 72 cu. ft. **6.** 540 gal. **7.** 10,500 cu. ft.; 78,750 gal. **8.** 67,500 gal.; \$28.80.

Page 59. — **1.** 32 cu. in. **2.** 42 cu. in. **3.** $49\frac{1}{2}$ cu. ft. **4.** $12\frac{1}{2}$ cu. ft. **5.** $75\frac{3}{7}$ cu. ft. **6.** $226\frac{2}{7}$ cu. in. **7.** $28\frac{2}{7}$ cu. in. **8.** 6930 cu. in. **9.** $28\frac{2}{7}$ cu. in. **10.** 192$\frac{1}{2}$ cu. ft.; 1444 gal.

Page 60. — **1.** 247$\frac{1}{2}$ cu. ft.; 34,650 lb. **2.** $553\frac{1}{7}$ cu. in.; 2.4 gal. **3.** 40 gal.; 61 gal. **4.** 712,800 cu. ft.; \$891. **5.** 2227$\frac{1}{2}$ gal. **6.** $125\frac{5}{7}$ cu. ft.; $942\frac{8}{7}$ gal. **7.** 3 gal.

Page 61. — **1.** 1571 cu. ft.; 1964 cu. ft.; 2357 cu. ft.; 2750 cu. ft. **2.** 2829 cu. ft.; 3394 cu. ft.; 4526 cu. ft. **3.** 4620 cu. ft.; 5390 cu. ft.; 6160 cu. ft.; 7241 cu. ft.; 7643 cu. ft.; 8448 cu. ft. **4.** 50 cows. **5.** 159 da. **6.** 94 cu. ft.

Page 62. — **1.** 39 cu. ft.; 39 cows. **2.** Yes; no. **3.** 10 ft. in diameter.
1. 6 A. **2.** 119 T. **3.** 21,120 gal. **4.** 61,097 cu. ft.; 1741 T. **5.** One is 4 times the other.

Page 63. — **1.** $37\frac{5}{7}$ sq. in. **2.** $39.60; $60.34. **3.** 90 sq. in. **4.** 24 sq. ft. **5.** 2600 sq. in. **6.** 880 sq. in. **7.** 660 sq. in. **8.** $942\frac{8}{7}$ sq. ft.

Page 65. — *See page* xx.

Page 66. — **1.** 10 cu. in. **2.** $13\frac{1}{3}$ cu. ft. **3.** 21 cu. ft. **4.** 7 cu. in. **5.** 30 cu. ft. **6.** $43\frac{1}{3}$ cu. ft. **7.** $85,517,958\frac{2}{3}$ cu. ft. **8.** $26\frac{2}{3}$ bu.

Page 67. — **1.** 192 sq. in. **2.** 3 gal. **3.** 6 gal. **4.** 884,756 sq. ft.; 1106 da.

Page 68. — **1.** $6\frac{2}{7}$ cu. yd. **2.** $3\frac{2}{3}$ cu. in.; 63 cones. **3.** $184\frac{4}{5}$ bu. **4.** 19,800 cu. ft.; 566 T. **5.** $565\frac{5}{7}$ cu. in. **6.** 13,860 cu. in. **7.** 36,960 cu. ft. **8.** 770,000 cu. in.

Page 69. — **1.** 616 sq. in. **2.** 154 sq. in. **3.** $314\frac{2}{7}$ sq. in. **4.** $1018\frac{2}{7}$ sq. ft. **5.** 5544 sq. in. **6.** 2464 sq. in. **7.** 15,400 sq. ft. **8.** 22,176 sq. ft. **9.** $169,161\frac{1}{7}$ sq. in. **10.** 1,540,000 sq. in. **11.** $201,142,857\frac{1}{7}$ sq. mi. **12.** 693 sq. ft.; $519.75.

Page 70. — **2.** $11,498\frac{2}{3}$ cu. in. **3.** $91,989\frac{1}{3}$ cu. in. **4.** $113,142\frac{8}{7}$ cu. in. **5.** 38,808 cu. in. **6.** $4190\frac{10}{21}$ cu. in., or 4190.5 cu. in. **7.** $7241\frac{1}{7}$ cu. in. **8.** $179\frac{2}{3}$ cu. ft., or 179.7 cu. ft. **9.** 310.5 cu. ft. **10.** 92.0 cu. ft. **11.** $4\frac{4}{21}$ cu. ft., or 4.2 cu. ft. **12.** 1.8 cu. in. **13.** 2.8 cu. in. **14.** $179\frac{2}{3}$ cu. in., or 179.7 cu. in. **15.** $606\frac{3}{8}$ cu. in., or 606.4 cu. in. **16.** 350.9 cu. in. **17.** $33\frac{11}{21}$ cu. in., or 33.5 cu. in.; $1767\frac{8}{7}$ cu. in., or 1767.9 cu. in.

Page 71. — **1.** 704 lb. **2.** 5632 lb.; 8 times. **3.** $15\frac{5}{7}$ gal. **4.** $1527\frac{3}{7}$ cu. ft.; about 3 T. **5.** 8.18 cu. in.; 17.98 cu. in.; about $2\frac{1}{5}$ times. **7.** 19.8 gal.; 13.2 gal.

Page 72. — **1.** $\frac{1}{6}$ cu. ft. **3.** $\frac{1}{4}$ cu. ft.; 15.6 lb.
1. $\frac{1}{3}$ cu. ft. **2.** 20.8 lb.; 20.8 lb. **3.** $\frac{1}{2}$ cu. ft.; 31.25 lb.; 10.45 lb. **4.** 312.5 lb.; 687.5 lb.; 1062.5 lb.; 1437.5 lb.; 1812.5 lb.

Page 73. — **1.** 375 lb. **2.** 468.75 lb.; 93.75 lb. **3.** 750 lb.; 281.25 lb. **4.** 3375 lb.; 3000 lb. **5.** 1723.75 lb. **6.** 61.5 cu. ft.; 3598.75 lb.

Page 74. — **1.** 24 sq. ft. **2.** 24 cu. ft. **3.** 88 ft.

Page 75. — **1.** 8 ft. **2.** 3 ft. **3.** $7\frac{1}{2}$ ft. **4.** 12 ft.

Page 76. — **2.** 17 ft. **3.** $12\frac{1}{2}$ ft. **4.** $16\frac{1}{4}$ ft. **5.** 17 yd. **6.** $l = \frac{A}{w}$.

Page 77. — **1.** 9 ft. **2.** 5 ft. **3.** 18 ft. **4.** 8 ft. **5.** 6 ft. **6.** 10 ft. **7.** 13 ft. **8.** 50 ft.; 36 ft.; 200 ft.

Page 78. — **2.** 40 ft. **3.** 12 in. **4.** 5 in. **5.** 9 yd. **6.** 26 ft. **7.** 216. **8.** 96. **9.** $37\frac{1}{2}$. **10.** 85. **11.** 40. **12.** 7.

Page 79. — **1.** $6\frac{1}{4}$ ft. **2.** $l = \frac{V}{wh}$; $w = \frac{V}{lh}$. **3.** 216. **4.** 19. **5.** 5. **6.** 2 ft.

Page 80. — **1.** 63 in. **2.** 28 in. **3.** $24\frac{1}{2}$ in.; 42 in.; 49 in.; $3\frac{1}{2}$ ft. **5.** $22\frac{10}{11}$ ft.; $11\frac{5}{11}$ ft. **6.** 7 ft.; $4\frac{5}{11}$ ft. **7.** $h = \frac{V}{\pi r^2}$. **8.** 30 ft.; 37 ft.; 62 ft.; 21.5 ft.

Page 81. — *See page* xx.

Page 82. — **1.** 404 sq. ft. **2.** 17 qt.; \$10.20. **3.** 220 sq. ft.; 9 qt.; \$5.40. **4.** \$.60 loss. **5.** 6 quart cans; 2 gallon cans. **6.** \$.40 loss.

Page 84. — **1.** 180 boys. **2.** 27 boys. **3.** 60%. **4.** 18 meets. **5.** 200 races.

1. \$23. **2.** 450 ft. **3.** 85%. **4.** \$8.37. **5.** \$68. **6.** \$1.65. **7.** \$3.96. **8.** \$408. **9.** \$3230. **10.** 30. **11.** 68. **12.** \$200. **13.** \$600. **14.** \$120. **15.** 300. **16.** 5650. **17.** $22\frac{1}{2}$. **18.** $3\frac{1}{2}$.

Page 85. — **1.** \$9.75; \$55.25. **2.** \$6.20; \$18.60. **3.** \$.75; \$.85; \$1.60. **4.** \$1.50; 25%. **5.** 20%. **6.** 10%. **7.** \$11.40; \$26.60. **8.** \$1.20; \$10.80. **9.** \$2.80; \$11.20. **10.** \$15; \$30. **11.** \$12.69; \$38.06. **12.** \$1.79; \$16.06. **13.** \$1.50; \$6. **14.** \$1.73; \$4.02. **15.** \$14.90; \$44.70.

Page 86. — **1.** $\frac{1}{3}$; $\frac{3}{5}$; $\frac{3}{8}$; $\frac{3}{10}$; $\frac{4}{5}$; $\frac{2}{5}$; $\frac{1}{5}$; $\frac{1}{4}$. **2.** $\frac{3}{4}$; $\frac{1}{2}$; $\frac{3}{5}$; $\frac{7}{8}$; $\frac{1}{8}$; $\frac{1}{6}$; $\frac{2}{3}$; $\frac{5}{6}$. **3.** About $\frac{1}{6}$; about 30 pupils. **4.** About 6000 voters. **5.** 2800 pupils; 2816 pupils. **6.** 350; 364. **7.** 440; 429. **8.** 200; 201.6. **9.** 91; 86.45. **10.** 205; 188.6. **11.** 144; 138.24. **12.** 91; 87.36. **13.** 235; 253.8. **14.** 200; 202.5.

Page 87. — **1.** .135; \$62.10; \$51.30; \$67.50. **2.** .146. **3.** .361. **4.** .174. **5.** .993. **6.** .8325. **7.** .6233. **8.** .082. **9.** .035. **10.** .004. **11.** .0018. **12.** \$127.50. **13.** \$12.75. **14.** \$131.75. **15.** \$13.18. **16.** \$24.82. **17.** \$19.38. **18.** \$54.31. **19.** \$5.93. **20.** \$8.70. **21.** \$16.40. **22.** \$914.40. **23.** \$280.50. **24.** \$1,271,000. **25.** 91,060,000 T.

Page 88. — **1.** 15.7%. **2.** 43.2%. **3.** 62.5%. **4.** 75.5%. **5.** 12.5%. **6.** 4.8%. **7.** 6.7%. **8.** 9.5%. **9.** 0.3%. **10.** 96.75%. **11.** 49.33%. **12.** 66.67%. **13.** 8.75%. **14.** 9.09%. **15.** 6.25%. **16.** $2\frac{1}{2}$%. **18.** 64.3%. **19.** 7.5%. **20.** 57.3%. **21.** 21.3%. **22.** 24.9%.

Page 89. — **2.** Cincinnati, 56.6%; St. Louis, 54.2%; Chicago, 52.1%; Brooklyn, 50%; New York, 48.9%; Boston, 40%; Philadelphia, 39.8%. **3.** Boston, 40%; Philadelphia, 40%.

Page 90. — **1.** New York, 65.3%; Cleveland, 56%; Philadelphia, 52.04%; Detroit, 52%; Washington, 51.1%; Chicago, 50.5%; St. Louis, 41.8%; Boston, 30.9%. **2.** New York, 65.7%; Cleveland, 56.4%; Philadelphia, 51.52%; Detroit, 51.49%; Chicago, 51%; Washington, 50.5%; St. Louis, 41.4%; Boston, 31.6%. **3.** Pittsburgh, 57.4%; St. Louis, 56.5%; Cincinnati, 56.0%; New York, 51.79%; Chicago, 51.75%; Brooklyn, 47.0%; Boston, 41.2%; Philadelphia, 38.2%. **4.** Pittsburgh, 58.04%; St. Louis, 57.98%; Cincinnati, 57.1%; Chicago, 51.7%; New York, 50%; Brooklyn, 47.1%; Boston, 40.2%; Philadelphia, 37.7%.

Page 91. — **2.** Brown, 34.2%; Donohue, 33.8%; Cuyler, 33.25%; Roush, 33.24%; Williams, 33.16%; Walker, 32.62%; Gibson, 32.61%; Goslin, 35.07%; McNulty, 35.14%; Collins, 34.8%; Welch, 33.7%; Bennett, 33.3%; Simmons, 32.20%, Miller, 32.16%. **3.** 26.2%. **4.** Cincinnati, 29.1%; Pittsburgh, 28.9%; Philadelphia, 28.7%; Chicago, 28.5%; St. Louis, 28.37%; New York, 28.41%; Boston, 28.35%; Brooklyn, 27.0%. **5.** 77.8%. **6.** 73.3%.

Page 92. — **1.** 23.7%. **2.** 8.6%. **3.** 4.1%; 6.1%.

Page 93. — **1.** 51.4%. **2.** 13.4%; 86.5%. **3.** 32.4%; 67.6%. **4.** 78.4%; 21.6%. **5.** 17.3%; 82.7%. **6.** 97.5%. **7.** New Jersey, New York, Rhode Island; Mississippi. **8.** In 1920.

Page 94. — **1.** \$.36 a month. **2.** \$4. **3.** \$1. **4.** \$6.30. **5.** \$.09. **6.** \$4.20. **7.** \$4.50. **8.** \$.51. **9.** \$.70. **10.** \$.72. **11.** \$.89. **12.** \$.98. **13.** \$.33. **14.** 0.4%. **15.** 50%; same.

Page 95. — **1.** 4. **2.** 3.54. **3.** 5. **4.** 1.15. **5.** 2.09. **6.** 1.62. **7.** 7.8. **8.** 4.68. **9.** 2.01. **10.** 12. **11.** 284%. **12.** 305%. **13.** 175%. **14.** 462%. **15.** 1808%. **16.** 1005%. **17.** 412%. **18.** 193%. **19.** 300%. **20.** 1500%. **21.** 494 boys. **22.** 300% as great. **23.** \$1085. **24.** \$1650. **25.** \$1031.24. **26.** \$1347.36. **27.** \$633.60. **28.** \$501.32. **29.** \$1127. **30.** \$2260.80. **31.** \$10,680. **32.** \$6600. **33.** \$2745. **34.** \$7862.50. **35.** \$75. **36.** \$15,750.

Page 96. — **1.** 250%; $6000. **2.** 400%; $8. **3.** 75%. **5.** 200; $500. **6.** 60; $1200. **7.** $36. **8.** 180; $36. **10.** 20%. **11.** 25% increase. **12.** 6.3% decrease. **13.** 16.1% increase. **14.** 9% decrease. **15.** 115% increase. **16.** 12.5% increase. **17.** 15.0% decrease. **18.** 18.8% increase. **19.** 11.4% decrease.

Page 97. — **4.** $18; 60% increase. **5.** 7¢; 100% increase. **6.** 64¢; 86.5% decrease. **7.** 210 schools; 46.7% increase.

Page 98. — **1.** $98. **2.** $182. **3.** February, $112, $208; March, $140, $260. **4.** $119, $221. **5.** $145.25, $269.75. **6.** $131.25, $243.75. **7.** $102.55, $190.45. **8.** $162.75, $302.25. **9.** $66.50, $123.50. **10.** $85.75, $159.25. **11.** $161.70, $300.30. **12.** $178.50, $331.50. **13.** $4295; $1503.25. **14.** 9%. **15.** $6\frac{1}{2}$%. **16.** 20%. **17.** $33\frac{1}{3}$%. **18.** 45%. **19.** $22\frac{1}{2}$%. **20.** 25%. **21.** 18%. **22.** 30%. **23.** $15\frac{1}{4}$%. **24.** 35%.

Page 99. — **1.** $180; 40%. **2.** $600; 5%. **3.** $600; 25%. **4.** $650; 25.2%. **5.** $324; 27%. **6.** $925; 26.4%. **7.** $1605; 15.3%. **8.** $2270; 8.9%. **9.** $9200; 20.1%. **10.** $15,000; 23.1%.

Page 100. — **2.** $830; 20%. **3.** $12\frac{1}{2}$%. **4.** $410; $12\frac{1}{2}$%; 20%. **5.** $217; 8.7%; 14.6%. **6.** $825; 15%; 23.1%. **7.** $710; 10.4%; 17.1%. **8.** $7430; 16.5%; 26.9%. **9.** $4608; 8%; 13.6%. **10.** $19,825; 22.6%; 41.4%. **11.** $12,200; 16.7%; 31.3%.

Page 101. — **1.** $640; $12\frac{1}{2}$%; 25%; $62\frac{1}{2}$%. **2.** 24%. **3.** 27%. **4.** $33\frac{1}{3}$%. **5.** 30%. **6.** Ex. 4, 25%; ex. 5, 32%; ex. 6, 20%; ex. 7, 28.5%, or $28\frac{1}{2}$%; ex. 8, 22%; ex. 9, $33\frac{1}{3}$%; ex. 10, 23%; ex. 11, 30%. **7.** 25%; 35%; 40%; 25¢, 35¢, 40¢.

Page 102. — **1.** $6. **2.** $20. **3.** $100. **4.** $40. **5.** $50. **6.** $810. **7.** $800. **8.** $450. **9.** $700. **10.** $375. **11.** $62.50.

Page 103. — **1.** Profit, 15%. **2.** Expenses, 28%. **3.** Cost, 40%. **4.** $7.06; $14.12; cost. **6.** 32%; 38%; former. **7.** $40; 10%; 15%; 75%. **8.** $161; 23%; 7%; 70%. **9.** $300; $33\frac{1}{3}$%; 10%; $56\frac{2}{3}$%. **10.** $66; 15%; 45%; 40%. **11.** $72; 16%; 36%; 48%. **12.** $250; $16\frac{2}{3}$%; $33\frac{1}{3}$%; 50%.

Page 104. — **2.** $38.76. **3.** $168; $672. **4.** $553.20; $368.80. **5.** $47.10; $266.90. **6.** $90; $270. **7.** $110.40; $165.60. **8.** $99.45; $665.55. **9.** $42.75; $242.25. **10.** $90; $450. **11.** $62; $434. **12.** $208; $416. **13.** $11.82; $66.97. **14.** $5.70; $39.90. **15.** $121.35; $225.35. **16.** $108.24; $324.71. **17.** $42.80; $214.01. **18.** Jones; $.20. **19.** Former; $1.20.

Page 105. — **1.** $899.64. **2.** $342. **3.** $274.95. **4.** $672.28. **5.** $523.80. **6.** $16,074. **7.** $58,101.75. **8.** $46,800. **9.** $66,500. **10.** Single discount of 25%; chain discount.

Page 106. — **1.** $1012.50. **3.** $182.40. **4.** $425.32. **5.** $398.29. **6.** $432.25. **7.** $445.50. **8.** $6811.98. **9.** $3675.60. **10.** $1243.54. **11.** $6933.62. **12.** $1689.11. **13.** $581.40. **14.** Single discount is 2.675% better.

Page 107. — **1.** $60. **2.** 64. **3.** 50. **4.** 120. **5.** 80. **6.** 150. **7.** 7200. **8.** 125. **9.** 237½. **10.** 80. **11.** 70. **12.** 16. **13.** 60.

Page 108. — **1.** $35. **2.** $60. **3.** 50¢. **4.** 9500. **5.** $500. **6.** $2.50. **7.** $700. **8.** $34,000. **9.** $13.50. **10.** $24.80. **11.** $32. **12.** $1020.41. **13.** $1480. **14.** $30,100. **15.** $1250. **16.** $1950. **17.** $12. **18.** $50. **19.** $13,636.36. **20.** $150.50.

Page 109. — **1.** $10.82. **2.** $45. **3.** $75,000. **4.** $300. **5.** $160. **6.** $780. **7.** $2880. **8.** $500. **9.** $500. **10.** $13.90. **11.** $70. **12.** $154.50. **13.** $1483. **14.** $3915.60.

Page 110. — **1.** $20. **2.** $400.

Page 111. — *See page* xxi.

Page 112. — **1.** 45,000. **2.** 5300 pupils. **3.** $500. **4.** 400 pupils. **5.** $2. **6.** $2.50. **7.** $400.

Page 115. — **2.** Total, $87.78.

Page 119. — **2.** $204.50. **3.** $322.70.

Page 122. — **2.** $.27; $274.77. **7.** $684.31. **10.** Sight draft; $812.69.

Page 123. — **1.** $2.05. **2.** $18.12. **3.** $25.15. **4.** $62.20. **5.** $80.20. **6.** $80.42. **7.** $75.20. **8.** $11.62. **9.** $1.25. **10.** $2.87. **11.** $4.82. **12.** $5.22. **13.** $20.17. **14.** $33.95. **15.** $99.72. **16.** $100.22. **17.** $150.40. **18.** $100 and $20.12, fee $.37; $100 and $35.50, fee $.37; $100, $100, and $40.35, fee $.62; $76.60, fee $.20. **19.** $1.53.

Page 124. — **1.** $2; $2.33; $1.67. **2.** $2.10; $2.45; $1.75. **3.** $12; $14; $10. **4.** $4.32; $5.04; $3.60. **5.** $12; $14; $10. **6.** $1.25; $1.46; $1.04. **7.** $1.90; $2.22; $1.58. **8.** $5.50; $6.42; $4.58. **9.** $5.92; $6.91; $4.93. **10.** $3.13; $3.65; $2.61. **11.** $5.50; $6.42; $4.58. **12.** $.50; $.58; $.42. **13.** $4.20; $4.90; $3.50. **14.** $4.32; $5.04; $3.60. **15.** $1.56; $1.82; $1.30. **16.** $9; $10.50; $7.50. **17.** $3.80; $4.43; $3.17. **18.** $37.20; $43.40; $31.

Page 125. — **1.** 180 da. **2.** 139 da. **3.** 143 da. **4.** 168 da. **5.** 146 da. **6.** 35 da. **7.** 123 da. **8.** 165 da. **9.** 73 da. **10.** 43 da. **11.** 51 da. **12.** 124 da. **13.** 60 da.; $.90. **14.** 72 da.; $12. **15.** $2.50. **16.** $7.35. **17.** $4.14. **18.** $.50. **19.** $7.15. **20.** $7.50.

Page 127. — **1.** \$.67; \$1; \$1.33; \$1.67. **2.** \$2; \$.1667, or \$.17; \$2.17. **3.** \$3.65. **4.** \$2; \$.20; \$.02; \$2.22; \$20. **5.** \$.60. **6.** \$.27. **7.** \$1.05. **8.** \$.25. **9.** \$.53. **10.** \$1.20. **11.** \$3.30. **12.** \$3.87. **13.** \$.47. **15.** \$2.92; \$2.08; \$3.75. **16.** \$1.13; \$3.10. **17.** \$4.40. **18.** \$5.58. **19.** \$2.66. **20.** \$2.59. **21.** \$5.53. **22.** \$14.85. **23.** \$26.92. **24.** \$38.04. **25.** \$4.88. **26.** \$6.75. **27.** \$1.23. **28.** \$6.63.

Page 128. — **1.** \$350. **2.** \$.90; \$89.10; \$90. **3.** \$3; \$597. **4.** \$.64; \$119.36. **5.** \$2.76; \$177.24. **6.** \$11.28; \$538.72. **7.** \$14; \$1386. **8.** \$8; \$1592. **9.** \$6.25; \$1243.75. **10.** \$37.50; \$2462.50.

Page 129. — **2.** 46 da.; \$11.50. **3.** \$23; \$15.50.

Page 130. — **1.** June 9; \$378.88. **2.** \$213.41; \$214.

Page 132. — **1.** \$485.28. **2.** \$520.87. **3.** \$609.10. **4.** \$862.75. **5.** \$1224. **6.** \$1648. **7.** \$2006.67. **8.** \$3771.60. **9.** 36 da.; \$3.40; \$481.88; \$485.28. **10.** \$1615.04; \$1648. **11.** Ex. 3, \$604.33; ex. 4, \$855.27; ex. 5, \$1211.96; ex. 7, \$2003.33; ex. 8, \$3618.22. **12.** Aug. 4. **13.** \$446.62; \$450. **14.** May 31; June 4; Nov. 14; July 30; Nov. 2.

Page 133. — *See page* xxi.

Page 134. — **1.** \$212.93. **5.** \$538.21. **7.** 2. **8.** 40. **9.** 8. **10.** 2. **11.** 45. **12.** 90. **13.** 125. **14.** 45. **15.** 120. **16.** 200. **17.** 150. **18.** 25. **19.** 120. **20.** 60. **21.** 70. **22.** 18. **23.** 180. **24.** 2000. **25.** 375. **26.** 400. **27.** 520.

Page 135. — **1.** \$8.32; \$8.48. **2.** 3%. *1st half year:* \$24, \$824; *2d half year:* \$24.72, \$848.72; *3d half year:* \$25.44, \$874.16; *4th half year:* \$26.22, \$900.38; *5th half year:* \$27, \$927.38; *6th half year:* \$27.81, \$955.19; *7th half year:* \$28.65, \$983.84; *8th half year:* \$29.49, \$1013.33.

Page 136. — **3.** \$65.80. **4.** \$185.03; \$601.66.

Page 137. — **1.** 6 mo.; 6 periods; 2%; \$56.31. **2.** \$450.20. **3.** \$358.22. **4.** \$965.42. **5.** \$823.40. **6.** \$574.78. **7.** \$784.17. **8.** \$369.58. **9.** \$1444.24. **10.** \$267.28. **11.** \$895.54. **12.** \$695.35. **13.** \$548.85. **14.** \$1098.21. **15.** \$1985.92. **16.** \$1271.86. **17.** 2 *yr.:* \$519.57, \$39.57; 4 *yr.:* \$562.40, \$82.40; 6 *yr.:* \$608.76, \$128.76; 8 *yr.:* \$658.94, \$178.94; 9 *yr.:* \$685.56, \$205.56; 10 *yr.:* \$713.26, \$233.26. **18.** 3 mo.; 20 periods; 1%; \$122.02. **19.** \$964.99.

Page 138. — **2.** \$592,096,000; \$192,096,000. **3.** \$199.99. **4.** \$1199.93; \$899.95. **5.** \$197.99. **6.** \$203.28. **7.** \$199.99, or about \$200; \$400. **8.** \$30.42.

Page 139. — **1.** \$6400; \$12,800; \$25,600; \$51,200; \$102,400. **3.** \$36.50; \$73; \$146. **4.** \$60. **5.** \$4; \$16; \$1024. **6.** \$20; \$80; \$5120. **7.** \$80; \$320; \$20,480. **8.** \$140; \$560; \$35,840. **9.** \$800; \$3200; \$204,800. **10.** \$1000; \$4000; \$256,000.

Page 140. — **1.** \$.60; \$1.20; \$1.80; \$3. **2.** \$.44. **3.** \$3.13. **4.** \$10.33; \$103.30. **5.** \$1957.52; \$1882.52. **6.** \$1957.52. **8.** \$61.94.

Page 142. — **1.** \$140.83; \$774.23; \$1458.21. **2.** \$3276. **3.** \$113.14. **4.** \$344.97. **5.** \$512.95. **6.** \$1157.79. **7.** \$20,950.43. **8.** \$45,113.12.

1. \$20.60. **2.** 4 periods; \$42.04; \$64.34; \$87.55; \$111.69; \$247.83; \$413.79; \$616.10; \$862.71. **3.** \$136.14; \$165.96; \$202.31; \$246.61; 20th to 25th year.

Page 143. — **1.** \$366.45; \$446.69; \$544.53. 3 times; $3\frac{1}{2}$ times; 4 times. **2.** \$172.25. **3.** \$150; \$22.25. **4.** \$164.25; \$26,078.60. **5.** \$18; \$471.30. **6.** \$183.18. **7.** \$976.10. **8.** 2896.56. **9.** \$5392.82. **10.** \$11,070.60. **11.** \$33,885.50

Page 144. — **1.** \$1418.52. **2.** \$1418.52. **3.** \$97.99. **4.** \$199.99. **5.** \$200.

Page 145. — *See page* xxi.

Page 146. — **1.** \$.53; \$3.57; \$10.05; \$25.89; \$95.27; \$178.42. **2.** \$12; \$24; \$48; \$72; \$14.76. **3.** \$840.78. **4.** \$14.26.

Page 147. — **1.** \$54; 36%. **2.** \$72; 6%. **3.** 10%. **4.** 6%. **5.** 40%. **6.** 24%. **7.** 8%. **8.** 48%. **9.** 7%. **10.** 5%.

Page 148. — **1.** 5 mo. **2.** $4\frac{1}{2}$ mo.; $4\frac{1}{2}$ mo. **5.** \$5.25. **6.** \$13. **7.** \$46.

Page 150. — **1.** \$27; \$324; 60%. **2.** 33%. **3.** 38%. **4.** 40%. **5.** 578%. **6.** \$135; 27%. **7.** 23%; 40%; Star.

Page 151. — **1.** \$15. **2.** \$11. **3.** \$22. **4.** 57%. **5.** \$21.87. **6.** About 17%.

Page 153. — **1.** *5th month:* \$5832, \$42, \$29.16, \$71.16; *6th month:* \$5790, \$42, \$28.95, \$70.95; *7th month:* \$5748, \$42, \$28.74, \$70.74; *8th month:* \$5706, \$42, \$28.53, \$70.53; *9th month:* \$5664, \$42, \$28.32, \$70.32; *10th month:* \$5622, \$42, \$28.11, \$70.11; *11th month:* \$5580, \$42, \$27.90, \$69.90; *12th month:* \$5538, \$42, \$27.69, \$69.69; *13th month:* \$5496, \$42, \$27.48, \$69.48; *14th month:* \$5454, \$42, \$27.27, \$69.27; *15th month:* \$5412, \$42, \$27.06, \$69.06; *16th month:* \$5370, \$42, \$26.85, \$68.85.

Page 153 (*continued*). — **2.** \$2016; \$3984; \$19.92. **3.** \$204. 139*th month:* \$204, \$42, \$1.02, \$43.02; 140*th month:* \$162, \$42, \$.81, \$42.81; 141*st month:* \$120, \$42, \$.60, \$42.60; 142*d month:* \$78, \$42, \$.39, \$42.39; 143*d month:* \$36, \$36, \$.18, \$36.18. 143 mo.

Page 155. — **4.** 5*th month:* \$5758.20, \$90, \$28.79, \$61.21; 6*th month:* \$5696.99, \$90, \$28.48, \$61.52; 7*th month:* \$5635.47, \$90, \$28.18, \$61.82; 8*th month:* \$5573.65, \$90, \$27.87, \$62.13; 9*th month:* \$5511.52, \$90, \$27.56, \$62.44; 10*th month:* \$5449.08, \$90, \$27.25, \$62.75; 11*th month:* \$5386.33, \$90, \$26.93, \$63.07; 12*th month:* \$5323.26, \$90, \$26.62, \$63.38; 13*th month:* \$5259.88, \$90, \$26.30, \$63.70; 14*th month:* \$5196.18, \$90, \$25.98, \$64.02; 15*th month:* \$5132.16, \$90, \$25.66, \$64.34; 16*th month:* \$5067.82, \$90, \$25.34, \$64.66; 17*th month:* \$5003.16, \$90, \$25.02, \$64.98; 18*th month:* \$4938.18, \$90, \$24.69, \$65.31; 19*th month:* \$4872.87, \$90, \$24.36, \$65.64; 20*th month:* \$4807.23, \$90, \$24.04, \$65.96; 21*st month:* \$4741.27, \$90, \$23.71, \$66.29; 22*d month:* \$4674.98, \$90, \$23.37, \$66.63; 23*d month:* \$4608.35, \$90, \$23.04, \$66.96; 24*th month:* \$4541.39, \$90, \$22.71, \$67.29. **5.** \$4474.10. **6.** 1*st month:* \$8000, \$75, \$40, \$35; 2*d month:* \$7965, \$75, \$39.83, \$35.17; 3*d month:* \$7929.83, \$75, \$39.65, \$35.35; 4*th month:* \$7894.48, \$75, \$39.47, \$35.53; 5*th month:* \$7858.95, \$75, \$39.29, \$35.71; 6*th month:* \$7823.24, \$75, \$39.12, \$35.88; 7*th month:* \$7787.36, \$75, \$38.94, \$36.06; 8*th month:* \$7751.30, \$75, \$38.76, \$36.24; 9*th month:* \$7715.06, \$75, \$38.58, \$36.42; 10*th month:* \$7678.64, \$75, \$38.39, \$36.61; 11*th month:* \$7642.03, \$75, \$38.21, \$36.79; 12*th month:* \$7605.24, \$75, \$38.03, \$36.97. **7.** 1*st month:* \$1500, \$25, \$7.50, \$17.50; 2*d month:* \$1482.50, \$25, \$7.41, \$17.59; 3*d month:* \$1464.91, \$25, \$7.32, \$17.68; 4*th month:* \$1447.23, \$25, \$7.24, \$17.76; 5*th month:* \$1429.47, \$25, \$7.15, \$17.85; 6*th month:* \$1411.62, \$25, \$7.06, \$17.94; 7*th month:* \$1393.68, \$25, \$6.97, \$18.03; 8*th month:* \$1375.65, \$25, \$6.88, \$18.12. 9*th month:* \$1357.53, \$25, \$6.79, \$18.21; 10*th month:* \$1339.32, \$25, \$6.70, \$18.30; 11*th month:* \$1321.02, \$25, \$6.61, \$18.39; 12*th month:* \$1302.63, \$25, \$6.51, \$18.49. **8.** 1*st month:* \$3000, \$60, \$15, \$45; 2*d month:* \$2955, \$60, \$14.78, \$45.22; 3*d month:* \$2909.78, \$60, \$14.55, \$45.45; 4*th month:* \$2864.33, \$60, \$14.32, \$45.68; 5*th month:* \$2818.65, \$60, \$14.09, \$45.91; 6*th month:* \$2772.74, \$60, \$13.86, \$46.14. **9.** 1*st month:* \$3000, \$60, \$15, \$75; 2*d month:* \$2940, \$60, \$14.70, \$74.70; 3*d month:* \$2880, \$60, \$14.40, \$74.40; 4*th month:* \$2820, \$60, \$14.10, \$74.10; 5*th month:* \$2760, \$60, \$13.80, \$73.80; 6*th month:* \$2700, \$60, \$13.50, \$73.50.

Page 156. — **1.** (6) 146 mo.; (7) 71 mo.; (8) 58 mo. **2.** 100 mo. **3.** 62 mo. **4.** 71 mo. **5.** 80 mo.

Page 157. — **1.** $10; $120; $600; $1200; $1390. **2.** $2000; $1390; $610. **3.** $1390; $2000.

Page 158. — **2.** $38.50; 33 shares. **3.** $71.50; $9938.50; $3338.50. **5.** 24 shares; $26.

Page 159. — **1.** $21. **2.** $63; $5040. **3.** $840; $840. **4.** $1680; $840. **6.** $142.50. **7.** $2400; $2400.

Page 160. — **3.** $335.06. **4.** $7.50. **5.** $403.06. **6.** $805.88; $810.42; $1193.99. **7.** About $300 (by rule, p. 139); $299.98 (by table, p. 136). **8.** 35 shares; $37.92.

Page 163. — **1.** 8000 shares. **2.** 7%. **3.** $7; $840. **4.** $525,000; $26,250. **5.** 4½%. **6.** 4%; $200. **7.** 7%; $350. **8.** 5½%; $275. **9.** 5%; $250. **10.** 6%; $300. **11.** 4%; $200. **12.** 6%; $300. **13.** 7½%; $375.

Page 167. — **3.** $6492.50. **4.** $2521.25. **5.** $1993.13. **6.** $23,025. **7.** $1939. **8.** $2930. **9.** $8070. **10.** $7905. **11.** $8242.50. **12.** $101,550.

Page 168. — **1.** $857.50. **2.** $525; $1382.50. **3.** $1377.50.

Page 169. — **1.** $210, loss. **2.** $2775, profit. **3.** $7000, loss. **4.** $2220, profit. **5.** $912.50, loss.

2. 5%. **3.** 6¼%. **4.** 7½%. **5.** 8¾%. **6.** 5%. **7.** 10%. **8.** 6¼%. **9.** 4%. **10.** 5%. **11.** 8%. **12.** 8%. **13.** 6%. **14.** 5.8%. **15.** 5%. **16.** 8.5%. **17.** 4.2%. **18.** 5.6%. **19.** 6.0%. **20.** 8.2%. **21.** 6.5%.

Page 171. — **1.** $17.50. **2.** $25. **3.** $35. **4.** $30. **5.** $32.50. **6.** $20. **8.** $200. **9.** $6,000,000; 1949. **10.** $5000.

Page 173. — **1.** $857; $4285; $12,855. **2.** $987; $50. **3.** $1032; $50. **4.** $1057; $70. **5.** $1027; $55. **6.** $5360; $212.50. **7.** $4585; $300. **8.** $19,140; $800. **9.** $1896.50; $80. **11.** $22.50; $1034.50. **12.** $8.67; $895.67.

Page 174. — **1.** $890; $50; 5.6%. **2.** 4.6%. **3.** 5.2%. **4.** 4.4%. **5.** 4.9%. **6.** 5.8%. **7.** 6.8%. **8.** 5.9%. **9.** 4.6%. **10.** Former; 0.2% higher.

Page 175. — **2.** 7.8%. **3.** 6¼%. **4.** 5.6%. **5.** 4.5%. **6.** 5.1%. **7.** 5.6%. **8.** 6.3%.

Page 176. — **13.** 4%; 5%; 6%. **14.** $120; $150; $180. **16.** $4925.63. **17.** $7956. **18.** $15,843.75. **19.** $25,950. **20.** Ex. 16, $4899.37; ex. 17, $7924; ex. 18, $15,768.75; ex. 19, $25,850. **21.** 5%; 6%. **23.** 4.4%; 5.6%. **24.** 7.8%. **25.** $947; $50. **26.** $15,592.50; $60.

Page 177. — *See page* xxi.

Page 178. — **3.** 25,142$\frac{8}{7}$ mi. **5.** 7th year, \$15; 8th year, \$30; 9th year, \$45. **6.** 86.7%; 81.8%; Riverdale. **8.** 450 pupils. **9.** \$4.80. **10.** \$5.85. **11.** \$2.70. **12.** \$55. **13.** \$11.13. **14.** \$112.50. **15.** 406. **16.** 8. **17.** 9. **18.** \$150. **19.** \$600. **20.** \$700. **21.** 560. **22.** 80. **23.** 20.

Page 179. — **1.** \$22. **2.** \$18. **3.** \$12.04. **4.** \$45. **5.** \$148.50. **6.** \$31.50. **7.** \$272. **8.** \$134.40. **9.** \$284.40.

Page 180. — **3.** \$5.25. **4.** \$40.60. **5.** \$9.60. **6.** \$18.04. **7.** \$36.25. **8.** \$3315. **9.** \$140. **10.** \$559.15.

Page 181. — **1.** \$21. **2.** \$42. **3.** \$537.20; \$644.64. **4.** \$114. **5.** \$3. **6.** \$91.35. **7.** \$56.25. **8.** \$378.40. **9.** \$396.68. **10.** \$116.25. **11.** \$177.48. **12.** \$3675. **13.** \$4797.76. **14.** \$4819.50. **15.** \$5175; \$3175, gain.

Page 183. — **1.** \$20.14. **2.** \$80.56. **3.** \$92.82. **4.** \$227.25. **5.** \$159.95. **6.** \$296.64. **7.** \$237.15. **8.** \$90.90. **9.** \$228.50. **10.** \$444.96. **11.** \$658.75. **12.** \$1299.48.

Page 185. — **4.** \$225.30. **5.** \$98.61. **6.** \$503.50. **7.** \$249.25.

Page 186. — **1.** \$9000; \$6000; \$4000. **3.** \$26,350. **1.** 54.2%. **2.** \$1461.54. **3.** 216$\frac{2}{3}$%. **4.** \$197,625; \$118,575.

Page 188. — **1.** \$730.05. **2.** \$43.50. **3.** \$4253.25. **4.** \$1022.40.

Page 189. — **1.** \$21.09; \$210.90. **2.** \$146.01; \$64.89; \$6.49. **3.** \$30.05. **4.** \$4.47. **5.** \$15.98. **6.** \$13.88; \$14.93.

Page 191. — \$62.60.

Page 193. — *See page* xxii.

Page 195. — **1.** \$375. **2.** \$680; \$305. **3.** \$425. **4.** \$850. **5.** \$7500; \$15,000. **6.** \$1950; \$1840. **7.** About 29.4%.

Page 196. — **2.** \$5175. **3.** \$80.90. **6.** Public liability, 3,680,000 cars; property damage, 2,990,000 cars; collision, 460,000 cars; fire and theft, from 8,050,000 to 11,500,000 cars.

Page 198. — **1.** 2.95%. **2.** 29.5 mills on \$1; \$2.95 on \$100; \$29.50 on \$1000. **3.** 3.86%; 38.6 mills on \$1; \$3.86 on \$100; \$38.60 on \$1000. **4.** \$2.20 on \$100; 2.2%; 22 mills on \$1; \$22 on \$1000 **5.** \$3.75, etc. **6.** \$3.85, etc. **7.** \$3.66, etc. **8.** \$4.32, etc. **9.** \$2.65, etc. **10.** \$2.24, etc. **11.** \$2.70, etc. **12.** \$3.95, etc. **13.** \$3.08, etc. **14.** \$4.22; \$3.38. **15.** 80%.

Page 199. — **4.** \$3.54. **5.** \$1.469, \$.538, \$.267, \$.100, \$.050, \$.038, \$.019; \$3.83.

Page 200. — **1.** \$513.30. **2.** \$112.24; \$19.28; \$8. **3.** \$13,575; \$534.86. **4.** \$804. **5.** \$1989.82. **6.** \$1675.22. **7.** \$3788 80. **8.** \$7020.75. **9.** \$17,549. **10.** \$4784. **11.** \$5370.75. **12.** Hatton: \$262.35, \$220.38; Lakewood: \$275.40, \$231.34.

Page 201. — **2.** \$398.25. **3.** \$433.65; \$330.99. **4.** \$1.77; \$1.77; \$1.77; \$6000 *column:* \$226.56, \$228.33, \$230.10; \$7000 *column:* \$253.11, \$254.88, \$256.65, \$258.42, \$260.19, \$261.96, \$263.73, \$265.50. **5.** \$54.87; \$56.64; \$58.41; \$60.18; \$61.95; \$63.72; \$65.49; \$67.26; \$69.03.

Page 202. — **1.** 317 da.; \$554.18. **2.** \$40.19. **3.** \$601.98. **4.** \$61.76. **5.** \$7.69. **6.** \$14.45. **7.** \$330.41. **8.** \$34.83. **9.** \$11.61.

Page 203. — **1.** \$4.80. **2.** \$44. **3.** \$15.40. **4.** \$58.95. **5.** \$58. **6.** \$41. **7.** \$78.60. **8.** \$137.75. **9.** \$6. **10.** \$65.77. **11.** \$34.85. **12.** \$545.12.

Page 204. — **2.** \$4.40; \$4.95; \$2; \$3. **3.** \$3.50; \$4.50. **4.** \$615. **5.** \$16,800. **6.** \$75.16.

Page 206. — **8.** 2.455 m. **9.** 3 ft. $2\frac{1}{4}$ in.

Page 207. — **1.** 464.2 yd. **2.** 7.8 mi. **3.** 428.8 ft. **4.** 22 yd. **5.** 147 mi. **6.** 78.9 mi. **7.** Former; 2261.1 ft.

Page 208. — **1.** 155.0 ft.; 151.4 ft. **2.** 110 yd.; 880 yd.; 220 yd.; 550 yd. **3.** (*a*) 6 ft. 4 in.; (*b*) 13 ft. 10 in.; (*c*) 218 ft. 1 in.; (*d*) 52 ft. 1 in.; (*e*) 168 ft. 7 in.; (*f*) 130 ft.; (*g*) 25 ft. 4 in.; (*h*) 49 ft. 11 in. **4.** 10 km., 6 mi.; 3 km., 1.8 mi.; 1.6 km., .96 mi.; .4 km., .24 mi.

Page 209. — **2.** 48 l. **3.** 60 cu. dm.; 60 l. **4.** 3840 cu. in.; 66.5 qt.

Page 210. — **2.** 2.5 g.; 12.5 g. **3.** 2 dimes; 200 nickels. **4.** 12,000 cu. dm.; 12,000 l.; 12,000 kg. **5.** 324 cu. ft.; 2430 gal.; 20,169 lb.

Page 211. — *See page* xxii.

Page 212. — **1.** \$8.21. **2.** \$3642. **3.** \$120. **4.** \$67.416, or \$67.42. **5.** \$3.35. **6.** $\frac{1}{2}$ kg. **7.** 102.96 lb., or about 103 lb.

Page 215. — **1.** No; AC and AB are both shorter; AB is shortest. **2.** AB is perpendicular; AC and AD are not perpendicular; AB is the shortest distance.

Page 219. — **5.** 60°. **6.** Angle $C = 70°$.

Page 222. — **4.** Angle $h = 110°$, angle $f = 70°$, angle $d = 70°$; **110°, 70°.**

Page 224. — **1.** Yes; yes. **2.** No; yes. **3.** Yes.

Page 227. — **3.** The first two figures are symmetric; the third figure is not symmetric. **4.** The second figure has 4 axes of symmetry; the third figure has 6 axes of symmetry.

Page 228. — **1.** Yes; the right side exactly matches the left side.

Page 229. — **1.** Fig. 1 and Fig. 2 are symmetric; Fig. 3 is not symmetric.

Page 230. — **2.** Yes; yes. **3.** Yes; yes; no; yes; yes; no.

Page 234. — **7.** $\frac{1}{12}$. **8.** $\frac{21}{2}$. **9.** $\frac{3}{64}$. **10.** $\frac{1}{10}$. **11.** $\frac{1}{6}$. **12.** $\frac{8}{1}$. **13.** $\frac{4}{1}$. **14.** $\frac{30}{1}$. **15.** $\frac{28}{1}$. **16.** $\frac{1}{16}$. **17.** $\frac{20}{1}$. **18.** $\frac{1}{8}$. **19.** $\frac{1}{12}$; $\frac{1}{36}$. **20.** $\frac{1}{20}$; $\frac{1}{10}$.

Page 235. — **1.** $\frac{3}{4}$; $\frac{6}{8}$; yes; $\frac{3}{4} = \frac{6}{8}$. **2.** $\frac{2}{3} = \frac{4}{6}$.

Page 236. — **1.** 10. **2.** 21. **3.** 3. **4.** 2. **5.** 2. **6.** $2\frac{1}{4}$. **7.** 6. **8.** 128.

Page 237. — **2.** 15 in. **3.** 9 in; 12 in.; 15 in.; $7\frac{1}{2}$ in. **4.** 30 in. **5.** 6 in.

Page 238. — **1.** 3. **2.** 3. **3.** 2. **4.** 36. **5.** 40. **6.** 18. **7.** 8. **8.** 42. **9.** $28\frac{1}{2}$. **10.** \$15. **11.** 18 games.

Page 239. — *See page* xxii.

Page 240. — **1.** 14. **2.** 6. **3.** 3. **4.** $2\frac{1}{2}$. **5.** 10. **6.** 39. **7.** 11. **8.** 20. **9.** 3. **10.** 1. **11.** 4. **12.** 1. **13.** 8. **14.** 3. **15.** 9. **16.** $3\frac{1}{2}$. **17.** 7. **18.** 8. **19.** 14. **20.** 18. **21.** 21. **22.** 25. **23.** 2. **24.** 14. **25.** 13,181,000 people.

Page 242. — **5.** $7\frac{1}{2}$ in. **6.** $BC = 8$ ft.; $AC = 9$ ft. **7.** $MO = 10\frac{1}{2}$ in.; $NO = 10\frac{1}{2}$ in.

Page 244. — **1.** 66 ft. **2.** 35 ft. **3.** 180 ft. **4.** 25 ft. **5.** 15 ft. **6.** $37\frac{1}{2}$ ft. **7.** $28\frac{3}{4}$ ft. **8.** 45 ft.

Page 245. — **2.** 6 ft. **3.** $2\frac{3}{4}$ ft. **4.** 10 ft.; 12 ft.

Page 246. — **1.** 630 ft. **2.** 75 ft. **3.** 180 ft. **4.** 660 ft. **5.** 160 ft. **6.** 80 ft. **7.** 600 ft. **8.** $1687\frac{1}{2}$ ft. **9.** 2440 ft. **10.** $687\frac{1}{2}$ ft. **11.** 750 ft. **12.** 400 ft.

Page 248. — **1.** About 6000 ft. **2.** About 4 mi.

Page 249. — **1.** $1\frac{1}{2}$ in. **2.** 200 ft.; 2400 in.; 1 in. **3.** 300 ft.; 3600 in.; $1\frac{1}{2}$ in. **5.** Ratio = $\frac{1}{2400}$.

Page 250. — **10.** $7\frac{1}{2}$ in. **11.** 528 ft.

Page 251. — **4.** 196. **5.** 400. **6.** 225. **7.** 324. **8.** 625. **9.** 256. **10.** 900. **11.** 8100. **12.** 10,000. **13.** 40,000. **14.** 1; 2; 3; 8; 4; 9; 7; 10; 12; 11; 13; 14; 30. **15.** 9. **16.** 20. **17.** 15.

Page 253. — **4.** 5.2. **5.** 7.2. **6.** 9.3. **7.** 28.1. **8.** 30.2. **9.** 48.3.

Page 254. — **1.** 11.4 ft. **2.** 27.4 ft.; 25.5 ft.; 24.5 ft. **3.** 6.8. **4.** 6.5. **5.** 9.5. **6.** 9.8. **7.** 7.1. **8.** 7.5. **9.** 7.7. **10.** 4.9. **11.** 13.2. **12.** 13.6. **13.** 13.8. **14.** 13.9. **15.** 21.2. **16.** 21.4. **17.** 21.7. **18.** 21.9. **19.** 38.1. **20.** 45.0. **21.** 56.8. **22.** 94.2. **23.** 4.6 in.

Page 255. — **2.** 6.93. **3.** 4.12. **4.** 6.63. **5.** 7.81. **6.** 8.60. **7.** 8.06. **8.** 14.14. **9.** 20.69. **10.** 11.31. **11.** 11.79. **12.** 11.83. **13.** 14.73. **14.** 18.49. **15.** 23.02. **16.** 24.35. **17.** 30.74. **18.** 32.94. **19.** 44.92. **20.** 56.83. **21.** 68.08. **22.** 82.55.

Page 257. — **1.** 75 ft. **2.** 96.2 ft. **3.** 99 ft. **4.** 43.4 ft. **5.** 25 ft. **6.** 10.82 ft. **7.** 21.63 ft. **8.** 53.01 ft. **9.** 13.45 mi. **10.** 69.46 ft. **11.** 38.08 ft.

Page 258. — **1.** 7.1 mi. **2.** 19.4 mi. **3.** 4.30 ft. **4.** 4.3 mi. **5.** 7.1 mi. **6.** 4.5 mi. **7.** 12.2 mi. **8.** 9.2 mi. **9.** 8.6 mi. **10.** 13.4 mi. **11.** 5.7 mi. **12.** 8.0 mi.

Page 259. — *See page* xxii.

Page 260. — **1.** 8.29 mi. **2.** 208.71 ft. **3.** 96.3 ft. **5.** 5.3 ft.

Page 262. — **11.** 144; 86; $111\frac{1}{2}$; 102.

Page 263. — **1.** + 13; − 11; + 10; + 3; + 7; − 7; + 10. **2.** − 10; − 5; + 9; − 14; + 8; − 8; + 9. **4.** $-7\frac{1}{4}$. **5.** + \$53.

Page 264. — **1.** 0. **3.** + \$400. **4.** + 5; − 8; − 3; 0; + 2; + 1; − 2. **5.** 0; − 6; + 1; 0; + 5; − 6; 0. **6.** 0; + 1; + 4; − 4; 0; + 3; + 5.

Page 265. — **3.** (*a*) + 9; (*b*) + 5; (*c*) − 7; (*d*) + 7; (*e*) − 14; (*f*) 0. **4.** 10 B.C.; 31 A.D. **5.** + 15; − 1; + 6; − 8; + 4; + 2; + 14. **6.** 0; − 4; − 13; 0; − 11; + 3; − 4.

Page 266. — **1.** \$30 gain. **2.** \$263.83. **3.** + 3. **4.** − 4. **5.** − 16. **6.** + 32. **7.** + 24. **8.** − 10. **9.** − 315. **10.** + 300. **11.** + \$13.67. **12.** + \$3.08.

Page 268. — **1.** (*a*) + 6°; (*b*) + 12°; (*c*) − 6°; (*d*) − 12°. **2.** 34 yr. **3.** 38 yr. **4.** 40°; 8°. **5.** 12 mi. **6.** 45 points. **7.** + 3; + 5; − 4; − 15; − 4; − 13; − 7. **8.** + 16; − 1; + 9; + 6; − 3; + 1; 0. **9.** − 6; + 4; + 6; + 13; + 2; − 3; 0.

Page 269. — **4.** + 2. **5.** − 14. **6.** + 105. **7.** 0. **8.** − \$.89. **9.** − 12. **10.** − 3. **11.** − 27. **12.** + 105. **13.** + \$.15. **14.** + 5. **15.** 0. **16.** − 64. **17.** + 56. **18.** + \$7.84. **19.** + 11. **20.** − 14. **21.** − 18. **22.** − 102. **23.** − \$.13. **24.** − 12. **25.** − 2. **26.** − 19. **27.** 0. **28.** + \$10.04. **29.** − 3. **30.** + 14. **31.** − 114. **32.** − 105. **33.** − \$.53. **34.** − 8. **35.** 0. **36.** + 111. **37.** + 33. **38.** + \$15.37.

Page 270. — **1.** $-12\frac{1}{4}$. **2.** 99°. **3.** \$17.02. **4.** $7\frac{3}{4}$ points. **5.** 100°. **7.** + 2; − 6; − 3; + 5; 0; − 2; 0. **8.** + 1; − 3; − 6; + 5; − 3; + 4; 0. **9.** + 8; − 9; + 10; + 8; − 18; − 10; + 3. **10.** + 16; − 12; + 6; + 16; − 2; − 12; − 4.

Page 272. — **3.** 210 ft. **4.** 14 yr. **5.** 88 ft. **6.** 48,200. **7.** 18 mi.; $22\frac{1}{2}$ mi.; 24 mi. **8.** $D = MG$. **9.** $C = .24\,n$. **10.** 1. **11.** $10\frac{1}{2}$. **12.** 112. **13.** 36. **14.** 24. **15.** $16\frac{1}{2}$. **16.** \$1450. **17.** \$5.50. **18.** \$4.

Page 273. — **1.** \$72. **3.** 112,165 lb.; 56.08 T. **4.** 216.3 ft. **5.** 528 sq. in., 1584 cu. in. **6.** 528 sq. in.; 1848 cu. in. **7.** $49\frac{1}{2}$ sq. ft.; $37\frac{1}{8}$ cu. ft. **8.** 49.28 sq. ft.; 34.496 cu. ft. **9.** 15 cu. in. **10.** $30\frac{1}{5}$ cu. in. **11.** 62 cu. ft. **12.** 19.25 cu. ft. **13.** $234\frac{3}{8}$ cu. ft. **14.** 1.49 cu. ft. **15.** $320\frac{5}{8}$ cu. in. **16.** 1604.94 cu. yd. **17.** 5544 sq. ft.; 38,808 cu. ft. **18.** 154 sq. in.; $179\frac{2}{3}$ cu. in. **19.** 1386 sq. yd.; 4851 cu. yd. **20.** 745.36 sq. ft.; 1913.09 cu. ft. **21.** 154 sq. ft.; $179\frac{2}{3}$ cu. ft.

Page 275. — **1.** Protein, .32 oz.; carbohydrate, .48 oz.; 92 calories. **2.** Protein, .16 oz.; carbohydrate, .8 oz.; 110 calories. **3.** Protein, .16 oz.; carbohydrate, 1.44 oz.; 184 calories. **4.** Protein, .16 oz.; carbohydrate, 3.52 oz.; fat, .096 oz.; 447 calories. **5.** Protein, 1.44 oz.; carbohydrate, 2.4 oz.; fat, 1.92 oz.; 932 calories. **6.** Protein, 1.04 oz.; fat, .8 oz.; 324 calories. **7.** Protein, 1.12 oz.; fat, 1.44 oz.; 496 calories. **8.** Protein, .16 oz.; fat, 13.6 oz.; 3486 calories. **9.** Protein, 1.68 oz.; fat, .24 oz.; 254 calories. **10.** Protein, .64 oz.; carbohydrate, 5.76 oz.; 736 calories. **11.** Protein, 3.04 oz.; fat, 2.88 oz.; 1084 calories. **12.** Protein, .28 oz.; carbohydrate, 2.08 oz.; fat, .84 oz.; 485 calories. **13.** Protein, 1.02 oz.; carbohydrate, 1.7 oz.; fat, 1.36 oz.; 660 calories.

Page 276. — **1.** Breakfast, 560 calories; luncheon, 800 calories; dinner, 1034 calories; total, 2394 calories. **2.** Protein calories, 64, 106, 156; fat calories, 199, 322, 388; carbohydrate calories, 297, 372, 490. **3.** 326 protein calories, 13.6%; 909 fat calories, 38%; 1159 carbohydrate calories, 48.4%.

Page 277. — **1.** Yes. **2.** 2894 calories; 490 protein calories; 1.9% more.

Page 278. — **2.** 1360 calories; 1496 calories; 1632 calories; 1700 calories. **3.** 2848 calories. **4.** 1836 calories. **5.** 2538 calories. **6.** 2808 calories. **7.** 638 calories. **9.** Spinach, 92 calories; bread, 1182 calories; butter, 3486 calories.

ANSWERS FOR IMPROVEMENT TESTS

Page 23. — **Test 1A.** — **1.** 6613. **2.** 5802. **3.** 7321. **4.** 7811. **5.** 6410.

Test 1B. — **1.** 329,281. **2.** 412,438. **3.** 364,816. **4.** 406,033.

Test 1C. — **1.** 187,308. **2.** 688,977. **3.** 77,411. **4.** 774,518. **5.** 584,958. **6.** 693,567. **7.** 286,999. **8.** 432,980. **9.** 472,330. **10.** 380,996. **11.** 259,379. **12.** 335,574.

Page 49. — **Test 2A.** — **1.** 2268.21. **2.** 3541.67. **3.** 212.853. **4.** 357.496.

Test 2B. — **1.** 1551.67. **2.** 60.7984. **3.** 4.46309. **4.** 109.057. **5.** 724.848. **6.** 53.0175. **7.** 467.348. **8.** 6685.32. **9.** 17,784.8. **10.** 2387.34. **11.** 62.6256. **12.** 859.195.

Test 2C. — **1.** 354. **2.** .918. **3.** 3.282. **4.** 494.1. **5.** 845.74. **6.** .00063. **7.** .9035. **8.** 9. **9.** 46.58. **10.** 10.872. **11.** .062. **12.** 32,280.

Test 2D. — **1.** .67. **2.** 4.76. **3.** 91.95.

Page 65. — **Test 3A.** — **1.** 19. **2.** $23\frac{1}{6}$. **3.** $6\frac{7}{20}$. **4.** $16\frac{23}{24}$. **5.** $24\frac{1}{3}$.

Test 3B. — **1.** $2\frac{1}{2}$. **2.** $1\frac{1}{4}$. **3.** $3\frac{11}{16}$. **4.** $4\frac{5}{12}$. **5.** $2\frac{1}{6}$. **6.** $4\frac{1}{6}$. **7.** $2\frac{9}{10}$. **8.** $3\frac{3}{8}$. **9.** $3\frac{3}{10}$. **10.** $3\frac{1}{15}$. **11.** $1\frac{23}{24}$. **12.** $1\frac{3}{16}$.

Test 3C. — **1.** 366. **2.** $11.81. **3.** $31.63. **4.** $21\frac{7}{8}$. **5.** $\frac{3}{4}$. **6.** $2\frac{1}{8}$. **7.** $53\frac{1}{3}$. **8.** $3\frac{5}{6}$. **9.** $16\frac{1}{3}$.

Test 3D. — **1.** $\frac{7}{16}$. **2.** 48. **3.** $\frac{2}{3}$. **4.** $1\frac{7}{20}$. **5.** $2\frac{1}{8}$. **6.** $\frac{1}{5}$. **7.** $\frac{5}{8}$. **8.** $3\frac{1}{3}$. **9.** $16\frac{1}{2}$. **10.** $1\frac{9}{16}$. **11.** $\frac{9}{20}$. **12.** $4\frac{1}{5}$.

Page 81. — **Test 4A.** — **1.** 31,027. **2.** 60,987. **3.** 34,983. **4.** 40,344. **5.** 36,986.

Test 4B. — **1.** 775,824. **2.** 343,461. **3.** 577,189. **4.** 489,153. **5.** 144,957. **6.** 586,865. **7.** 169,478. **8.** 221,238. **9.** 133,329. **10.** 356,589. **11.** 239,696. **12.** 298,697.

Test 4C. — **1.** 373,596. **2.** 102,384. **3.** 393,737. **4.** 859,876. **5.** 194,775.

Test 4D. — **1.** 625. **2.** 709.1. **3.** 419.1. **4.** 589.

Page 111. — **Test 5A.** — **1.** $19\frac{2}{3}$. **2.** $12\frac{15}{16}$. **3.** $17\frac{1}{6}$. **4.** $18\frac{1}{4}$. **5.** $23\frac{5}{8}$.

Test 5B. — **1.** $2\frac{2}{3}$. **2.** $8\frac{3}{16}$. **3.** $\frac{1}{10}$. **4.** $3\frac{1}{12}$. **5.** $1\frac{1}{10}$. **6.** $1\frac{2}{3}$. **7.** $1\frac{1}{4}$. **8.** $2\frac{7}{12}$. **9.** $6\frac{17}{24}$. **10.** $1\frac{7}{8}$. **11.** $\frac{1}{10}$. **12.** $4\frac{1}{15}$.

Test 5C. — **1.** $19{,}531\frac{1}{4}$. **2.** 3650. **3.** 8967. **4.** $9\frac{1}{3}$. **5.** $\frac{1}{25}$. **6.** $\frac{3}{4}$. **7.** $26\frac{2}{3}$. **8.** 1. **9.** $29\frac{9}{16}$.

Test 5D. — **1.** $6\frac{1}{2}$. **2.** $1\frac{9}{16}$. **3.** 32. **4.** $18\frac{2}{3}$. **5.** $\frac{3}{8}$. **6.** $2\frac{1}{6}$. **7.** $11\frac{2}{3}$. **8.** $\frac{5}{32}$. **9.** $5\frac{1}{2}$. **10.** $\frac{4}{9}$. **11.** $\frac{2}{15}$. **12.** $1\frac{11}{12}$.

Page 133. — **Test 6A.** — **1.** 8208. **2.** 6262. **3.** 7330. **4.** 6540. **5.** 5496.

Test 6B. — **1.** 5,477,359. **2.** 7,725,581. **3.** 4,156,287. **4.** 6,984,542. **5.** 3,365,860. **6.** 5,459,894. **7.** 4,129,479. **8.** 1,462,297. **9.** 3,537,819.

Test 6C. — **1.** 271,890. **2.** 115,231. **3.** 688,350. **4.** 455,323. **5.** 419,952.

Test 6D. — **1.** 519. **2.** 679. **3.** 357. **4.** 865. **5.** 784.

Page 145. — **Test 7A.** — **1.** .99. **2.** .03. **3.** 2. **4.** .001. **5.** .005. **6.** .1. **7.** .09. **8.** 1.06. **9.** .055. **10.** .0025. **11.** $\frac{3}{5}$. **12.** $\frac{1}{100}$. **13.** $\frac{1}{8}$. **14.** $\frac{4}{25}$. **15.** $\frac{19}{20}$. **16.** $83\frac{1}{3}\%$. **17.** 900%. **18.** 8%. **19.** 21.5%, or $21\frac{1}{2}\%$. **20.** 9.2%, or $9\frac{1}{5}\%$. **21.** 90%. **22.** 50%. **23.** $6\frac{1}{4}\%$. **24.** 0.6%, or $\frac{3}{5}\%$. **25.** 225%.

Test 7B. — **1.** \$1.73. **2.** \$1.18. **3.** \$202.50. **4.** \$330.80. **5.** \$216.88. **6.** \$1101.90. **7.** \$116.74. **8.** \$1.26.

Test 7C. — **1.** 6. **2.** $\frac{1}{2}$. **3.** 28. **4.** 120. **5.** $66\frac{2}{3}$. **6.** 205.

Test 7D. — **1.** 15%. **2.** 100%. **3.** 285%.

Page 177. — **Test 8A.** — **1.** 72,977. **2.** 41,801. **3.** 68,406. **4.** 41,151. **5.** 53,639.

Test 8B. — **1.** 385,758. **2.** 175,669. **3.** 598,561. **4.** 197,588. **5.** 418,994. **6.** 245,329. **7.** 244,567. **8.** 795,480. **9.** 239,458. **10.** 113,476. **11.** 372,679. **12.** 386,239.

Test 8C. — **1.** 4,141,187,986. **2.** 115,143,589. **3.** 71,110,085.

Test 8D. — **1.** 327.21. **2.** 902.17. **3.** 311.42.

Page 193. — **Test 9A.** — **1.** $17\frac{9}{16}$. **2.** $25\frac{7}{12}$. **3.** $20\frac{11}{12}$. **4.** $8\frac{2}{5}$. **5.** $27\frac{2}{3}$.

Test 9B. — **1.** $1\frac{3}{10}$. **2.** $2\frac{5}{6}$. **3.** $7\frac{1}{8}$. **4.** $3\frac{1}{3}$. **5.** $1\frac{1}{16}$. **6.** $\frac{5}{8}$. **7.** $6\frac{7}{15}$. **8.** $4\frac{9}{16}$. **9.** $4\frac{1}{20}$. **10.** $\frac{7}{12}$. **11.** $6\frac{1}{12}$. **12.** $3\frac{1}{24}$.

Test 9C. — **1.** $28.31. **2.** $32.43. **3.** $46.87. **4.** $\frac{11}{16}$. **5.** 15. **6.** $166\frac{1}{2}$. **7.** $\frac{3}{8}$. **8.** $110\frac{1}{4}$. **9.** $28\frac{3}{4}$.

Test 9D. — **1.** $\frac{7}{16}$. **2.** $\frac{3}{4}$. **3.** $34\frac{4}{5}$. **4.** 1. **5.** $2\frac{13}{16}$. **6.** $\frac{14}{17}$. **7.** $2\frac{1}{2}$. **8.** $\frac{7}{12}$. **9.** $6\frac{1}{2}$. **10.** $2\frac{2}{7}$. **11.** $\frac{5}{24}$. **12.** 7.

Page 211. — **Test 10A.** — **1.** 430,537. **2.** 282,778. **3.** 471,450. **4.** 358,152.

Test 10B. — **1.** 5167. **2.** 6782. **3.** 7588. **4.** 6453.

Test 10C. — **1.** 67,513,715. **2.** 66,549,780. **3.** 29,387,268. **4.** 37,904,852. **5.** 14,297,443. **6.** 48,695,676. **7.** 35,393,549. **8.** 22,981,646. **9.** 41,891,558.

Page 239. — **Test 11A.** — **1.** 317.782. **2.** 29.4516. **3.** 35,932.6. **4.** 3947.55.

Test 11B. — **1.** 28.2646. **2.** 65,444.8. **3.** 1738.75. **4.** 3.87654. **5.** 355.722. **6.** 66,695.3. **7.** 77.8819. **8.** 2.72931. **9.** 4962.86. **10.** 419.834. **11.** 58,174.9. **12.** 769.318.

Test 11C. — **1.** .625. **2.** .15. **3.** .725. **4.** .822. **5.** .929. **6.** .68. **7.** .938. **8.** .488.

Test 11D. — **1.** 158.928. **2.** 8922.96. **3.** .008464. **4.** .215589. **5.** 48.97.

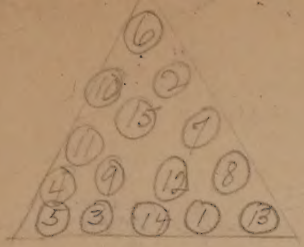
Page 259. — **Test 12A.** — **1.** 23.8%. **2.** 117.8%.

Test 12B. — **1.** 36.5%. **2.** 131.7%.

Test 12C. — **1.** 450. **2.** 4950. **3.** 640. **4.** 411. **5.** 1567.

Test 12D. — **1.** $5.33. **2.** $404.17. **3.** $499.80. **4.** $8.07. **5.** $658. **6.** $1617.96.

Test 12E. — **1.** $7.92. **2.** $18.13. **3.** $3.13.


6
10
2
15
7
11
4
9
12
8
5
3
14
1
13